# Part VII: AXIOMS
# The axioms of integrodynamics

Foundations of the Physics of Integrity

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**Abstract**

This paper formalises the axiomatic foundations of *Integrodynamics*: the unified dynamical theory governing contradiction, symmetry, adaptation, and integrity across epistemic, procedural, and institutional systems. Building on the six prior papers of the Integrity series, we consolidate their implicit assumptions into a minimal and complete axiom set from which the full integrity field theory follows.

Integrity is treated not as a moral property but as a conserved structural quantity governed by free-energy dissipation, symmetry constraints, and evidential collapse under stochastic contradictions. Eight axioms are stated defining contradiction observability, free-energy monotonicity, conservation of evidential structure, symmetry-stabilised dynamics, bounded adaptation, and probabilistic irreversibility. From these axioms, the core integrodynamic law is derived, unifying contradiction, symmetry, adaptation, implausibility, and integrity into a single field-theoretic architecture.

This paper establishes Integrodynamics as a complete structural theory of integrity, with testable predictions, conservation laws, and a well-defined phase structure.

**Keywords:** integrodynamics; axiomatic systems; integrity theory; free-energy formalism; structural integrity; contradiction curvature; symmetry constraints; procedural symmetry; bounded adaptation; integrity entropy; evidential collapse; statistical irreversibility; legitimacy invariance; Lyapunov structure; phase structure; field-theoretic integrity; structural falsifiability; integrity diagnostics; epistemic entropy; institutional dynamics.

# Contents

# 1 Purpose of the axiomatisation

The preceding six papers establish the operational machinery of integrity diagnostics across epistemic, procedural, statistical, and institutional domains. What remains absent is a formal axiomatic closure: a minimal set of primitive assumptions from which all prior constructions follow as consequences rather than independent inventions.

This paper supplies that closure. The objective is not philosophical justification, but structural completion. The axioms stated herein render the theory:

- closed under derivation,
- resistant to interpretive drift,
- invariant under domain translation,
- and fully falsifiable as a single integrated system.

# 2 Primitive Objects of Integrodynamics

Let an integrodynamic system consist of the following primitive elements:

- A behaviour field: $b(x,t)$,
- An integrity potential functional: $U[b]$,
- An integrity entropy functional: $S_I[b]$,
- An integrity free-energy functional:

$$\mathscr{F}[b] = U[b] - \Theta S_I[b],$$

- A declared admissible symmetry family: $\mathscr{G}$,
- A coherence current: $J_I$,
- A bounded adaptive update operator: $\mathscr{P}'$.

All derived quantities, diagnostics, and institutional observables arise from these primitives. This paper introduces no new empirical claims.

## 2.1 Structural Assumptions on Admissible Symmetries

The declared admissible symmetry family $\mathscr{G}$ acts on the configuration space $X$ and satisfies the following algebraic properties:

1. **Identity.** There exists an element $e \in \mathscr{G}$ such that
$$e(x) = x \qquad \text{for all } x \in X.$$
2. **Closure.** If $g_1, g_2 \in \mathscr{G}$, then their composition
$$g_2 \circ g_1 \in \mathscr{G}.$$
3. **Optional Invertibility (Group Case).** If each $g \in \mathscr{G}$ is bijective and $g^{-1} \in \mathscr{G}$, then $\mathscr{G}$ forms a group. In general, $\mathscr{G}$ need only be a monoid, allowing integrodynamic systems whose admissible processes are irreversible.

These structural assumptions ensure that symmetry-based invariance (Axiom V), contradiction observability under admissible transformations (Axiom I), and symmetry-stabilised low-energy states (Axiom IV) are all well-defined. They also guarantee the compositional testability of integrity configurations under sequences of admissible procedures.

# 3 The Axioms of Integrodynamics

## Axiom I — Observability of contradiction

If a system's declared rationale and its observable behaviour cannot be jointly satisfied under admissible transformations, the resulting inconsistency manifests as measurable structural curvature.

Contradiction is therefore an observable quantity, not a logical failure.

## Axiom II — Integrity as Free Energy

The integrity state of a system is represented by a free-energy functional:

$$\mathscr{F}[b] = U[b] - \Theta S_I[b],$$

whose evolution under admissible dynamics satisfies:

$$\dot{\mathscr{F}} \leq 0.$$

Integrity cannot increase without compensating entropy export.

## Axiom III — Conservation of Evidential Structure

In closed systems, the sum of operational integrity and epistemic entropy is approximately conserved:

$$I^* + H \approx \text{const.}$$

Integrity loss therefore reappears as disorder elsewhere in the informational environment.

## Axiom IV — Symmetry-Stabilised Low-Energy States

Procedural symmetry corresponds to local minima of the integrity free-energy landscape. Persistent asymmetry corresponds to curvature away from stable basins.

Symmetry without intervention is the unique passive equilibrium of integrity.

## Axiom V — Legitimacy as Invariance

An institution is legitimate if and only if its observable behaviour is invariant under its declared admissible symmetry group.

Legitimacy is therefore a demonstrated invariance, not a narrative claim.

## Axiom VI — Bounded Adaptation

All adaptive updates must remain within the symmetry-preserving feasible region defined by declared commitments.

Unbounded adaptation constitutes constitutional phase transition, not learning.

### Axiom VII — Statistical Irreversibility

When the probability of an observed multi-domain outcome under the innocent stochastic null model collapses below a defined threshold, directional structure is established as positive evidence.

Null collapse is an irreversible evidential transition.

### Axiom VIII — Deferred Intent

Intent is never assumed. It becomes the least-complex explanatory hypothesis only after symmetry violation, procedural deviation, and stochastic innocence have all failed.

## 4 The Integrodynamic Law

From Axioms I–VIII, the governing law of integrodynamics follows:

> *Integrity evolves as a conserved free-energy field whose stable states correspond to symmetry-preserving dynamics, whose drift generates evidential curvature, and whose stochastic irreversibility establishes structural directionality.*

Formally, the law may be expressed as:

$$\dot{\mathscr{F}} = -\Phi_{\text{contradiction}} - \Phi_{\text{asymmetry}} - \Phi_{\text{evidential}},$$

with each dissipation term non-negative under admissible dynamics.

## 5 Theorems and Derived Results

In this section we record several basic consequences of the axioms. The proofs are structural: they establish properties that hold in any domain where the axioms are satisfied, independently of the specific interpretation of $b$, $U$, $S_I$, and $\mathscr{G}$.

## 5.1 Lyapunov Structure and Monotonicity

**Definition 5.1** (Admissible dynamics)**.** A trajectory $t \mapsto b(\cdot, t)$ is said to follow *admissible dynamics* if it respects all declared institutional and procedural constraints and satisfies the integrodynamic law.

**Theorem 5.2** (Second Law of Integrity)**.** *Under Axioms II and III, the free-energy functional $\mathscr{F}$ is a Lyapunov function for any closed integrodynamic system evolving under admissible dynamics. In particular,*

$$\dot{\mathscr{F}}(t) \leq 0$$

*for all $t$, with equality if and only if the system is in an integrity-equilibrium state.*

*Proof.* Axiom II states that for admissible dynamics the integrity state is represented by a free-energy functional $\mathscr{F}[b]$ whose evolution satisfies $\dot{\mathscr{F}} \leq 0$. This is precisely the Lyapunov condition: $\mathscr{F}$ is non-increasing along trajectories. In a closed system, Axiom III implies that $I^* + H$ is approximately constant, so any strict reduction in $\mathscr{F}$ corresponds to a reallocation between ordered integrity and epistemic entropy rather than an external injection of order.

If $\dot{\mathscr{F}} < 0$ on a time interval, the system is moving down the free-energy landscape and is therefore out of equilibrium. Conversely, if $\dot{\mathscr{F}} = 0$ along a trajectory, then by Axiom II no further integrity dissipation is possible under admissible dynamics and the system resides at an equilibrium point of $\mathscr{F}$. Hence $\mathscr{F}$ is a Lyapunov function and the claim follows. □

**Corollary 1** (Irreversibility of Integrity Dissipation)**.** *Under the assumptions of Theorem 5.2, there exists no admissible trajectory that returns the system from a state $b_2$ to a state $b_1$ with $\mathscr{F}[b_2] < \mathscr{F}[b_1]$ while preserving the closed-system condition.*

*Proof.* Suppose for contradiction that there exists an admissible trajectory from $b_1$ to $b_2$ with $\mathscr{F}[b_2] < \mathscr{F}[b_1]$, and another admissible trajectory returning from $b_2$ to $b_1$ in a closed system. Along the first trajectory, Theorem 5.2 implies $\mathscr{F}$ is non-increasing, so the inequality $\mathscr{F}[b_2] < \mathscr{F}[b_1]$ is compatible with admissibility. Along the reverse trajectory, $\mathscr{F}$ would have to increase from $\mathscr{F}[b_2]$ back to $\mathscr{F}[b_1]$, contradicting the monotonicity condition $\dot{\mathscr{F}} \leq 0$ for admissible dynamics. Hence no such reversible cycle exists, and integrity dissipation is irreversible in closed systems. □

## 5.2 Symmetry and Stability

**Definition 5.3** (Symmetric configuration)**.** A configuration $b^\star$ is said to be *symmetric* if it is invariant under the declared admissible symmetry group $\mathscr{G}$, i.e. $g \cdot b^\star = b^\star$ for all $g \in \mathscr{G}$.

**Theorem 5.4** (Symmetry Stabilisation Theorem)**.** *Assume Axioms II and IV. Let $\mathscr{F}$ be invariant under the action of $\mathscr{G}$ and let $b^\star$ be a symmetric configuration. If $b^\star$ is a local minimiser of $\mathscr{F}$ within the space of admissible configurations, then $b^\star$ is Lyapunov-stable under admissible dynamics. In particular, any sufficiently small admissible perturbation that preserves the symmetry constraints cannot decrease $\mathscr{F}$.*

*Proof.* By Axiom IV, procedural symmetry corresponds to local minima of the free-energy landscape and persistent asymmetry corresponds to curvature away from those minima. Let $b^\star$ be a symmetric configuration that is a local minimiser of $\mathscr{F}$. Consider any admissible perturbation $b^\star + \delta b$ that preserves the symmetry constraints; by local minimality we have

$$\mathscr{F}[b^\star + \delta b] \geq \mathscr{F}[b^\star]$$

for all sufficiently small such perturbations. Since $\mathscr{F}$ is $\mathscr{G}$-invariant, group actions do not change the value of $\mathscr{F}$, so the minimum is attained on the entire orbit of $b^\star$ under $\mathscr{G}$.

Under Axiom II, admissible dynamics can only move the system in directions that weakly decrease $\mathscr{F}$. In a neighbourhood of $b^\star$, any direction that would strictly decrease $\mathscr{F}$ corresponds to a departure from the symmetric basin and is therefore forbidden by the symmetry constraints. Hence trajectories starting sufficiently close to $b^\star$ cannot move to states of strictly lower $\mathscr{F}$ while respecting admissibility, and Theorem 5.2 implies that they cannot move to states of higher $\mathscr{F}$ either. This establishes Lyapunov stability of $b^\star$. □

**Corollary 2** (Uniqueness of Passive Equilibrium)**.** *Under the hypotheses of Theorem 5.4, any passive equilibrium attainable under admissible dynamics is symmetric. Equivalently, symmetry without intervention is the unique passive equilibrium of integrity.*

*Proof.* Let $b^\dagger$ be a passive equilibrium attainable under admissible dynamics, i.e. a state with $\dot{\mathscr{F}} = 0$. If $b^\dagger$ were not symmetric, Axiom IV would classify it as lying in a region of curvature away from any symmetric basin. In such a region, the free-energy gradient is non-zero and admissible dynamics must move the system towards lower values of $\mathscr{F}$, implying $\dot{\mathscr{F}} < 0$ and contradicting the assumption of passivity. Therefore every passive equilibrium must be symmetric, and symmetry without intervention is the unique form of passive equilibrium. □

## 5.3 Legitimacy and Invariance

**Definition 5.5** (Legitimacy)**.** An institution is *legitimate* if its observable behaviour is invariant under its declared admissible symmetry group $\mathscr{G}$.

**Theorem 5.6** (Legitimacy–Invariance Equivalence)**.** *Assume Axiom V. Then legitimacy is equivalent to invariance: an institution is legitimate if and only if its observable behaviour is invariant under its declared admissible symmetry group $\mathscr{G}$.*

*Proof.* The "only if" direction is immediate from Axiom V, which states that an institution is legitimate if and only if its observable behaviour is invariant under $\mathscr{G}$. For the "if" direction, suppose that the institution's behaviour is invariant under $\mathscr{G}$. Then, again by Axiom V, this invariance is both necessary and sufficient for legitimacy as defined within the integrodynamic framework. No additional narrative or justificatory layer is required; invariance itself is the certificate of legitimacy. □

**Remark 1.** Theorem 5.6 shows that within Integrodynamics legitimacy is a *derived* property rather than a primitive assumption. Once the symmetry group and observables are specified, legitimacy is determined by invariance alone.

## 5.4 Statistical Irreversibility and Evidential Collapse

**Definition 5.7** (Evidential threshold)**.** Let $p_0$ denote the probability, under the innocent stochastic null model, of observing a given multi-domain outcome. A pre-specified threshold $\varepsilon > 0$ is called an *evidential threshold* if outcomes with $p_0 \le \varepsilon$ are treated as statistically irreconcilable with the null.

**Theorem 5.8** (Evidential Irreversibility)**.** *Assume Axiom VII and fix an evidential threshold $\varepsilon$. Let $\mathscr{H}_0$ denote the innocent null model and $\mathscr{H}_1$ a directional alternative. If an observed outcome $D$ satisfies $\mathbb{P}(D \mid \mathscr{H}_0) \le \varepsilon$ and $\mathbb{P}(D \mid \mathscr{H}_1) \gg \mathbb{P}(D \mid \mathscr{H}_0)$, then:*

*(i) the Bayes factor* $B(D) = \dfrac{\mathbb{P}(D \mid \mathscr{H}_1)}{\mathbb{P}(D \mid \mathscr{H}_0)}$ *is large, and*

*(ii) under any prior that assigns non-zero probability to $\mathscr{H}_1$, the posterior odds in favour of $\mathscr{H}_1$ are strictly increased by observing $D$.*

*Consequently, the evidential update favouring directionality is irreversible in the sense that no further observation of $D$ can restore the original posterior odds.*

*Proof.* By assumption, $\mathbb{P}(D \mid \mathscr{H}_0) \le \varepsilon$ and $\mathbb{P}(D \mid \mathscr{H}_1) \gg \mathbb{P}(D \mid \mathscr{H}_0)$, so the Bayes factor

$$B(D) = \frac{\mathbb{P}(D \mid \mathscr{H}_1)}{\mathbb{P}(D \mid \mathscr{H}_0)}$$

is strictly greater than 1 and typically large. Let $\pi_0$ and $\pi_1$ denote the prior probabilities of $\mathscr{H}_0$ and $\mathscr{H}_1$ respectively, with $\pi_1 > 0$. The posterior odds after observing $D$ are:

$$\frac{\mathbb{P}(\mathscr{H}_1 \mid D)}{\mathbb{P}(\mathscr{H}_0 \mid D)} = \frac{\pi_1}{\pi_0} \cdot B(D).$$

Since $B(D) > 1$, the posterior odds are strictly larger than the prior odds $\pi_1/\pi_0$. This establishes (ii).

Axiom VII asserts that once the null probability falls below the evidential threshold, the null is treated as statistically irreconcilable and the outcome is classified as positive evidence of directional structure. Observing $D$ repeatedly cannot change its likelihoods under $\mathscr{H}_0$ or $\mathscr{H}_1$, so the Bayes factor associated with $D$ cannot be reduced by further evidence of the same outcome. Hence the evidential shift in favour of $\mathscr{H}_1$ is irreversible with respect to that outcome class. □

**Corollary 3** (Directionality as Structural Evidence). *Under the conditions of Theorem 5.8, directional structure becomes the least-complex explanation for $D$ consistent with the axioms. Any attempt to maintain $\mathscr{H}_0$ requires auxiliary assumptions whose cumulative complexity exceeds that of adopting $\mathscr{H}_1$.*

*Proof.* By Theorem 5.8, observing $D$ irreversibly shifts the posterior odds in favour of $\mathscr{H}_1$. To retain $\mathscr{H}_0$ as the preferred explanation, one must introduce additional auxiliary hypotheses (for example, extreme parameter values, hidden selection mechanisms, or ad hoc constraints) that increase the effective complexity of the null. Axiom VIII requires that intent or directionality be considered only after symmetry-based, procedural, and stochastic explanations have failed. Once the null has collapsed below the evidential threshold, the simplest remaining model consistent with the axioms is the directional alternative $\mathscr{H}_1$. Thus directionality becomes the least-complex structural explanation. □

# 6 Derived Phase Structure

The axioms imply the existence of three qualitative regimes of integrity:

- **Symmetry (solid):** $\nabla \mathscr{F} = 0$ (procedural honesty),

- **Adaptation (liquid):** bounded operation of $\mathscr{P}_{\text{prime}}$ (legitimate drift),
- **Contradiction (plasma):** $\nabla \cdot J_I \to \infty$ (narrative collapse).

These regimes correspond to the phase structure observed in numerical simulations of the integrity field and are consistent with Theorems 5.2 and 5.4: solid phases arise at symmetric minima of $\mathscr{F}$, liquid phases correspond to bounded drift within the feasible region, and plasma phases manifest as high-curvature regions with divergent coherence currents.

## 7 Position Within the Integrity Series

This paper serves as the axiomatic keystone of the Integrity programme:

| | |
|---|---|
| Paper I | Epistemic contradiction detection |
| Paper II | Procedural symmetry and convergence |
| Paper III | Institutional adaptation and bounded asymmetry |
| Paper IV | Unified integrity field formulation |
| Paper V | The Physics of Integrity |
| Paper VI | Statistical evidential ignition |
| **Paper VII** | **Axiomatic closure of Integrodynamics** |

No new machinery is introduced here. The contribution is structural: the theory is now closed, self-consistent, and irreducible.

## 8 Falsifiability and Structural Failure Conditions

A theory qualifies as physically meaningful only if it admits conditions under which it can be false. The purpose of this section is to state explicitly the observational and structural circumstances under which the axioms of Integrodynamics would be violated. These conditions are not philosophical counterexamples but operational failure modes.

If any of the conditions below are empirically demonstrated under admissible measurement, the theory is falsified in its present form.

## 8.1 Failure of Free-Energy Monotonicity

By Axiom II, admissible dynamics satisfy $\dot{\mathscr{F}} \leq 0$. The theory is falsified if there exists an admissible closed-system trajectory such that

$$\dot{\mathscr{F}}(t) > 0$$

holds on a non-zero-measure time interval without compensating entropy export. Such a finding would directly violate the Second Law of Integrity and invalidate the Lyapunov structure established in Theorem 5.2.

## 8.2 Failure of Symmetry–Stability Correspondence

By Axiom IV and Theorem 5.4, persistent procedural symmetry uniquely corresponds to locally stable low-energy configurations. The theory is falsified if a system can be shown to exhibit:

- persistent, uncorrected asymmetry under admissible dynamics, and
- absence of measurable curvature or coherence dissipation,

simultaneously. Such behaviour would contradict the identification of asymmetry with free-energy curvature and destabilisation.

## 8.3 Failure of Legitimacy–Invariance Equivalence

By Axiom V and Theorem 5.6, legitimacy is equivalent to invariance under the declared admissible symmetry group. The theory is falsified if an institution is demonstrated to retain full legitimacy while violating its declared symmetry constraints in sustained observable behaviour.

This includes regimes in which:

- decision symmetry is formally declared,
- outcome symmetry is persistently violated, and
- no integrity penalty is empirically detectable.

## 8.4 Failure of Bounded Adaptation

By Axiom VI, adaptive updates must remain within a declared symmetry-preserving feasible region. The theory is falsified if a system is shown to:

- undergo unbounded parameter drift,
- cross a constitutional phase boundary, and
- simultaneously preserve integrity invariants,

under admissible dynamics. Such behaviour would contradict the definition of adaptation as symmetry-preserving bounded flow.

## 8.5 Failure of Statistical Irreversibility

By Axiom VII and Theorem 5.8, collapse of the stochastic null model constitutes an irreversible evidential transition. The theory is falsified if there exists an observed outcome $D$ such that:

- $\mathbb{P}(D \mid \mathscr{H}_0) \leq \varepsilon$ under the innocent null model,
- $\mathbb{P}(D \mid \mathscr{H}_1) \gg \mathbb{P}(D \mid \mathscr{H}_0)$ under a directional alternative, and
- posterior odds fail to shift in favour of $\mathscr{H}_1$ under Bayesian updating,

for any non-degenerate prior assignment. Such a result would contradict the evidential ignition mechanism of Integrodynamics.

## 8.6 Failure of Deferred Intent

By Axiom VIII, intent enters inference only after symmetry, procedural, and stochastic innocence have collapsed. The theory is falsified if intent attribution is shown to be operationally necessary *prior* to violations of Axioms IV–VII in order to preserve internal consistency.

This excludes any formulation in which motive is required as a primitive explanatory input.

## 8.7 Summary of Falsification Modes

The axioms of Integrodynamics are therefore jointly falsified if any of the following are demonstrated under admissible observation:

- sustained increase of $\mathscr{F}$ in a closed system,
- persistent asymmetry without curvature or dissipation,
- legitimacy without invariance,
- unbounded adaptation without structural penalty,
- stochastic null collapse without posterior convergence,
- or primitive dependence on intent.

In each case, failure is structural rather than narrative. The theory makes no attempt to withstand such counterexamples by reinterpretation; violation constitutes direct falsification.

# 9 A Minimal Integrodynamic Toy Model

To illustrate the axioms in the simplest possible setting, we consider a one-dimensional integrodynamic system with a single scalar commitment variable $b(t) \in \mathbb{R}$. This toy model is not intended to be realistic; its purpose is to demonstrate explicitly how the free-energy structure, symmetry, and monotonicity conditions can be realised in a concrete dynamical law.

## 9.1 Definition of the model

Let $b : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denote a scalar behaviour variable. We define:

- a quadratic integrity potential

$$U(b) = \frac{\lambda}{2} b^2, \qquad \lambda > 0,$$

- an integrity entropy functional

$$S_I(b) = \log\big(1 + b^2\big),$$

- and the corresponding free-energy functional

$$\mathscr{F}(b) = U(b) - \Theta S_I(b) = \frac{\lambda}{2} b^2 - \Theta \log\big(1 + b^2\big),$$

  with $\Theta > 0$.

The declared admissible symmetry group is the two-element group

$$\mathscr{G} = \{+1, -1\},$$

acting on $b$ by sign flip $g \cdot b = gb$. By construction, $\mathscr{F}(b)$ is invariant under this action:

$$\mathscr{F}(-b) = \mathscr{F}(b),$$

so the model has an explicit $\mathbb{Z}_2$ symmetry.

## 9.2 Gradient-flow dynamics

We now impose an admissible dynamics in the form of gradient flow on $\mathscr{F}$:

$$\dot{b}(t) = -\kappa \frac{\mathrm{d}\mathscr{F}}{\mathrm{d}b}\big(b(t)\big), \qquad \kappa > 0. \tag{9.1}$$

Differentiating $\mathscr{F}$ with respect to $b$ yields

$$\frac{\mathrm{d}\mathscr{F}}{\mathrm{d}b}(b) = \lambda b - \Theta \frac{2b}{1 + b^2} = b\left(\lambda - \frac{2\Theta}{1 + b^2}\right).$$

Substituting into (9.1) gives the explicit scalar ODE

$$\dot{b}(t) = -\kappa\, b(t)\left(\lambda - \frac{2\Theta}{1 + b(t)^2}\right). \tag{9.2}$$

This defines a one-dimensional integrodynamic evolution consistent with Axiom II.

## 9.3 Free-energy monotonicity

We now verify that $\mathscr{F}$ is strictly non-increasing along trajectories of (9.2). By the chain rule,

$$\dot{\mathscr{F}}(t) = \frac{d\mathscr{F}}{db}\big(b(t)\big) \cdot \dot{b}(t).$$

Using (9.1) we obtain

$$\dot{\mathscr{F}}(t) = \frac{d\mathscr{F}}{db}\big(b(t)\big) \cdot \left(-\kappa \frac{d\mathscr{F}}{db}\big(b(t)\big)\right) = -\kappa \left(\frac{d\mathscr{F}}{db}\big(b(t)\big)\right)^2.$$

Since $\kappa > 0$ and the square term is non-negative, we have

$$\dot{\mathscr{F}}(t) \leq 0$$

for all $t$, with equality if and only if $\frac{d\mathscr{F}}{db}(b(t)) = 0$. Thus, in this toy model, $\mathscr{F}$ is a Lyapunov function in the strict sense of Theorem 5.2: the integrity free energy cannot increase under the admissible dynamics (9.2).

## 9.4 Equilibria and symmetry

Equilibria of the toy dynamics satisfy $\dot{b} = 0$, which from (9.2) occurs when either

$$b = 0 \quad \text{or} \quad \lambda - \frac{2\Theta}{1 + b^2} = 0.$$

The point $b = 0$ is always an equilibrium. Its symmetry is immediate: for every $g \in \mathscr{G}$, $g \cdot 0 = 0$, so the configuration is invariant under the declared symmetry group. A short calculation shows that near $b = 0$,

$$\frac{d^2\mathscr{F}}{db^2}(0) = \lambda - 2\Theta,$$

so for parameter regimes with $\lambda > 2\Theta$ the origin is a strict local minimum of $\mathscr{F}$ and therefore a stable symmetric equilibrium.

Additional equilibria may exist if the algebraic condition $\lambda - 2\Theta/(1 + b^2) = 0$ admits non-zero solutions. In such cases one obtains symmetric pairs $\pm b_\star$ with equal free energy, again reflecting the underlying $\mathbb{Z}_2$ symmetry. Their stability depends on the sign of the second derivative $d^2\mathscr{F}/db^2$ at those points; for typical parameter values, the symmetric configuration at $b = 0$ remains the global minimum.

## 9.5 Interpretation

This toy model exhibits, in the simplest possible form, the structural features required by the axioms:

- Axiom II (integrity as free energy) is realised via $\mathscr{F}(b)$ and the gradient-flow dynamics (9.1), with $\dot{\mathscr{F}} \leq 0$ holding identically.
- Axiom IV (symmetry-stabilised low-energy states) is realised by the $\mathbb{Z}_2$ symmetry and the symmetric equilibrium at $b = 0$, which is a local (and for suitable parameters global) minimum of $\mathscr{F}$.
- Theorem 5.4 is concretely instantiated: the symmetric equilibrium at $b = 0$ is Lyapunov-stable whenever $\lambda > 2\Theta$.
- Deviations $b \neq 0$ generate curvature in the free-energy landscape and hence non-zero dissipation $\dot{\mathscr{F}} < 0$ until the system relaxes back towards a symmetric basin.

Although highly simplified, the model shows that the axioms of Integrodynamics can be realised in a concrete dynamical system with explicit equations of motion, rather than remaining purely conceptual.

# 10 Conclusion

With the axioms stated and their basic consequences derived, Integrodynamics becomes a complete dynamical theory of integrity rather than a collection of compatible methods. Contradiction becomes curvature, symmetry becomes stability, adaptation becomes bounded flow, and statistical collapse becomes irreversibility. All subsequent applications inherit their validity solely through these foundations.