Technical Clarification on the Goldbach Interaction Operator

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Purpose of This Document

This note clarifies a common source of misunderstanding regarding the object referred to in the main paper as the "Goldbach matrix." It is intended to prevent incorrect interpretations by reviewers and readers familiar with standard graph-theoretic adjacency matrices.

What This Operator Is *Not*

The object used throughout this work is **not**:

- the adjacency matrix of a full Goldbach graph,
- a standard undirected or weighted network Laplacian,
- a graph where each prime is connected to all of its Goldbach partners across multiple even integers.

Consequently, standard intuitions regarding spectral growth with system size do *not* apply.

Correct Definition: Goldbach Interaction Operator

For a fixed even integer N, let

$$\mathcal{P}_N = \{p \text{ prime} \mid p < N\}.$$

We define the *Goldbach Interaction Operator* W_N acting on $\mathbb{R}^{|\mathcal{P}_N|}$ by

$$(W_N)_{ij} = \begin{cases} 1, & \text{if } p_i + p_j = N, \\ 0, & \text{otherwise.} \end{cases}$$

This operator encodes a *single arithmetic constraint* and does not represent cumulative connectivity across multiple decompositions.

Structural Consequence

Because each prime p_i participates in at most one valid Goldbach pair for fixed N, the operator W_N decomposes into disjoint 2×2 permutation blocks (and isolated zeros).

As a result:

- W_N is a partial permutation operator,
- W_N is norm-bounded independently of N,
- the spectral radius satisfies

$$\rho(W_N) = \lambda_{\max}(W_N) = 1 \quad \text{for all } N.$$

This is a **structural property of the definition**, not an empirical observation.

Implication for Synchronization Dynamics

In Kuramoto-type and related oscillator systems, linear stability theory predicts a critical coupling of the form

$$\kappa_c \propto \frac{1}{\rho(W_N)}.$$

Since $\rho(W_N) \equiv 1$, the observed scaling

$$\kappa_c(N) \propto N$$

arises from the linear scaling of intrinsic frequency dispersion, not from spectral growth of the coupling operator.

Conclusion

All numerical results reported in the main work are consistent with this operator definition. Apparent paradoxes (e.g. constant $\lambda_{\max}$ or exact linear scaling) disappear once the operator is interpreted correctly.

This document serves purely as a clarification and does not modify the original results.