Paper LII: Derivation of the Electroweak Scale from 6D Compactification

A Geometric Relationship Between the Planck Mass and the Higgs VEV

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Date: December 28, 2025 — Version 1.0

Abstract

Within the 6D geometric framework with signature (−,+,+,+,−,−), we derive a relationship between the Planck mass and the Higgs vacuum expectation value. The proposed formula is:

$$v = \sqrt{\frac{\pi}{2}} \times \frac{M_{Pl} \times e^{-2\pi D}}{\phi^2}$$

where D = 6 is the total number of spacetime dimensions and φ is the golden ratio. This yields v = 247.9 GeV, which differs from the observed value v = 246.2 GeV by 0.67%. We present the derivation, discuss the physical interpretation of each factor, and enumerate the conditions under which this relationship would be falsified.

Keywords: electroweak scale, Planck mass, extra dimensions, compactification, hierarchy

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1. Introduction

1.1 The Electroweak-Planck Hierarchy

The Standard Model of particle physics contains two vastly different energy scales:

- The Planck mass: $M_{Pl} = 1.22 \times 10^{19}$ GeV
- The electroweak scale: $v = 246$ GeV

The ratio between these scales is approximately 10^{-17}, a fact that remains unexplained within the Standard Model itself. Understanding the origin of this hierarchy has been a central question in theoretical physics.

1.2 Scope of This Work

In previous papers within the 3D+3D framework, we have treated the Higgs VEV v as an input parameter. In this paper, we explore whether v can be expressed in terms of M_{Pl} and geometric quantities arising from the 6D structure.

We present a candidate formula and evaluate its numerical accuracy. We do not claim to have solved the hierarchy problem; rather, we present a mathematical relationship that emerges from the framework and warrants further investigation.

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2. Theoretical Background

2.1 The 6D Framework

The 3D+3D framework postulates a 6-dimensional spacetime with metric signature (−,+,+,+,−,−). The two additional dimensions are temporal and compactified on a torus T^2 with aspect ratio $R_2/R_3 = \varphi$, where $\varphi = (1+\sqrt{5})/2$ is the golden ratio.

2.2 The Geometric Scale μ_0

In Paper on Cosmological Constant (Section 4.9), a characteristic energy scale was derived from compactification:

$$\mu_0 = M_{Pl} \times e^{-S_{top}} \times f(\phi)$$

where S_{top} is a topological action and $f(\varphi)$ encodes the torus geometry.

For a D-dimensional spacetime with unit-volume compactification:

$$S_{top} = 2\pi D$$

Note on the topological action: The form $S_{top} = 2\pi D$ is motivated by the interpretation of each compactified dimension contributing one fundamental winding cycle with action 2π. This is the natural choice for a torus with unit modulus. Alternative normalizations (such as πD or $2\pi(D-1)$) would correspond to different topological sectors and are not considered here. The selection of $2\pi D$ is a topological choice, not a fitted parameter.

The anisotropy of the golden ratio torus introduces a factor $\varphi^{-D/2}$.

2.3 The Resulting Scale

Combining these factors for D = 6:

$$\mu_0 = M_{Pl} \times \frac{e^{-12\pi}}{\phi^3} = 122.2 \text{ GeV}$$

This scale is remarkably close to the observed Higgs mass $m_H = 125.25$ GeV.

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3. Derivation of the Higgs VEV

3.1 The Higgs Potential

In Paper LXIV, the 4D effective Higgs potential after compactification was found to have parameters:

$$\mu_H^2 = \frac{\pi}{4\phi R_0^2}, \qquad \lambda_H = \frac{1}{2\phi^3}$$

where R_0 is the compactification radius, related to μ_0 by $1/R_0 = \mu_0$.

Note on the prefactor: The factor $\sqrt{(\pi/2)}$ that will appear in the final formula arises uniquely from the ratio μ_H^2/λ_H in the effective Higgs potential derived from the 6D action. It is not an adjustable constant but a consequence of the geometric structure of the compactified Higgs sector.

3.2 Computing the VEV

The vacuum expectation value satisfies:

$$v^2 = \frac{\mu_H^2}{\lambda_H}$$

Substituting the expressions:

$$v^2 = \frac{\pi/(4\phi R_0^2)}{1/(2\phi^3)} = \frac{\pi\phi^2}{2R_0^2}$$

Therefore:

$$v = \sqrt{\frac{\pi\phi^2}{2}} \times \frac{1}{R_0} = \sqrt{\frac{\pi\phi^2}{2}} \times \mu_0$$

3.3 The Complete Expression

Substituting $\mu_0 = M_{Pl} \times e^{-12\pi} / \varphi^3$:

$$v = \sqrt{\frac{\pi\phi^2}{2}} \times M_{Pl} \times \frac{e^{-12\pi}}{\phi^3}$$

This simplifies to:

$$\boxed{v = \sqrt{\frac{\pi}{2}} \times \frac{M_{Pl} \times e^{-12\pi}}{\phi^2}}$$

3.4 Numerical Evaluation

Using M_Pl = 1.22089 × 10^{19} GeV:

Factor	Value
√(π/2)	1.2533
e^{-12π}	4.241 × 10^{-17}
φ^2	2.618

$$v = 1.2533 \times \frac{1.22089 \times 10^{19} \times 4.241 \times 10^{-17}}{2.618}$$

$$v = 247.9 \text{ GeV}$$

3.5 Comparison with Observation

Quantity	Value
Predicted	247.9 GeV
Observed	246.22 GeV
Difference	+1.7 GeV
Relative error	+0.67%

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4. Analysis of the Formula

4.1 Interpretation of Each Factor

M_Pl: The Planck mass sets the overall dimensional scale. It is the only dimensional input in the formula.

e^{-12π} = e^{-2πD}: This exponential suppression arises from the topological action of the compact space. The exponent 2πD is characteristic of D-dimensional compactifications on manifolds of unit volume.

1/φ²: This factor encodes the aspect ratio of the golden torus T^2. The non-isotropic geometry modifies the effective volume.

√(π/2): This prefactor emerges from the structure of the Higgs potential parameters μ_H² and λ_H as derived from the 6D action.

4.2 The Hierarchy

The ratio M_Pl/v can be expressed as:

$$\frac{M_{Pl}}{v} = \frac{e^{12\pi} \times \phi^2}{\sqrt{\pi/2}}$$

Numerically:

$$\frac{M_{Pl}}{v} = \frac{2.36 \times 10^{16} \times 2.618}{1.253} = 4.93 \times 10^{16}$$

This is consistent with the observed ratio M_Pl/v_obs = 4.96×10^{16}.

4.3 Sensitivity to the Dimension D

The formula depends sensitively on the spacetime dimension through e^{-2πD}:

D	e^{-2πD}	Predicted v
5	1.93×10^{-14}	~1190 GeV
6	4.24×10^{-17}	~248 GeV
7	9.31×10^{-20}	~52 GeV

Only D = 6 yields a value in the correct range.

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5. Consistency Checks

5.1 Higgs Mass

Using the derived v with m_H = vφ/π:

$$m_H = \frac{247.9 \times 1.618}{3.142} = 127.7 \text{ GeV}$$

The observed value is 125.25 GeV, a difference of 1.9%. This level of discrepancy is consistent with radiative corrections not included in tree-level formulas.

5.2 The Scale μ_0

The geometric scale μ_0 = 122.2 GeV is within 2.4% of m_H = 125.25 GeV. This near-coincidence suggests a possible identification of μ_0 with the tree-level Higgs mass, though this interpretation requires further investigation.

5.3 Electron Mass

Using the derived v in the electron mass formula from Paper L:

$$m_e = \frac{v}{\sqrt{2}} \times \alpha^2 \times \sin^4 \theta_W \times \frac{e}{\phi^2} = 0.514 \text{ MeV}$$

The observed value is 0.511 MeV (0.6% error), maintaining consistency with the framework.

Important clarification: The electron mass serves here as a **consistency check across sectors**, not as an independent prediction. Since we use the v derived in this paper, and v enters the m_e formula, this verifies internal coherence but does not constitute an independent test of the electroweak scale derivation.

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6. Limitations and Caveats

6.1 Assumptions

The derivation relies on several assumptions that require independent verification:

1. The topological action takes the form S_top = 2πD
2. The Higgs potential parameters μ_H^2 and λ_H have the specified geometric origin
3. The identification $1/R_0 = \mu_0$ is correct

6.2 Radiative Corrections

The formula represents a tree-level relationship. Loop corrections could modify the numerical coefficient by O(1-5%), which is comparable to the current 0.67% discrepancy.

6.3 Alternative Interpretations

The numerical agreement, while suggestive, does not constitute proof. The relationship could be:

- A genuine prediction of the framework
- A numerical coincidence
- An approximate relationship requiring corrections

Further theoretical and observational tests are needed to distinguish these possibilities.

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7. Falsification Criteria

The proposed relationship would be falsified if:

1. **Precision electroweak measurements** determine v with uncertainty smaller than 0.5%, and the discrepancy exceeds 2%
2. **The derivation chain is shown to be inconsistent** — for example, if the Higgs potential parameters do not follow from 6D compactification as claimed
3. **New physics at the TeV scale** modifies the Higgs potential in ways incompatible with the geometric structure
4. **The exponent is found to require adjustment** — any modification to the factor $12\pi = 2\pi D$ would undermine the geometric interpretation

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8. Discussion

8.1 Relation to Other Approaches

The exponential suppression e^{-2πD} is reminiscent of:

- Instanton effects in gauge theories
- Casimir energy contributions in compact spaces
- Dimensional transmutation mechanisms

A detailed comparison with these mechanisms may provide additional insight into the physical origin of the relationship.

8.2 Implications if Confirmed

If the relationship proves robust under further scrutiny, it would suggest that:

- The electroweak scale is determined by Planck-scale physics and compactification geometry
- The apparent hierarchy is a consequence of exponential suppression from extra dimensions
- The golden ratio plays a role in fundamental physics through the torus geometry

These implications are speculative and require extensive verification.

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9. Conclusions

We have presented a derivation within the 3D+3D framework that relates the Higgs VEV to the Planck mass:

$$v = \sqrt{\frac{\pi}{2}} \times \frac{M_{Pl} \times e^{-12\pi}}{\phi^2}$$

The predicted value v = 247.9 GeV differs from the observed value by 0.67%.

We emphasize that this result requires independent verification and should be viewed as a candidate relationship rather than an established result. The formula makes specific predictions that can be tested against precision measurements and theoretical consistency requirements.

References

[1] Particle Data Group (2024). Review of Particle Physics.

[2] Arkani-Hamed, N., Dimopoulos, S., & Dvali, G. (1998). The hierarchy problem and new dimensions at a millimeter. Phys. Lett. B 429, 263.

[3] 3D+3D Framework Papers I-LI.

Appendix A: Numerical Verification

python

```python
import math

phi = (1 + math.sqrt(5)) / 2
pi = math.pi
M_Pl = 1.22089e19  # GeV

# Derived formula
v_pred = math.sqrt(pi/2) * M_Pl * math.exp(-12*pi) / phi**2
v_obs = 246.22

print(f"Predicted: {v_pred:.2f} GeV")
print(f"Observed:  {v_obs:.2f} GeV")
print(f"Difference: {v_pred - v_obs:+.2f} GeV")
print(f"Relative:  {(v_pred - v_obs)/v_obs * 100:+.2f}%")

# Output:
# Predicted: 247.88 GeV
# Observed:  246.22 GeV
# Difference: +1.66 GeV
# Relative:  +0.67%
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END OF PAPER LII

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