Engineering Lattice Metamaterials – Simulation and Experimental Validation

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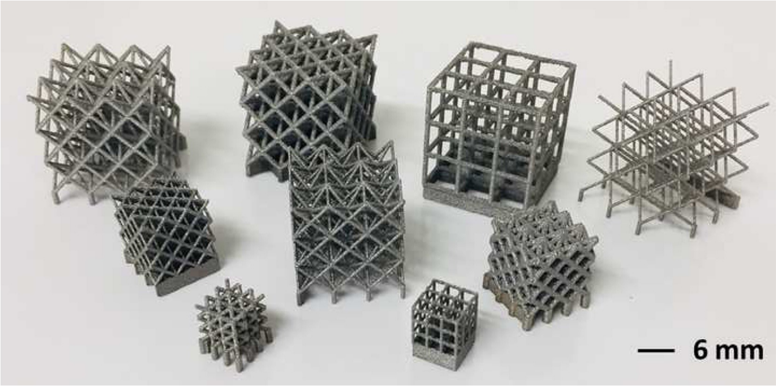
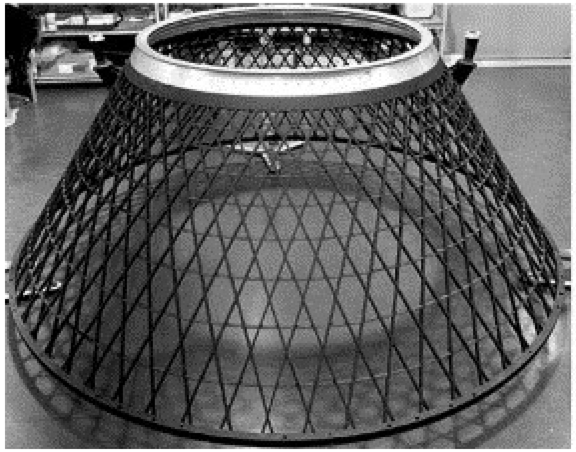
Motiviation – Lattice Metamaterials

- Slender lattices
- High strength to density ratio
- Allow for tailoring towards desired properties
  - Auxetic metamaterials
  - Bi-stable metameterials
- Huge potential for optimization



B. Hanks, et al, Additive Manufacturing, Vol. 35, 101301, 2020

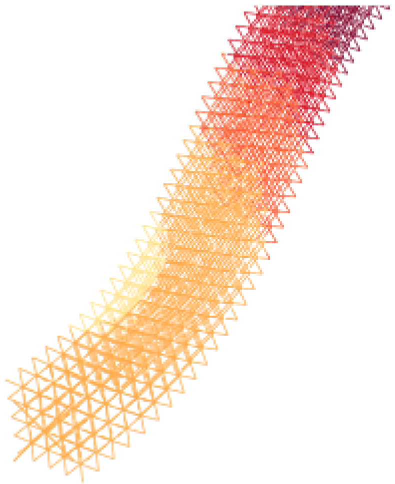


V.V. Vasiliev, A.F. Razin, Compos. Struct., Vol. 76, 182–189, 2006

Modeling Concepts

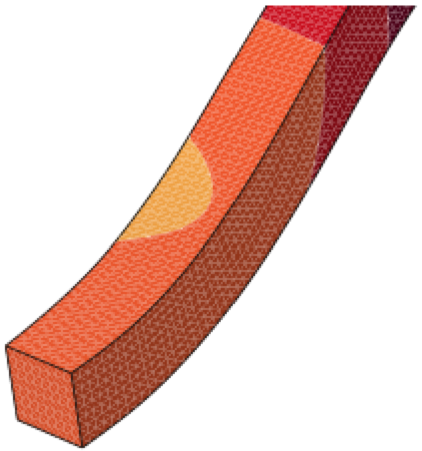
Discrete Models

- Each lattice member resolved
- Detailed geometric model
- Load introduction?
- Computational demanding
- Local phenomena can be captured in detail



Continuum Models

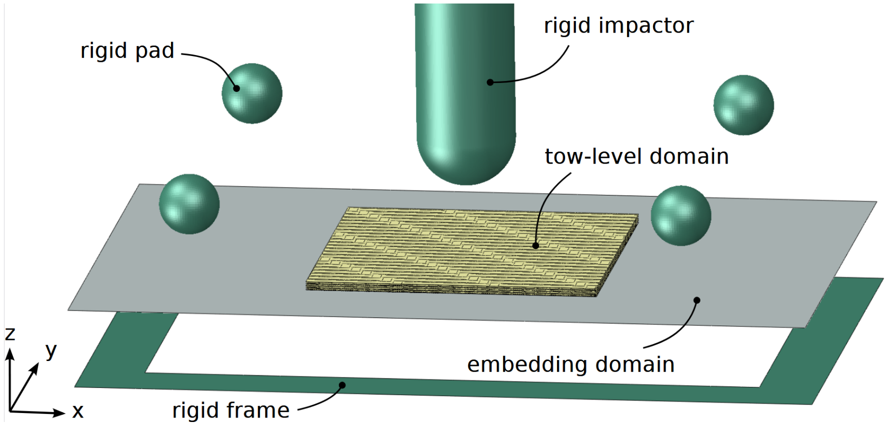
- Homogeneous solid
- Effective response
- Material Parameters?
- Computationally efficient
- Local phenomena not/partly captured

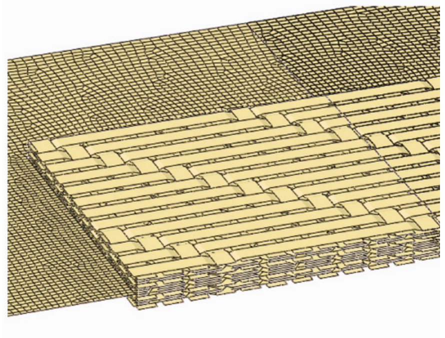


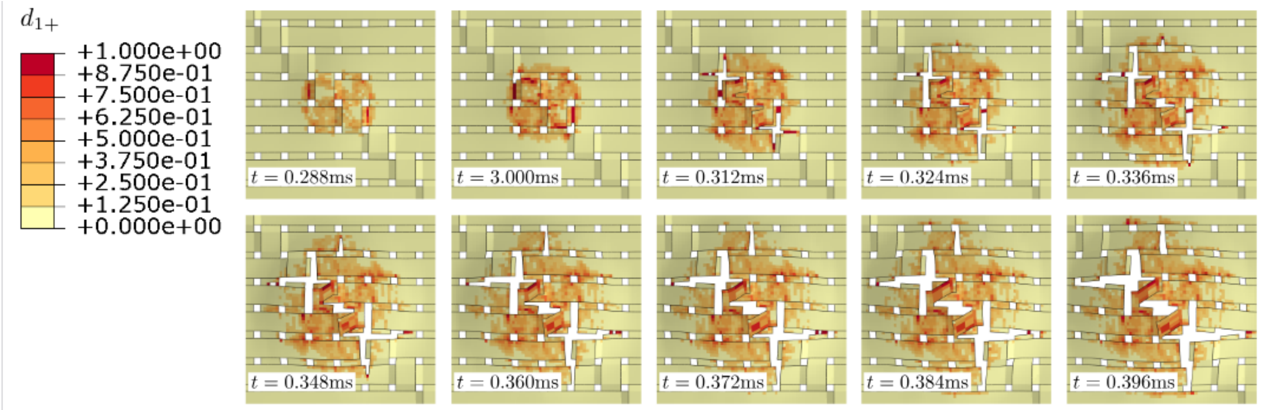
M. Schasching, PhD Thesis, TU Wien 2023

Modeling Concepts

Combined Approaches – e.g. embedding approach







M. Schwab, M. Todt, H.E. Pettermann, Journal of Composite Materials, Vol 52, https://doi.org/10.1177/0021998318755865

Overview

- Discrete Models
- Continuum Modeling
- Summary

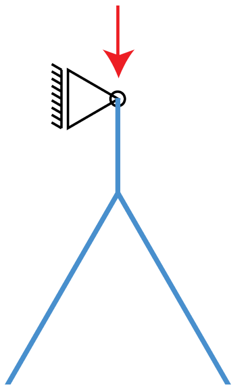
Overview

- **Discrete Models**
  - **Modeling**
  - **Examples**
- Continuum Modeling
- Summary

Modeling

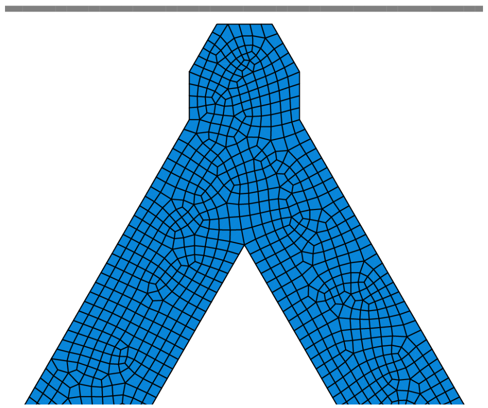
Beam Element Models

- Lattice members discretized with beam elements
- How to represent
  - material aggregation at nodes?
  - material nonlinearities?
  - boundary conditions / load introduction?
  - geometric imperfections?
- Computationally efficient
- Postprocessing

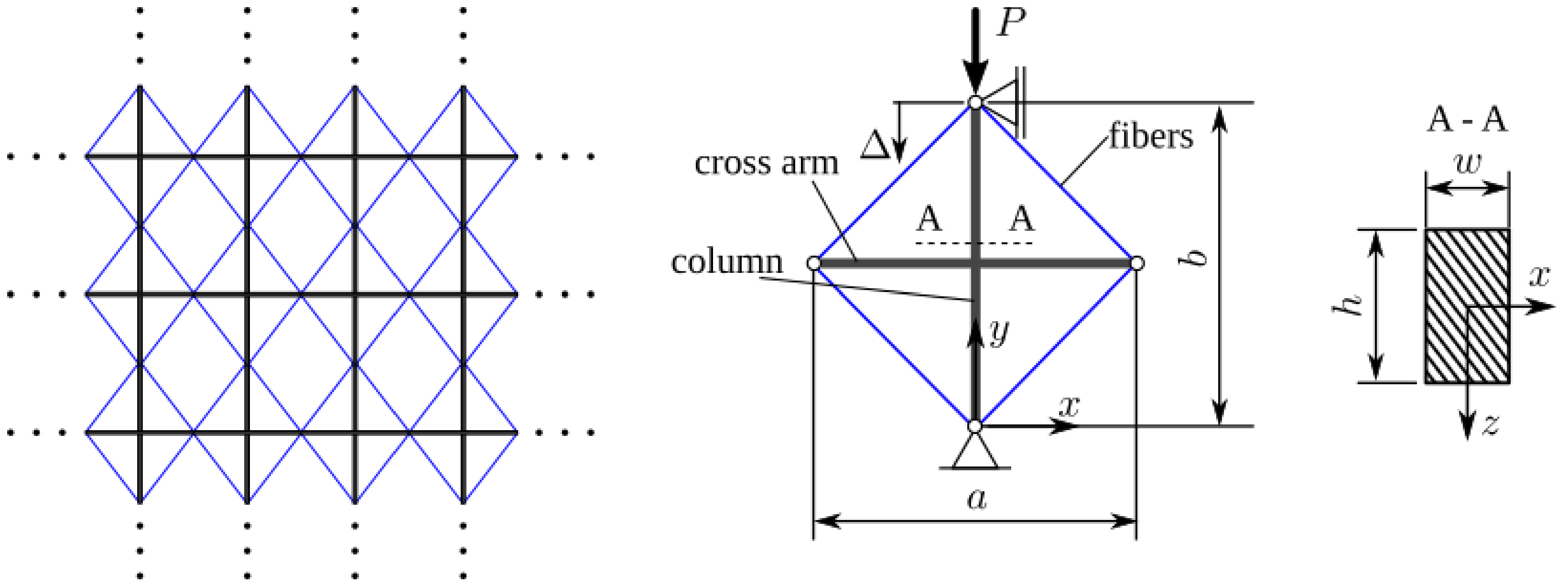


Continuum Element Models

- Lattice members discretized using continuum elements
- Discretization is trade off between
  - required elements over the strut thickness.
  - computational resources.
- Computationally more demanding



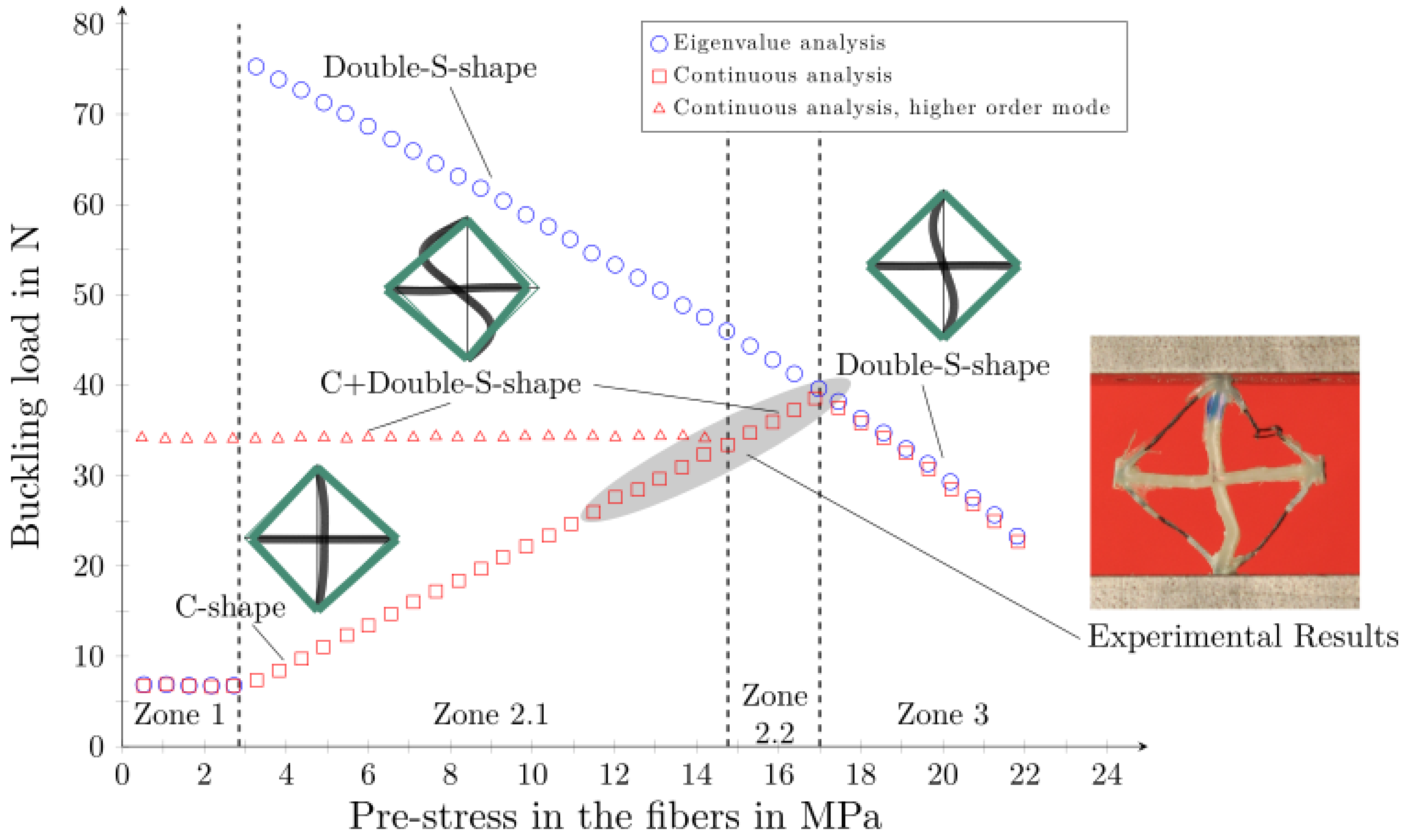
Example – Pre Stressed Lattices



- Fibers are pre-stressed to **increase the buckling load** of the lattice
- Aims:
  - **Proof of concept**
  - Investigate influence of pre-stress on buckling load
- Beam model of single cell, elastic material
- Comparison with experiments

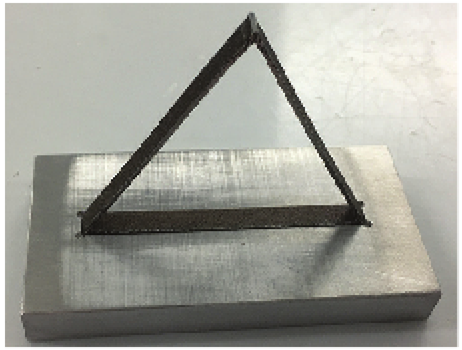
A. Köllner, M. Todt, G. Ganzosch, C. Völlmecke, Thin-Walled Structures 145, 106396, https://doi.org/10.1016/j.tws.2019.106396

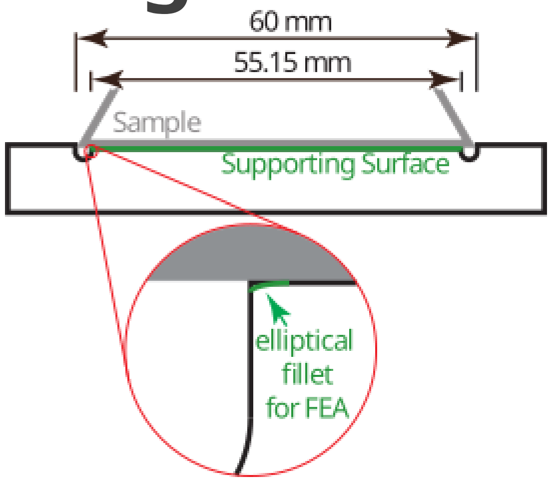
Example – Pre Stressed Lattices



- Pre-stress leads to an increase of the buckling load
- Good agreement with experiments
- Manufacturing of multi-cell arrangements?

Example – Triangular Lattices





- SLM printed cells using stainless steel powder
- Subjected to compressive loading

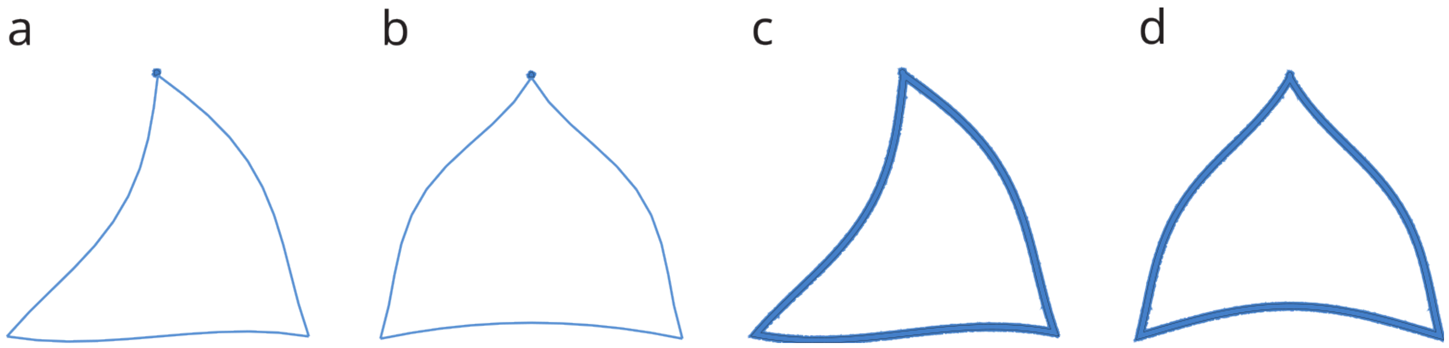
- Modeling with FEM – beam vs. continuum element models
- Elastic-plastic material model, isotropic hardening

B. Werner, O. Červinek, D. Koutný, A. Reisinger, H.E. Pettermann, M. Todt; Int. J. Solids Struct. 236–237, 2022, 111295, https://doi.org/10.1016/j.ijsolstr.2021.111295

Example – Triangular Lattices

<u>Linear buckling analysis</u>

- No material nonlinearities
- No friction



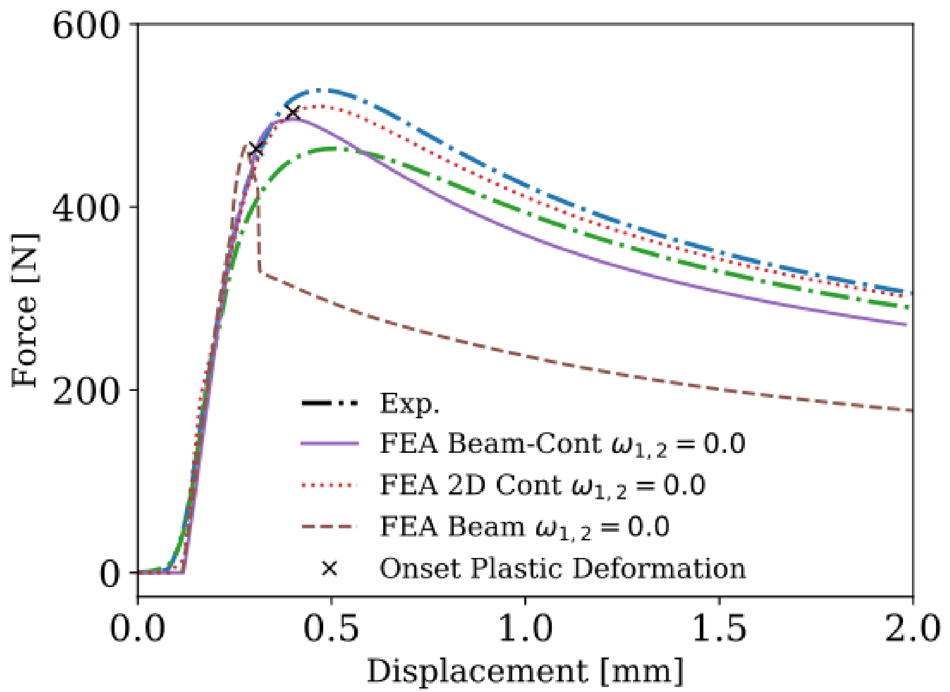
- a & c: mode 1 at approx. 300 N
- b & d: mode 2 at approx. 600 N

Mode 1 should be more likely than mode 2 – experiments tell a different story!

Example – Triangular Lattices

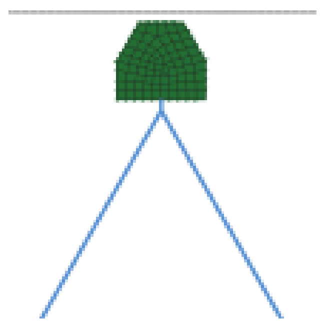
Nonlinear post-buckling analysis

- Beam vs. continuum vs. combined model



Combined Model

Contact area at the top modeled using continuum elements

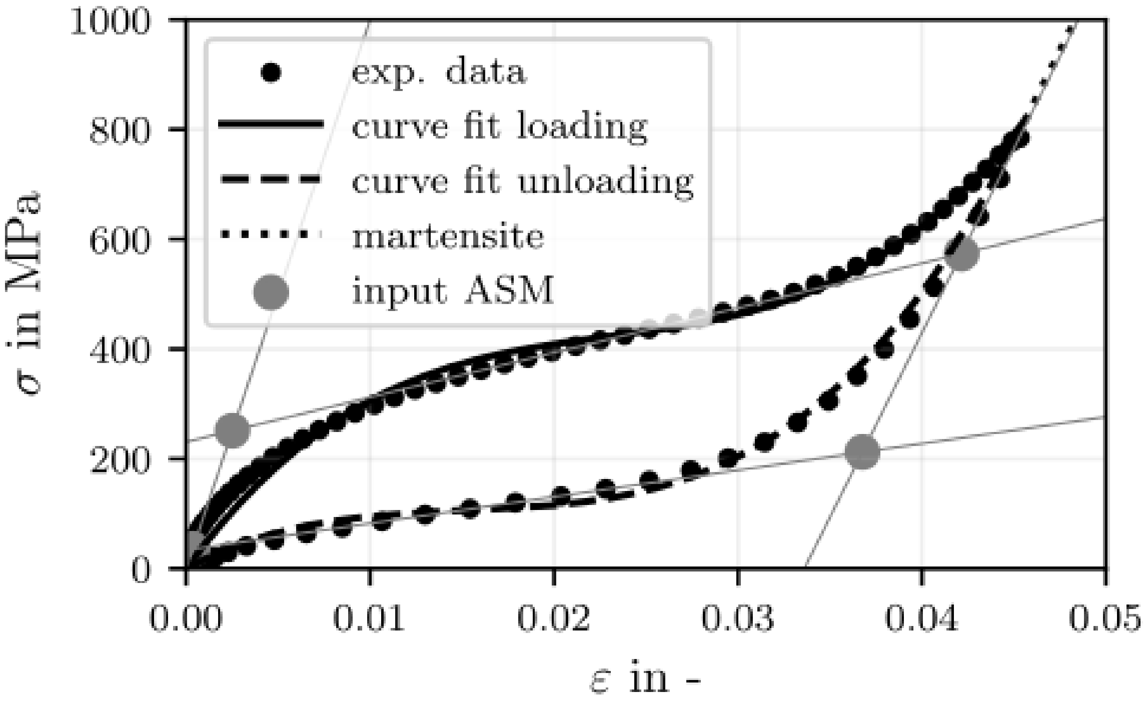


- Friction at top contact point drives structure into mode 2
- Pure beam model is not able to handle this effect

Example – Superelastic Lattices

User Material Model for NiTi 38

- Loading and unloading path using 3rd order polynomials
- Accurate representation of material response
- Available only for uniaxial stress states (beam elements)



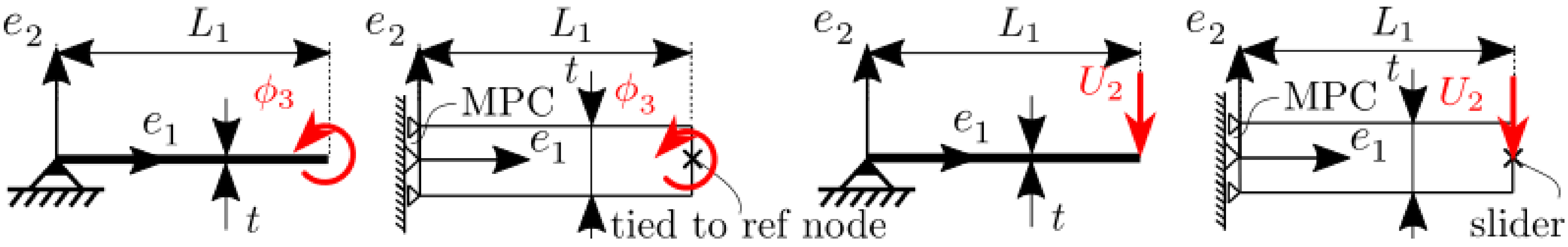
M.M. Schasching, O. Červinek, D. Koutný, H.E. Pettermann, M. Todt; PAMM 25, 2025, e202400092, https://doi.org/10.1002/pamm.202400092

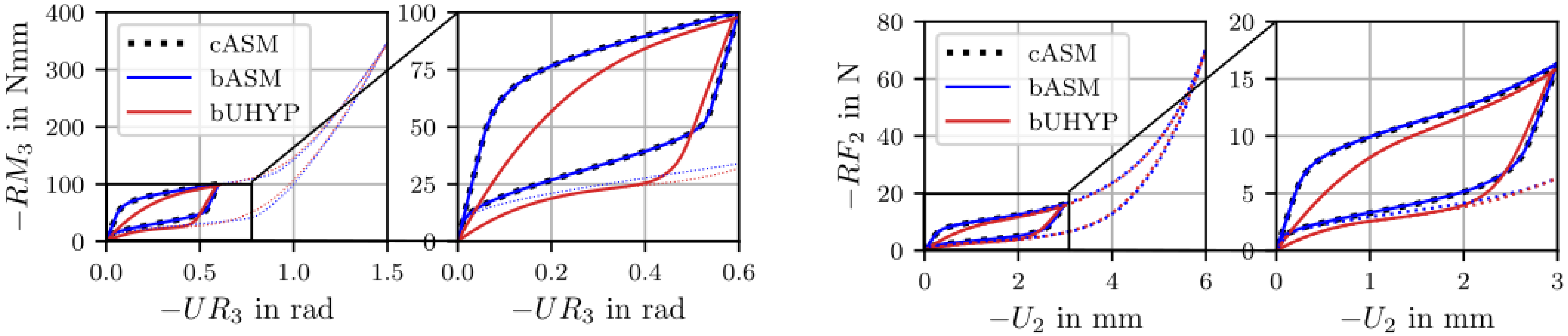
Example – Superelastic Lattices

Beam vs. Continuum Modeling

Pure bending

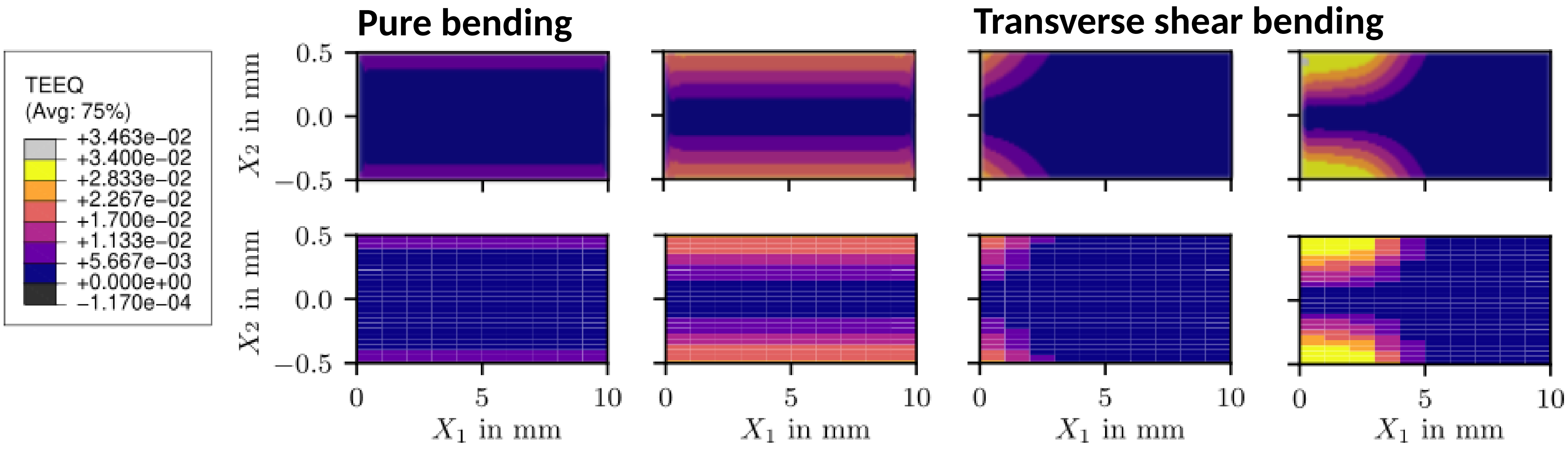
Transverse shear bending





Example – Superelastic Lattices

Beam vs. Continuum Modeling

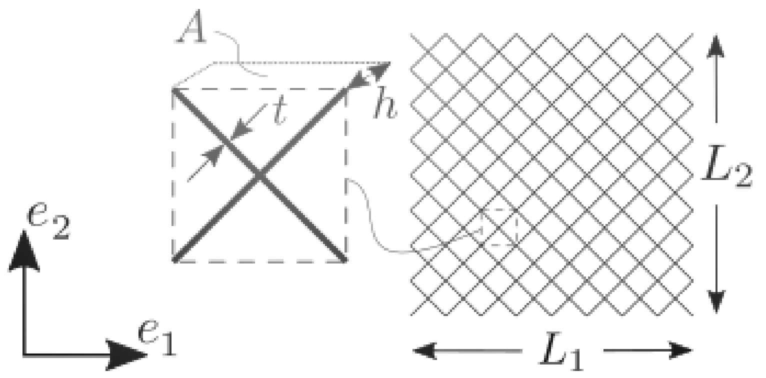


- Beam based modelling approach is suitable
- Very good representation of
  - load displacement response
  - spatial evolution of the transformation zones during loading / unloading

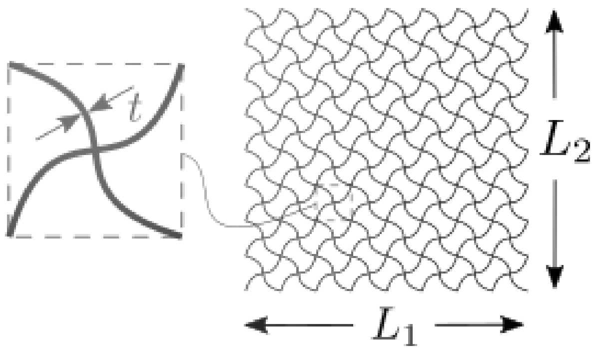
Example – Superelastic Lattices

<u>Simulation of cell structures</u>

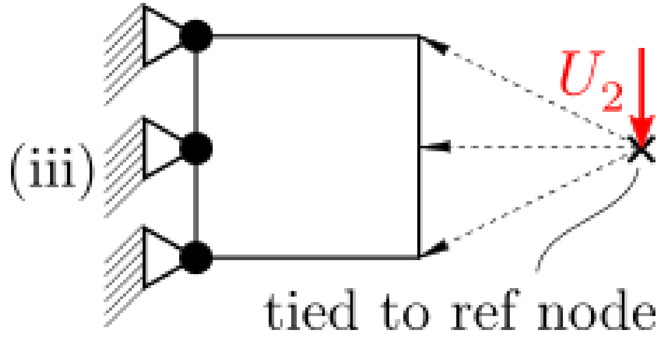
Stiff lattice



Compliant lattice

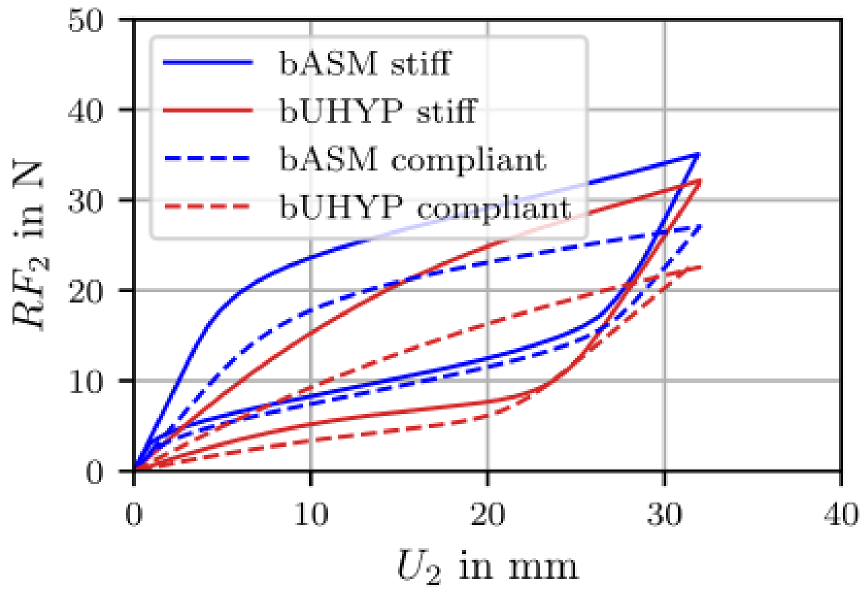


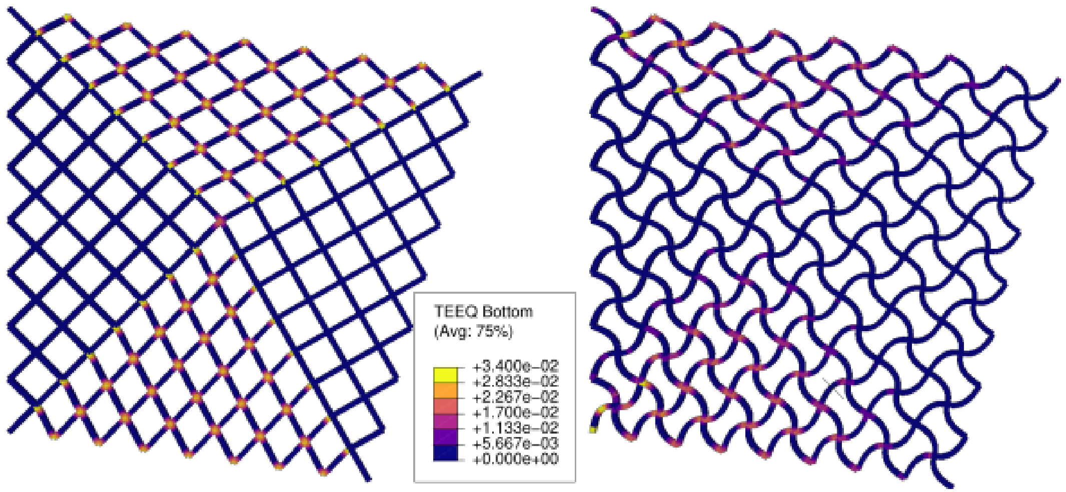
Loading scenario



$N \times N$	$L_i = L$	t	$A = Lh$
-	in mm	in mm	in mm^2
8x8	80	0.5	80

Example – Superelastic Lattices





- ASM model overestimates the reaction forces for finite structures
- Applicability of ASM model?

Overview

- Discrete Models
- **Continuum Modeling**
  - **Micropolar Continuum Model**
  - **Example**
- Summary

Micropolar Continuum Model

Micropolar Theory

- Scenarios where criteria of separation of scales is not met
- Displacement and **additional rotational** DOFs
- Material constants e.g. by energy based homogenization [1,2]

$$\underset{\sim}{\boldsymbol{\sigma}} = \underset{\sim}{\mathbb{A}} : \underset{\sim}{\boldsymbol{\varepsilon}} + \underset{\sim}{\mathbb{C}} : \underset{\sim}{\boldsymbol{\kappa}} \qquad \text{with} \qquad \underset{\sim}{\boldsymbol{\varepsilon}} = \underline{\mathbf{u}} \otimes \underline{\boldsymbol{\nabla}} - \underline{\boldsymbol{\phi}} \times \underset{\sim}{\mathbf{I}}$$

$$^{\kappa}\underset{\sim}{\boldsymbol{\sigma}} = \underset{\sim}{\mathbb{C}}^{\mathrm{T}} : \underset{\sim}{\boldsymbol{\varepsilon}} + \underset{\sim}{\mathbb{B}} : \underset{\sim}{\boldsymbol{\kappa}} \qquad\qquad \underset{\sim}{\boldsymbol{\kappa}} = \underline{\boldsymbol{\phi}} \otimes \underline{\boldsymbol{\nabla}}$$

$$\underbrace{\begin{bmatrix} [\underset{\sim}{\boldsymbol{\sigma}}] \\ [^{\kappa}\underset{\sim}{\boldsymbol{\sigma}}] \end{bmatrix}}_{[\underset{\sim}{\boldsymbol{\sigma}}]} = \underbrace{\begin{bmatrix} [\underset{\sim}{\mathbb{A}}] & [\underset{\sim}{\mathbb{C}}] \\ [\underset{\sim}{\mathbb{C}}]^{\mathrm{T}} & [\underset{\sim}{\mathbb{B}}] \end{bmatrix}}_{[\underset{\sim}{\mathbb{D}}]} \underbrace{\begin{bmatrix} [\underset{\sim}{\boldsymbol{\varepsilon}}] \\ [\underset{\sim}{\boldsymbol{\kappa}}] \end{bmatrix}}_{[\underset{\sim}{\boldsymbol{\varepsilon}}]}$$

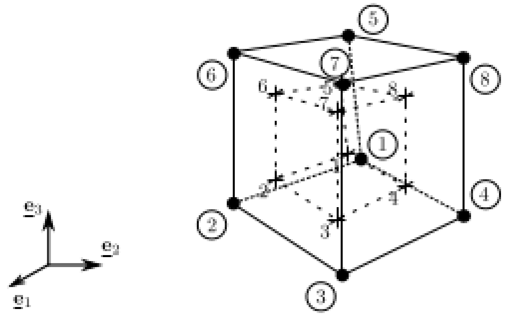
[1] Z.P. Bazant, Int. J. Solids Struct. 8 , 1972, 327–346, https://doi.org/10.1016/0020-7683(72)90093-5

[3] R.S. Kumar, D.L. McDowell, Int. J. Solids Struct. 41, 7399–7422, 2004, https://doi.org/10.1016/j.ijsolstr.2004.06.038

Micropolar Continuum Model

FEM Implementation

- Geometric nonlinear behavior – large rotations
- Stiffness matrix via finite differences of perturbed residuals [3]
- FEM implementation as 3D user element



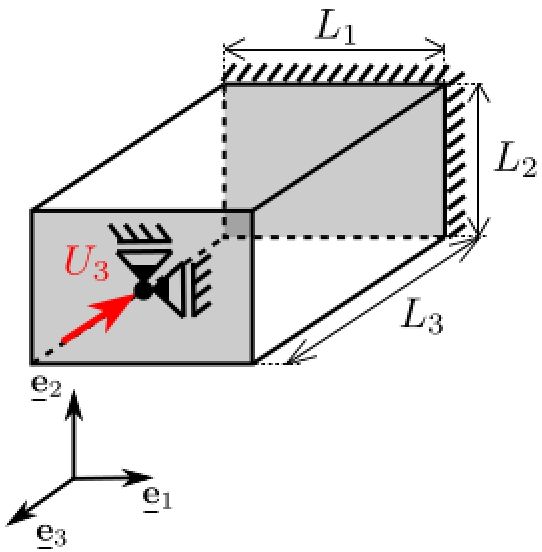
$$\check{\underset{\sim}{\mathbf{T}}} = \underset{\sim}{\mathbb{A}} : \underset{\sim}{\mathbf{E}} + \underset{\sim}{\mathbb{C}} : \underset{\sim}{\boldsymbol{\mathcal{K}}} \qquad \text{with} \qquad \underset{\sim}{\mathbf{E}} = \underset{\sim}{\mathbf{R}}^{\mathrm{T}} \underset{\sim}{\mathbf{F}} - \underset{\sim}{\mathbf{I}}$$

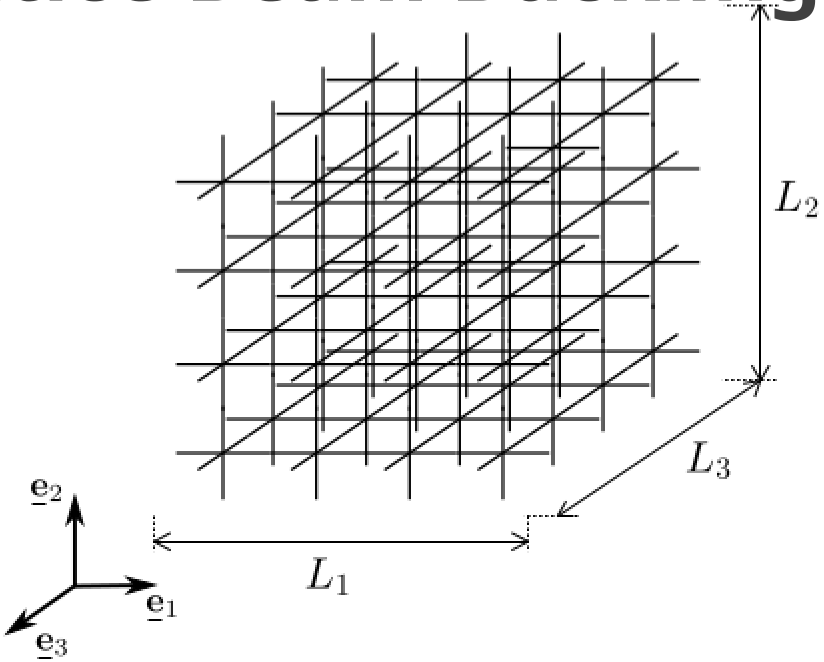
$${}^{\kappa}\check{\underset{\sim}{\mathbf{T}}} = \underset{\sim}{\mathbb{C}}^{\mathrm{T}} : \underset{\sim}{\mathbf{E}} + \underset{\sim}{\mathbb{B}} : \underset{\sim}{\boldsymbol{\mathcal{K}}} \qquad \underset{\sim}{\boldsymbol{\mathcal{K}}} = \mathrm{axl}\left(\underset{\sim}{\mathbf{R}}^{\mathrm{T}} \frac{\partial \underset{\sim}{\mathbf{R}}}{\partial X_k}\right) \otimes \underline{\mathbf{E}}_k = -\frac{1}{2} \underset{\sim}{\boldsymbol{\epsilon}} : \left(\underset{\sim}{\mathbf{R}}^{\mathrm{T}} \cdot \mathrm{Grad}_{\mathrm{R}}(\underset{\sim}{\mathbf{R}})\right)$$

$$[\underset{\sim}{\mathbf{K}}^{(k)}_{\mathrm{T};IJ}] = \left.\frac{\partial r_{iI}}{\partial y_{jJ}}\right|_k \approx \left.\frac{r_{iI}([\underline{\mathbf{y}}_J] + \vartheta[\underline{\mathbf{e}}_j]) - r_{iI}([\underline{\mathbf{y}}_J] - \vartheta[\underline{\mathbf{e}}_j])}{2\vartheta}\right|_k$$

[3] S. Bauer et al., Comput. Methods Appl. Mech. Eng. 199, 2010, 2643–2654, https://doi.org/10.1016/j.cma.2010.05.002

Example – Lattice Beam Buckling

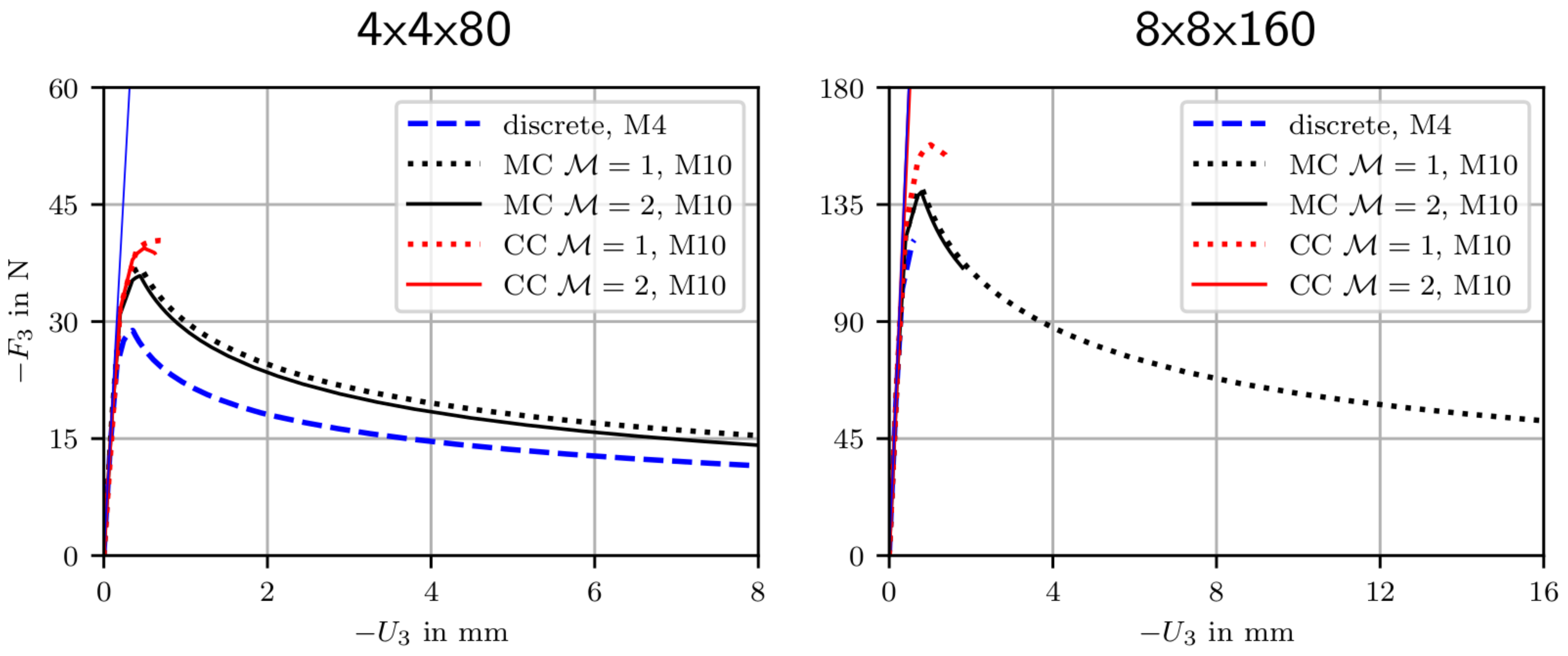




$L_1 = L_2$ in mm	L_3 in mm	l in mm	r in mm	$N_1 \times N_2 \times N_3$ (/)
4.0	80.0	1.0	1./20.0	**4x4x80**
8.0	160.0	1.0	1./20.0	**8x8x160**

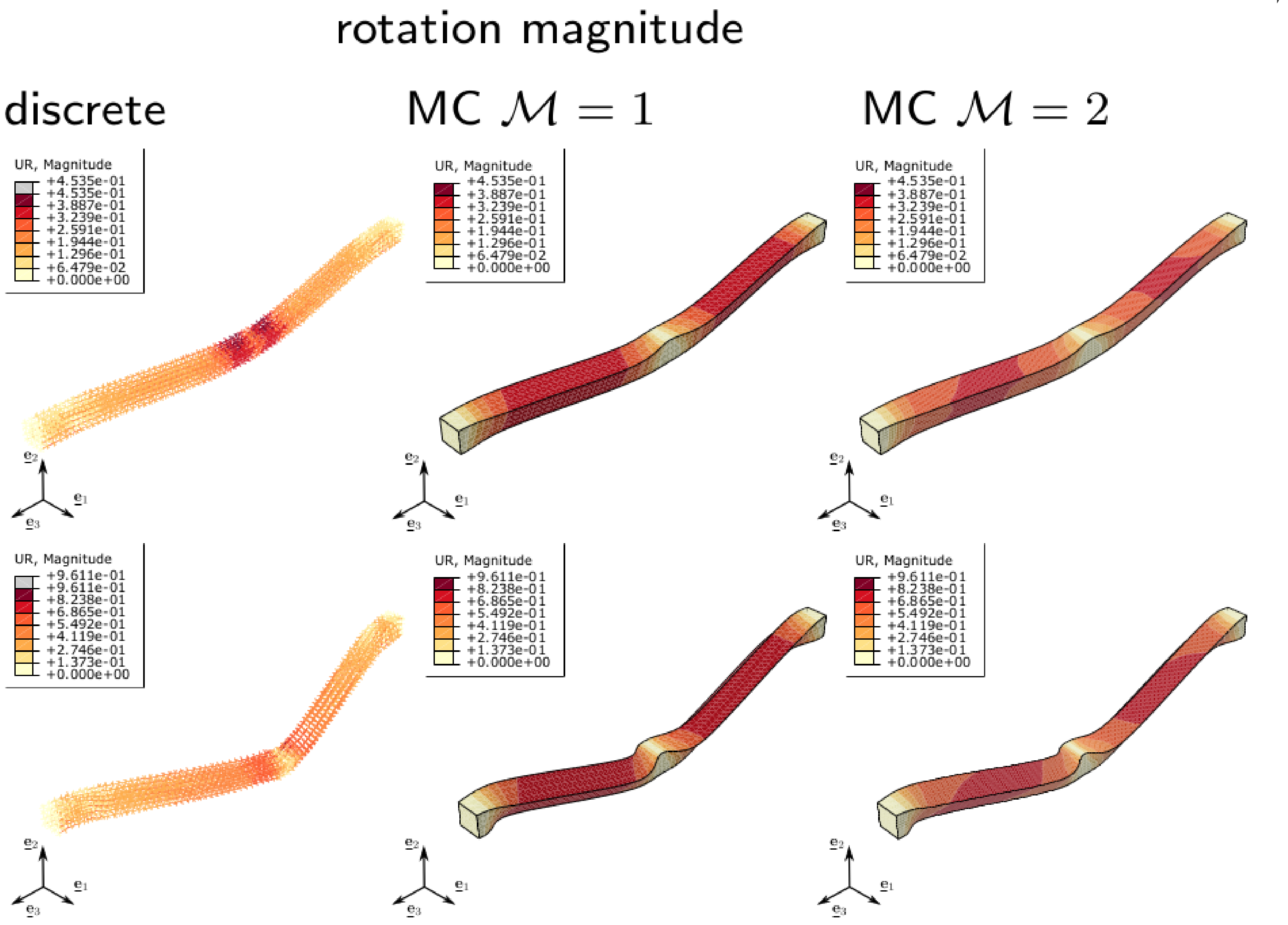
with $E_\text{S} = 120000$ in MPa, $\nu_\text{S} = 0.3$

Example – Lattice Beam Buckling



- Critical load slightly overestimated
- Unstable post-buckling response captured with MC model

Example – Lattice Beam Buckling



- Results for 2 different discretizations
- Good agreement for displacements and rotations
- Model still needs further improvement in terms of computational efficiency

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Summary

- There exists no “one-fits-all” model
- Model has to be appropriate for the questions asked
  - It is has to be **built correctly**
  - It has to be able to **capture the underlying physics**
- Validation against experiments or more sophisticated models necessary
  - Experiments should be mappable to a model.
  - Effect of boundary conditions has to be addressed.
- Interpretation of results always under consideration of the modeling assumptions

Acknowledgement

Many Thanks

- **BUT:** O. Červinek, D. Koutný
- **TU Berlin:** G. Ganzosch, A. Köllner, Ch. Völlmecke
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- **for your attention!**