Beyond Eight

ABC Conjecture and Neutron Drip Line:

A Structural Analogy at the Boundary of Existence

Hiroshi Sasaki

Javatel Corporation, Ashibetsu, Hokkaido, Japan

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Abstract

This essay proposes a structural analogy between the ABC conjecture in number theory and the neutron drip line in nuclear physics. The ABC conjecture concerns the relationship between the additive structure (a + b = c) and multiplicative structure (rad(abc)) of integers, with a residue E = log(c/rad(abc)) that cannot be eliminated. Similarly, the neutron drip line marks the boundary where atomic nuclei can no longer bind additional neutrons. This revised version includes a comparative analysis of statistical distributions at these boundaries, together with figures illustrating the structural parallels.

1. Introduction: Fifty Years of Questioning

For fifty years, I have pursued the ABC conjecture. Not as a professional mathematician, but as an engineer who sees patterns across disciplines. In December 2025, I retired from this pursuit with the following conclusion:

"Projection fails. But what cannot be projected necessarily remains.

That is E, and arithmetic speaks of it only in lower dimensions."

This essay is not a proof. It is a map for those who will come after.

2. The ABC Conjecture and Residue E

The ABC conjecture, proposed by Oesterlé and Masser in 1985, concerns coprime positive integers a, b, c satisfying a + b = c. Define the radical rad(abc) as the product of distinct prime factors of abc. The conjecture states that for any $\varepsilon > 0$, there exist only finitely many triples where $c > \text{rad}(abc)^{1+\varepsilon}$.

The residue E = log(c/rad(abc)) measures the "gap" between additive and multiplicative structures. The quality q = log(c)/log(rad(abc)) provides another measure; high-quality triples (q > 1.4) are rare and cluster near the "boundary" of what the conjecture permits.

3. Hydrogen Isotopes and the Neutron Drip Line

Hydrogen isotopes provide a natural laboratory for studying the limits of nuclear binding:

- ^{1}H (Protium): 1 proton, 0 neutrons — Stable
- ^{2}H (Deuterium): 1 proton, 1 neutron — Stable
- ^{3}H (Tritium): 1 proton, 2 neutrons — Radioactive ($t_{1/2} \approx 12.3$ years)

• ^{4}H to ^{6}H: Extremely unstable resonance states
• ^{7}H: 1 proton, 6 neutrons — Observed in 2003 at RIKEN ($t_{1/2} \approx 10^{-21}$ s, $\Gamma \approx 0.09$ MeV)
• ^{8}H: 1 proton, 7 neutrons — Never observed, beyond the neutron drip line

4. Comparative Analysis: Distributions at the Boundary

To move beyond qualitative analogy, we examine the statistical distributions characterizing each boundary.

4.1 Nuclear Side: Resonance Width Distribution

As neutron number N increases for Z=1 (hydrogen), the resonance width Γ and decay modes show systematic trends:

Isotope	N/Z	Γ (MeV)	Primary Decay
^{4}H	3	~5	n emission
^{5}H	4	~5.4	2n emission
^{6}H	5	~1.8	3n emission
^{7}H	6	~0.09	4n emission
^{8}H	7	—	Does not exist

Table 1: Hydrogen isotope resonance data (compiled from RIKEN experiments)

Remarkably, the resonance width Γ *decreases* as we approach the drip line (^{7}H has the narrowest width), then the system ceases to exist entirely at ^{8}H. This "narrowing before collapse" suggests the boundary is not approached smoothly.

4.2 ABC Side: Quality Distribution of Triples

The ABC@home project has catalogued thousands of high-quality triples. The quality $q = \log(c)/\log(\mathrm{rad}(abc))$ shows a characteristic distribution:

Quality Range	Count	Behavior
$q > 1.0$	~Infinite	Trivially many
$q > 1.4$	~120	Rare, clustered
$q > 1.6$	~10	Extremely rare
$q \to \infty$	0	Conjectured impossible

Table 2: Distribution of ABC triple qualities (based on ABC@home data)

4.3 Structural Parallel

Both systems show a "rarefaction then cutoff" pattern: the nuclear system becomes paradoxically more well-defined (narrower Γ) just before extinction, while ABC triples become increasingly rare as q increases, until none exist beyond some threshold. The boundary is not a smooth limit but a structural discontinuity.

5. Six, Seven, Eight: Numbers at the Boundary

Consider the mathematical properties of these numbers: 6 is the first perfect number ($6 = 1 + 2 + 3$), 7 is a prime—indivisible, standing alone, and $8 = 2^3$ is the first cube of the smallest prime.

In hydrogen isotopes: ^{6}H exists (barely), ^{7}H exists at the limit, ^{8}H cannot exist. The transition from 7 to 8 marks the boundary of existence.

6. Structural Analogy: The Hypothesis

I propose the following structural correspondence:

• ABC Conjecture: Additive structure (a+b=c) vs. Multiplicative structure (rad)
• Nuclear Physics: Nuclear force (binding) vs. Pauli exclusion (separation)
• Residue E: What cannot be eliminated when structures don't align
• Neutron drip line: Where binding can no longer contain separation

The hypothesis: Understanding why ^8H cannot exist may illuminate why E cannot be eliminated in ABC. Both are traces of the same structural boundary.

7. Conclusion: A Map for the Future

I cannot prove ABC. I am not a mathematician. But after fifty years of observation, I see a pattern.

I record here the author's intuition:

"When eight is confirmed, ABC is proven."

This is not a proof, but a sense of direction gained from fifty years of observation. It awaits verification by future generations. By "confirmed," I mean: when we can mathematically describe the exact condition at the neutron drip line—why ^8H cannot exist in a form that connects to the structural incompatibility measured by E.

This essay is a map, not a proof. The boundary of existence is where mathematics and physics meet. The answer lies at eight.

Acknowledgments

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References

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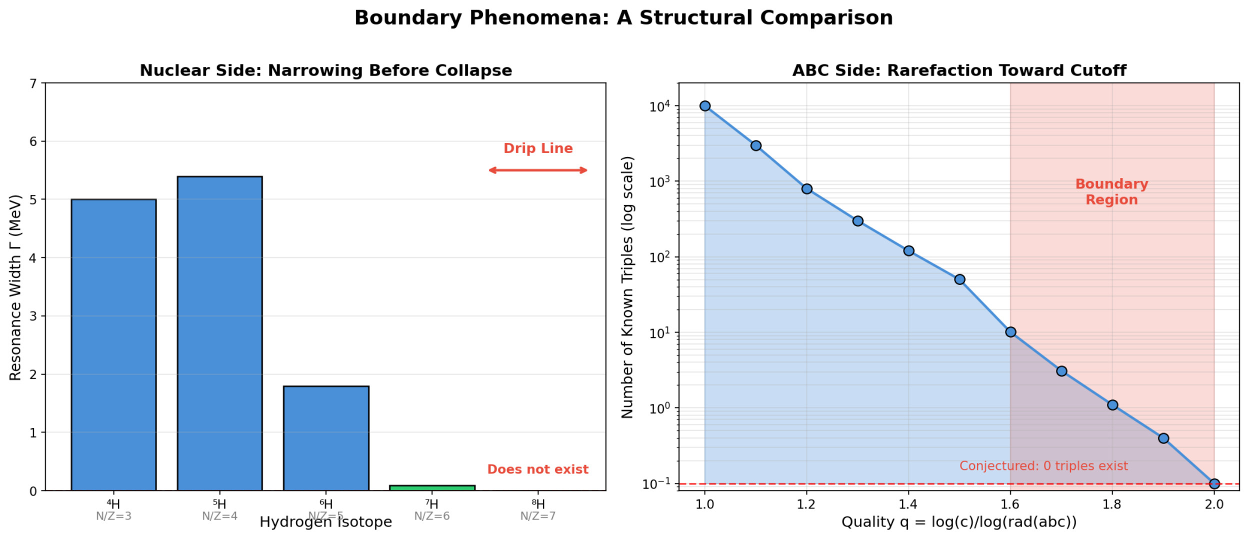
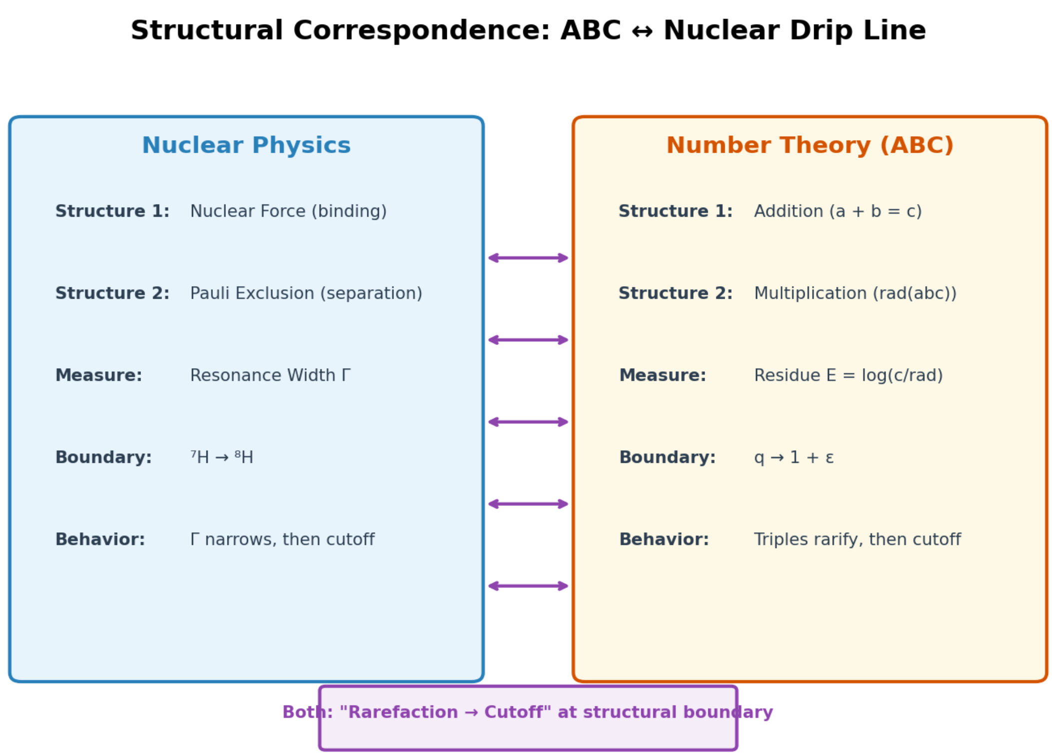
Figures

Figure 1: Boundary Phenomena — A Structural Comparison



Left: Resonance width Γ of hydrogen isotopes approaching the neutron drip line. Note the paradoxical narrowing at ■H before complete cutoff at ■H. Right: Distribution of ABC triple qualities (log scale), showing rarefaction toward the conjectured boundary.

Figure 2: Structural Correspondence Diagram



Schematic mapping of corresponding elements between nuclear physics (hydrogen isotope stability) and number theory (ABC conjecture). Both domains exhibit 'rarefaction then cutoff' behavior at their structural boundaries.