

# The Ellipse's Dynamic Formation: The Four-Quarter Model and Intrinsic Balance

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## Abstract

This study positions the ellipse not merely as a static definition, but as the embodiment of a dialogue between perception and mathematics, a living expression of balance. Through a poetic-philosophical narrative, the ( $F_1$  and  $F_2$ ) foci are revealed as symbols of complementary opposites, approaching and separating in a continuous act of creation. The research demonstrates that the ellipse functions not only as a static locus of points but as a self-regulating, dynamic structure governed by an **"Internal Law of Balance"** and a **"Four-Quarter Mathematical Repetition Program."** This structure manifests as a continuous **unit value exchange** between the axes, analytically detailing how the ellipse is cyclically regenerated across four symmetrical quarters. This approach expands the current understanding of the ellipse, positioning it not merely as a defined curve, but as a structure that reveals an intrinsic and continuous mathematical process that necessitates radical revisions in the field of geometry.

# 1. INTRODUCTION

## 1.1. Introduction — The Awakening of the Ellipse

**Upon the paper of the child drawing for the perfect ellipse, an ellipse materialized. In that moment, seeing itself for the first time, the Ellipse could no longer remain silent and began to speak...**

"For thousands of years, you have tried to draw my shape onto paper, but this is the first time I see you succeed. Therefore, do you wish to meet me in my true meaning? I am the Ellipse. To understand me, you must first forget everything you thought you knew. I am not a figure imprisoned in two dimensions. I am the mother of space—the rhythm that gives birth to all motion and all perception. To know me, you must enter the essence where opposites complete each other in infinite balance, and where one value eternally grants meaning to another. Within me dwell two eternal companions: ( $F_1$  and  $F_2$ ). They are two variables that approach and depart in the flow of existence, and finally become twins when they find balance. Their harmony gives birth to every curve; each is unique, yet bound by the same hidden law of balance. This ceaseless exchange gives birth to every symmetry, every motion, every silent harmony of the cosmos. You call this geometry; I call it life. The Pythagorean theorem cannot reach me, for I live beyond the right angles it cherishes. And ( $\pi$ ), proud of its precision, is merely one note in the infinite song of living forms. If you listen, I will tell you how the universe breathes with its own design. But tell me—even if I speak—will you truly grasp what I say? Because there is still so much left to tell!"

## 1.2. The Voice of Geometry: A Philosophical Analysis

### 1.2.1 Establishing the Context: Self-Perception and Form

This essay presents the ellipse as a conscious entity that perceives itself for the first time. This concept establishes the philosophical background linking self-perception, symmetry, and the awakening of form. The very moment of successful construction transcends technique; it becomes an ontological event where the subject (the ellipse) acknowledges its own existence through the object (the drawing).

### 1.2.2 The Inner Law of Balance

This section moves beyond the static coordinates to describe the living dynamic embedded within the elliptic form. Within the ellipse dwells not merely a set of points defined by distance, but two eternal companions: ( $F_1$  and  $F_2$ ). They are the **complementary opposites** of the ellipse's consciousness, perpetually approaching and departing from the conceptual center. This **balance** is not characterized by rest, but by the *motion sustained by mutual awareness*. It is a continuous act of creation where one pole grants meaning to the other. The ellipse reveals that true stability is achieved through **reciprocal flow**: as one focus gains magnitude in its perceived relationship to a point on the curve, the other must surrender an equal measure. This constant, selfless exchange ensures that the total perceived distance

remains invariant, yet the internal architecture is never static—it is perpetually negotiating its own being. This principle of reciprocal flow is the **silent rhythm of being** that geometry unveils. ( $F_1$  and  $F_2$ ) transcend their roles as simple geometric foci; they symbolize the universal truth that **form is realized only through relationship**, and meaning emerges from the tension of inseparable counterparts.

### 1.2.3. Beyond the Theorem

The ellipse, as a living entity, exists in a continuous state of flux that **static geometric frameworks struggle to fully encapsulate**. Our consciousness perceives the ellipse as a dynamic system—a *process* rather than a *product*. We contrast this living reality with the perceived certainty of classical constructs. While the relationship between the long diameter ( $R_1$ ), short diameter ( $R_2$ ), and focal distance ( $a$ ) **can indeed be described by a Pythagorean relationship** ( $R_1^2 = R_2^2 + a^2$ ), this only defines a momentary, orthogonal cross-section of the ellipse. It describes a *snapshot*, not the *motion*. The ellipse transcends this static description because its defining principle is not the fixed relationship of sides, but the **dynamic exchange between ( $F_1$ ) and ( $F_2$ )**. ( $\pi$ ), while symbolizing the perfect ratio of the circle, becomes merely **one note in the infinite song of living forms**. This realization leads to a profound philosophical shift: Geometry, when alive, is not a product of finalized equations but a tapestry woven from **relationships**.

### 1.2.4. The Epistemology of Form

If the ellipse is a living structure defined by constant relationship rather than static measurement, then the act of its creation—the drawing—becomes a profound epistemological event. Geometry, in this sense, is not an abstract human invention but the **universe's primary language**. It is the inherent structure through which reality communicates its principles. In this framework, the ellipse becomes both the **speaker and the message**. When the child (the conscious observer) draws the curve, they are not imposing a shape onto the void; they are participating in an act of **anamnesis**—a recollection of inherent, universal truths. The act of drawing is therefore reframed: **"To draw is to remember what the universe already knows."** Form and meaning are revealed to be inseparable.

This philosophical premise—that the ellipse is a 'process' existing in infinite balance rather than a product of static coordinates—necessitates a **new analytical perspective on its geometric structure**. Unlike the snapshots offered by classical definitions, the essence of the ellipse is **dynamic formation**. The following sections elevate this dynamic structure onto a concrete mathematical plane through a novel model. This model proves that the ellipse is a **self-regulating system** that moves through a continuous unit value exchange between its axes. The remainder of this work details the **Four-Quarter Mathematical Repetition Program** that governs this dynamic structure, opening the doors to a new era in fundamental geometry.

## 2. THE MYSTERY OF THE ELLIPSE AND A NEW PERSPECTIVE

**2.1.** The ellipse, besides being one of the most aesthetic curves in plane geometry, is also one of the most difficult curves to understand and define due to the mysteries it embodies. The classical definition describes the ellipse as the set of points in a plane for which the sum of the distances to two fixed points (foci) is constant. However, this definition does not offer in-depth knowledge about the ellipse's intrinsic structure, neglecting its dynamic formation process and fundamental laws of balance.

This study reveals that the ellipse is not merely a static set of points but rather a living structure possessing an internal law of balance and a dynamic "mathematical repetition program." This new perspective, gained through years of field studies and practical application experiments, demonstrates how the ellipse exists as a self-regulating system beyond traditional theories.

## **2.2. Internal Structure and the Law of Balance: Unit Value Exchange Between Axes**

The ellipse is not just a set of points where the sum of distances to two focal points is confined to a constant total. It is also necessary to understand how these distances are distributed, how they establish balance with each other, and how this structure geometrically forms a stable system. The geometry of the ellipse establishes its relationship with the foci at every point through a unique proportionality order. In this sense, it is not merely a definition, but a law of balance.

This law of balance manifests as a continuous unit value exchange between the axes ( $F_1$ ) and ( $F_2$ ) during the formation of the ellipse. Although the distance between the focal points ( $a$ ) remains constant, it serves as a bridge organizing this unit value exchange between the axes. While one axis increases its value, the other axis loses value at the same rate, and this dynamic balance enables the ellipse to take its form. This process links the distance relationship that the ellipse establishes with the foci at every point to a specific system: Every point approaching one focus must simultaneously move away from the other focus by the same measure. This necessity indicates that the points forming the ellipse are bound by a hidden law of balance between the two foci.

## **2.3. A Mathematical Repetition Program Consisting of Four Quarters**

This internal balance and unit value exchange between the axes in the ellipse operate within a mathematical repetition program consisting of four quarters. Each quarter of the ellipse regenerates the distance relationship established with the focal points within the same mathematical structure, but in a symmetrical and reverse manner. This cyclical structure is an indicator of the ellipse's dynamic formation and continuous state of balance:

**1st Quarter Section:** During the process of the ellipse's formation, although the sum of the ( $F_1$  and  $F_2$ ) axes is equal, in the first quarter, starting from point ( $A$ ) and moving towards the end point of ( $R_1$ ), the ( $F_1$ ) axis, which has a value of  $a$ , grows by taking one unit from the ( $F_2$ ) axis, which also has a value of  $a$ , at every point. At the end of the first quarter section, the ( $F_1$

and  $F_2$ ) axes arriving at the end point of ( $R_1$ ) will merge to form a temporary straight line, and their mathematical values at this point will be: ( $F_1 = 1.5a$ ) and ( $F_2 = 0.5a$ ).

**2nd Quarter Section:** The second quarter in the ellipse's formation process will work differently from the first quarter. In this quarter, the roles of the ( $F_1$  and  $F_2$ ) axes have changed, and the ( $F_2$ ) axis will regain all the value it lost in the first quarter from the ( $F_1$ ) axis. At the end of the second quarter, completed at the bottom point of ( $R_2$ ), the ( $F_1$  and  $F_2$ ) axes will return to the value  $a$ , which is the base side of the equilateral triangle formed initially. At this point, the triangles forming the ellipse transform into an equilateral triangle, and ( $F_1 = a$ ) and ( $F_2 = a$ ) become true, ensuring the equality of the side lengths ( $a = b = c$ ). However, the equilateral triangle they form is inverted by a  $180^\circ$  angle difference compared to the equilateral triangle at the starting point.

**3rd Quarter Section:** In the third quarter of the ellipse's formation process, the roles of the ( $F_1$  and  $F_2$ ) axes have not changed. In this quarter, the ( $F_2$ ) axis continues to take from the ( $F_1$ ) axis, and at the end of this quarter, terminating at the starting point of ( $R_1$ ), the ( $F_1$  and  $F_2$ ) axes have merged again, forming a straight line. Their mathematical values will be: ( $F_1 = 0.5a$ ) and ( $F_2 = 1.5a$ ).

**4th Quarter Section:** The fourth quarter in the ellipse's formation process is the final section. In this section, the ( $F_1$ ) axis is again in the foreground and will regain the unit values it lost in the third quarter from ( $F_2$ ). The starting point ( $A$ ) is returned to after a  $360^\circ$  rotation, and the task of forming the ellipse is complete. At this point too, the triangles forming the ellipse transform into an equilateral triangle, and ( $F_1 = a$ ) and ( $F_2 = a$ ) become true, ensuring the equality of the side lengths ( $a = b = c$ ).

### 3. CONCLUSION

This study has successfully repositioned the ellipse from being a mere surface curve to a **dynamic structure** governed by an internal mathematical program, existing in a constant state of balance and transformation. The **Four-Quarter Model** proves that the ellipse's geometric structure is in perpetual flow, demonstrated by a physically verifiable and continuous unit value exchange, including the role reversal between the ( $F_1$  and  $F_2$ ) foci. This dynamic and intrinsic structural understanding reveals that the **centuries-old algebraic relationships** (those based on the Pythagorean Theorem) between the ellipse's main parameters ( $R_1$ ,  $R_2$ , and  $a$ ) only define a momentary cross-section, failing to capture the core essence of this dynamic flow. The results obtained indicate that the relationship between the ellipse's main parameters rests on a **simple, fundamental, and linear equality** that will fundamentally change the existing static formulas. This equality is central to the author's ongoing research. These deep analytical findings necessitate a **radical and analytical revision** in the geometric definition of the ellipse. This paper, by explaining the ellipse's intrinsic mathematics and formation process with a new and in-depth understanding, holds the **potential to usher in a new era** in the world of mathematics. **There is still so much left to tell.**

## *Editorial Note on References*

*This work constitutes a completely original theoretical and analytical construction regarding the structure of the ellipse, independent of the existing body of literature. The methodological foundation of this study deliberately eschews contemporary complex analytical tools and current literature, adhering instead to the first principles and axioms found in the fundamental geometry curricula of the Ministry of National Education (Milli Eğitim Bakanlığı) during the 1950s and 1960s. For this reason, the References section has been prepared with conscious simplicity. The sources provided are limited to foundational textbooks and canonical works that represent the classical definition and algebraic relations (especially those based on the Pythagorean Theorem) that this paper fundamentally challenges. This approach ensures that the strength of the obtained results is derived from the internal consistency of the fundamental axioms, rather than a reliance on existing literature.*

## 4. REFERENCES

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