Technical Appendix: Spectral Validation of the Prime Synchronization Theorem

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1 Overview

This technical appendix provides the computational framework for the *Prime Synchronization Theorem*. It focuses on the extraction of spectral properties from the Goldbach-structured graph for $N = 30$ nodes, which is the proposed scale for the physical experimental verification.

2 Mathematical Foundation

The stability of the synchronized state in a network of N coupled oscillators is governed by the Laplacian matrix L of the underlying topology. According to Theorem 1, the critical coupling strength is given by:

$$\kappa_c(N) = \frac{\lambda_{max}(\Lambda)}{\lambda_2(\tilde{L})} \tag{1}$$

Where λ_2 (the algebraic connectivity) and λ_{max} (the spectral radius) are derived from the Goldbach Bridge graph.

3 Source Code for Verification

The following Python script (using `NetworkX`) generates the graph and computes the exact spectral values required for the experiment.

```python
import networkx as nx
import numpy as np
```

```python
from sympy import primerange

def generate_goldbach_bridge(n_limit):
    primes = list(primerange(2, n_limit + 1))
    G = nx.Graph()
    even_numbers = list(range(4, n_limit + 1, 2))
    G.add_nodes_from(even_numbers)
    for n in even_numbers:
        for p in primes:
            if p < n:
                q = n - p
                if q in primes:
                    G.add_edge(n, p)
                    G.add_edge(n, q)
    return G

N = 30
bridge_graph = generate_goldbach_bridge(N)
laplacian_matrix = nx.laplacian_matrix(bridge_graph).todense()
eigenvalues = sorted(np.linalg.eigvals(laplacian_matrix).real)
lambda_2 = eigenvalues[1]
lambda_max = eigenvalues[-1]

print(f"Lambda_2: {lambda_2:.4f}")
print(f"Lambda_max: {lambda_max:.4f}")
```

Listing 1: Python script for Goldbach Bridge spectral analysis

4 Experimental Target

For the experimental setup with $N = 30$ oscillators, the synchronization threshold κ_c predicted by the arithmetic distribution serves as the primary benchmark. A physical result matching this threshold within 5% error will constitute a successful bridge between number theory and non-linear dynamics.