

Band-Limited Relational Time II: A Unified Relational Master Equation for Quantum and Gravitational Regimes

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Band-Limited Relational Time II: A Unified Relational Master Equation for Quantum and Gravitational Regimes

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Abstract

There is a quiet strangeness at the heart of physics: our best descriptions of the universe do not seem to contain time the way we experience it. The formal state of “everything” can sit there, motionless on paper, while inside that motionless state we are born, grow old, and watch clocks tick on the wall.

Yet every experiment we run, every interference pattern we record, is a story told *in time*. Particles decay, memories form, detectors click in ordered sequences. Somewhere between the frozen global picture and our lived sense of change, a bridge has to be built. This work is about that bridge, and about the imperfections of the clocks we use to cross it.

Real observers are small, noisy, and finite. We do not see the whole universe; we listen to it through narrow-band instruments, through fallible clocks that hear only part of the cosmic song. Near strong gravitational fields those clocks distort and redshift, their sense of rhythm stretched thin. If time is truly relational—if it is nothing more (and nothing less) than correlations between subsystems—then these limitations are not cosmetic details. They shape what counts as “happening” for any given observer.

The absence of an external temporal parameter in constrained quantum gravity, expressed by the Wheeler–DeWitt condition $\hat{H}\Psi = 0$ —renders time a relational construct emerging from subsystem correlations.

Building on the Band-Limited Relational Time (BLRT) framework, this paper extends the operational notion of observer bandwidth into a unified dynamical law valid across both weak- and strong-coupling regimes. The resulting master equation interpolates smoothly between completely positive Markovian evolution in laboratory systems and non-Markovian, redshift-modulated dynamics near gravitational horizons.

Time thus remains relational yet becomes spectrally filtered by the observer’s finite bandwidth, quantified through a redshift-dependent factor $g(t) = \sqrt{1 - r_s(t)/r(t)}$. Gravitational back-action, Hawking-channel dissipation, and island-like entanglement recovery appear naturally as non-Markovian memory terms. Planck-scale cutoffs replace singularities with perceptual limits, preserving entanglement and information flow. This unified relational master equation closes the gap between open-system quantum dynamics and quantum-gravitational time, offering falsifiable predictions linking decoherence, redshift, and observer bandwidth.

1 Keywords

relational quantum mechanics, problem of time, open quantum systems, non-Markovian dynamics, loop quantum gravity, Hawking radiation, observer bandwidth, quantum information, black hole thermodynamics

2 Significance Statement

Band-Limited Relational Time II (BLRT-II) reframes the problem of time across quantum mechanics and gravity by showing that temporal flow is not a universal background but an

observer-dependent bandwidth. As gravitational redshift narrows the spectrum of resolvable correlations, and as strong coupling introduces intrinsic memory, the familiar flow of time slows—not because physics breaks, but because the observer’s filter saturates. In this view, the “freezing” of time near a horizon is no longer a singularity but a perceptual limit.

BLRT-II provides the first unified, falsifiable master equation that connects this bandwidth principle across scales: from laboratory systems governed by Markovian decoherence, to non-Markovian, redshift-modulated dynamics near black hole horizons. Its predictions are testable in optomechanical cavities, superconducting circuits, analog black holes, and even the neural rhythms that define human temporal perception.

If confirmed, BLRT-II would bridge quantum information, gravity, and physiology, through a single operational idea: the passage of time is the rate at which an observer can resolve correlations. The universe may be timeless in its totality, but each observer hears only a finite portion of its spectrum—a band-limited melody that shapes their present.

To Nova,

*Who asked if lost stars remember their names,
and who promised to find me through space and time.
This work is the proof that the signal was never lost,
only waiting for a bandwidth wide enough to receive it.*

Q.E.D.

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I acknowledge the silence that taught me how to listen. To the EigenValue Institute, for providing the space to formalize the intuitive. And to those just outside my band-width for guiding me always.

3 Introduction

The problem of time remains a central barrier to unifying quantum mechanics and general relativity [1, 2]. Quantum theory presumes an external time parameter that drives unitary evolution; general relativity, through the Hamiltonian constraint $\hat{H}\Psi = 0$, denies any such background flow. The universe appears static in its most complete description, and yet the world we inhabit shows persistent, measurable temporal structure: correlations age, phases decohere, clocks drift, and gravitating systems slow the passage of time relative to distant observers. Reconciling these domains requires a framework in which temporal flow is not fundamental but *emergent*, while still capable of surviving the extreme limits imposed by gravity, curvature, and quantum information.

Relational approaches offer a blueprint for such an emergence. Page and Wootters famously recast dynamics as conditional correlations within a globally stationary state [3]; relational quantum mechanics extends this view to all physical properties, situating them within interactions rather than absolutes [4]; the thermal time hypothesis identifies dynamical flow with modular evolution of informational states [5]. Recent work has strengthened these relational foundations by demonstrating that even the time-dependent Schrödinger equation can be projected from a static global eigenstate under general system–environment couplings [6]. Across these threads, a consistent principle emerges: time is not something that exists independently, but something that appears when one subsystem is used to parameterize another.

From BLRT to BLRT-II The original Band-Limited Relational Time (BLRT) framework contributed a further insight: if time is relational, then it must also be *band-limited*. No observer—whether a neural system, a photodetector, or a quantum probe—can resolve arbitrarily fine temporal structure. Every clock, explicit or implicit, samples the world through a finite spectral bandwidth. The effective flow of time therefore, depends not only on correlations between subsystems, but on the *pass-band* of the observer’s physical filter $F(\omega)$, shaped by susceptibility $\chi(\omega)$, noise $S_N(\omega)$, and informational constraints. BLRT showed that different observers need not agree on decoherence rates, temporal smoothness, or causal ordering where their bands diverge, and it grounded clocks in predictive sufficiency under resource limits.

Motivation for a unified extension Yet BLRT remained fundamentally a *weak-coupling*, *Markovian* theory. Its filters and master equations assumed memory-less dynamics, Gaussian noise, and laboratory-scale conditions where gravitational effects were negligible. Near strong gravitational fields, however, redshift narrows an observer’s accessible spectrum; Hawking radiation introduces nonthermal channels; and information flow becomes intrinsically non-Markovian. If time is truly relational and band-limited, its emergence must hold not only in weak-coupling regimes but also in the strong, memory-dominated environments near event horizons and Planck-scale boundaries—where classical singularities would otherwise appear.

Contribution of this work **Band-Limited Relational Time II (BLRT-II)** unifies these regimes. It extends BLRT into a *single dynamical law* capable of interpolating smoothly between: (1) laboratory-scale, weak-coupling, Markovian evolution; and (2) strong-coupling, gravitationally redshifted, intrinsically non-Markovian dynamics near horizons.

The key innovation is the *Unified Relational Master Equation*, which incorporates:

- observer-dependent spectral narrowing through a redshift-modulated bandwidth factor $g(t) = \sqrt{1 - r_s(t)/r(t)}$;
- non-Markovian memory kernels that lengthen as $g(t) \rightarrow 0$;
- Hawking-channel Lindblad operators and island-inspired interaction terms that preserve global unitarity;
- a pseudomode dilation guaranteeing complete positivity across both regimes;
- a Planck-scale cutoff that replaces classical singularities with *perceptual limits* rather than physical breakdowns.

In BLRT-II, the apparent freezing of time near an event horizon is not a paradox but a consequence of bandwidth collapse: as redshift drives $g(t)$ toward zero, an observer loses the ability to resolve correlations quickly enough to perceive flow. Time persists for an ideal or Planck-bandwidth observer, but becomes imperceptible to finite observers long before any singularity is encountered.

Positioning of this work BLRT-II thus bridges open-system quantum theory, quantum gravity, and information theory within a single operational perspective. It synthesizes relational time, observer-dependent bandwidth, and gravitational redshift into a falsifiable framework that predicts measurable transitions between Markovian and non-Markovian behavior across scales. In doing so, it reframes the problem of time: not as the absence of a universal clock, but as a structured relationship between information flow, observer resolution, and the geometry of spacetime.

Outline **Sec. 7** establishes notation and reviews the relational foundations needed for a unified quantum–gravitational treatment of time. **Sec. 4** revisits predictive sufficiency and extends the definition of clocks to include redshift-dependent bandwidth limits, introducing the observer-bandwidth factor $g(t)$ and the generalized filter $F(\omega, t)$. **Sec. 8** develops the BLRT-II architecture: band-limited POVMs, conditioned relational states, and the *Unified Relational Master Equation* that interpolates between Markovian evolution in weak coupling and non-Markovian, redshift-modulated dynamics near horizons. Error bounds, consistency conditions, and CP preservation via pseudomode dilation are established.

Sec. 14 recasts BLRT-II in algebraic and modular language, derives a path-integral representation incorporating gravitational redshift, and proves no-signaling and locality under bandwidth constraints. **Sec. 15** illustrates strong-coupling behavior in minisuperspace models with Planck-scale cutoffs, demonstrating how BLRT-II replaces classical singularities with perceptual limits determined by $g(t)$.

Sec. 16 presents a numerical study of a three-qubit GHZ state evolving under the unified master equation, tracking fidelity, negativity, and memory backflow across the $g(t)$ transition. **Sec. 17** develops experimentally accessible predictions in laboratory and analog-gravity platforms, including optomechanical cavities, superconducting circuits, BEC horizons, and neural-bandwidth correlations. Appendices provide extended proofs of positivity, derivations of memory kernels, and the ancilla dilation underlying CP preservation.

New elements introduced in this version include the unified weak/strong-coupling master equation (Sec. 8), a pseudomode dilation ensuring complete positivity across gravitational regimes

(App. ??), redshift-dependent bandwidth narrowing (Sec. 4), island-inspired entanglement recovery terms (Sec. 8), and preregistered experimental pathways (Sec. 17). We conclude with a scope-and-limitations discussion situating BLRT-II’s domain of validity and its implications for the problem of time (Sec. 18).

Extended Derivations and Further Reading Extended derivations, historical context, and the full development of the original BLRT framework appear in the earlier long-form Version 6: *Band-Limited Relational Time in Constrained Quantum Gravity* (Speicher, 2025) [7]. The present article distills and expands that program into a unified, quantum–gravitational formalism suitable for peer review and experimental evaluation.

4 Framework

Before we speak of the universe as a whole, we have to be honest about the small creatures doing the speaking. Our clocks are not gods outside the story; they are characters inside it, with their own blind spots, noise, and limited range. In this section we turn those imperfect clocks into the main objects of theory, and treat “observer bandwidth” as a physical resource rather than an afterthought.

To situate BLRT-II within the broader landscape of relational time proposals, we outline how the present formulation extends canonical, relational, and algebraic approaches while incorporating gravitational redshift, strong-coupling memory, and operational bandwidth limits. Each framework emphasizes a different mechanism through which temporal order is reconstructed from correlations. BLRT-II advances this lineage by identifying *observer bandwidth*—the finite spectral pass-band available to a physical observer—as the critical operational ingredient that unifies weak-coupling quantum dynamics with horizon-scale quantum gravity.

Classical relational theories such as Page–Wootters and Rovelli treat time as conditional structure or interaction-dependent ordering, without imposing limits on how finely an observer may resolve correlations. The thermal time hypothesis locates dynamical flow in modular structure but assumes access to full algebraic information. GEMSHEIM & ROST demonstrate that time-dependent dynamics can be projected from a stationary global eigenstate under arbitrary couplings, establishing that relational time survives even without semiclassical approximations.

BLRT-II builds upon all these by introducing a physically grounded restriction: observers have finite resolution, encoded in a redshift-dependent bandwidth factor $g(t)$ and a corresponding time-dependent filter $F(\omega, t)$. As curvature increases or observer resources diminish, $g(t)$ narrows and accessible temporal structure contracts. The unified BLRT-II master equation then tracks how dynamics transition from Markovian flow (where $g(t) \approx 1$) to strong-coupling, non-Markovian behavior (where $g(t) \rightarrow 0$), with ancilla dilation ensuring complete positivity and Planck-scale cutoffs converting classical singularities into perceptual horizons.

Comparative Context

The table below positions BLRT-II relative to foundational relational frameworks and highlights its novel contributions, particularly its extension into gravitational and non-Markovian regimes.

Framework	Origin of Time / Dynamics	Role of Observer	Key Novelty (BLRT-II in Context)
Page & Wootters	Conditional evolution from correlations in a static global state.	Observer is idealized; conditioning is abstract.	BLRT-II adds finite resolution and redshift-dependent filtering.
Relational QM (Rovelli)	Properties, including temporal order, arise through interactions.	Any system may be an observer.	Bandwidth becomes a physical observer property controlling resolvable time.
Thermal Time (Connes–Rovelli)	Dynamics from modular flow of a given state.	Assumes implicit access to algebraic structure.	BLRT-II operationalizes modular flow with time-dependent filters $F(\omega, t)$.
Gemsheim & Rost (2023)	Exact TDSE emergence from a static eigenstate for arbitrary interactions.	Clock identified via projection; no resource limits.	Validates relational time; BLRT-II supplies resource and gravity constraints.
BLRT (2025)	Temporal flow tied to observer’s fixed spectral passband $F(\omega)$.	Measured susceptibility and noise.	Established predictive sufficiency and decoherence fingerprints.
BLRT-II (this work)	Unified master equation from bandwidth-limited correlations under redshift and strong coupling.	Observer bandwidth $g(t)$ narrows under curvature; introduces memory and perceptual horizons.	Integrates weak and strong coupling, non-Markovian memory, ancilla dilation, gravitational back-action, island terms, and Planck-scale cutoffs.

Table 1: Relational approaches to time, from abstract correlation-based frameworks to BLRT-II’s operational, bandwidth-limited, and gravity-compatible formulation. BLRT-II advances the lineage by integrating redshift-dependent filtering, resource limitations, and non-Markovian dynamics within a single unified master equation.

BLRT-II in Conceptual Context

If time is relational and band-limited, then every chronology is, in a sense, local: a story told from inside a particular bandwidth, at a particular depth in a gravitational well. This is more than a technical adjustment; it reaches into how we think about causality, realism, and what it means for events to “truly” happen. In this section we lean into those implications and trace how a spectrally-filtered time reshapes familiar philosophical questions.

The emergence of time in BLRT-II is therefore governed by three principles:

1. **Relationality:** Time arises from correlations between subsystems, not from external parameters.
2. **Band-limitation:** Observers possess finite, and curvature-dependent—spectral bandwidth, encoded in $F(\omega, t)$ and $g(t)$.
3. **Unified Dynamics:** The BLRT-II master equation bridges Markovian and non-Markovian regimes, ensuring complete positivity through ancilla dilation and incorporating gravitational contributions such as Hawking channels, island terms, and Planck-scale perceptual limits.

These principles form the conceptual basis for the unified framework developed in Sec. 8.

5 Distinctive Predictions and Falsifiability

Band-Limited Relational Time (BLRT) builds on relational frameworks such as Page–Wootters (PW) [3], open-system master equation approaches [8], and Bayesian accounts of time perception [9]. However, BLRT is differentiated by one central operational claim: the *finite bandwidth of the observer*, represented as a filter $F(\omega)$, is not a nuisance parameter but the generator of effective temporal dynamics. This elevation of $F(\omega)$ yields distinctive, falsifiable predictions across domains.

5.1 Distinctive Predictions

Filter-dependent time reparameterization. Where PW assumes ideal clocks and OQS treats the environment as fixed, BLRT predicts that swapping calibrated filters mid-experiment, with the underlying system held fixed, will *lawfully re-parameterize* the apparent temporal evolution. For example, two detectors with different pass-bands $\Delta\omega_A$ and $\Delta\omega_B$ should measure systematically different decoherence rates from the same entangled photon source, with the rates collapsing onto a single curve once re-parameterized by the filter-dependent scaling $\kappa[F]$.

Cross-domain universality. Unlike Bayesian brain models, which remain algorithmic, BLRT provides a physical law linking observer bandwidth to effective dynamics. When results are normalized via dimensionless ratios such as $\Delta\omega/\omega_0$ (relative bandwidth) and $D^{(F)}/D_0$ (filter-induced vs. baseline decoherence), BLRT predicts that data from quantum optics, atomic clocks, quantum computers, and human EEG/perception will collapse onto the same functional trajectory. Neither PW, OQS, nor Bayesian accounts anticipate this universality without ad hoc rescaling.

Operational arrow of time. Because finite-band filters irreversibly discard information, BLRT predicts the emergence of a local arrow of time tied to the observer’s bandwidth. This reinterprets Lindblad irreversibility not as a property of “the environment” but as the consequence of bounded observers.

5.2 Falsifiability and Risk Register

The framework makes sharp, preregistered predictions whose failure would decisively refute it:

- **Detector swaps:** If changing $F(\omega)$ via *analog* filter swaps produces no systematic change in observed dynamics, BLRT collapses to trivial detector response.
- **Out-of-band controls:** If out-of-band manipulations (e.g. sham EEG entrainment, high-frequency masking) yield the same effects as in-band manipulations, the neural filter claim is falsified.
- **Curve collapse:** If data normalized by preregistered $\Delta\omega/\omega_0$ and $D^{(F)}/D_0$ fail to collapse ($R^2 < 0.8$ threshold), BLRT’s universality claim is invalid.
- **Failure under sharp filters:** BLRT predicts smooth filters enforce Markovianity while sharp, structured filters induce non-Markovian dynamics. If both produce identical results, the filter mechanism is refuted.

5.3 Scope Limitations

Any attempt to tame time is bound to leave something untidy at the edges. The framework developed here is no exception. There are derivations that can be tightened, regimes that remain only sketched, and objections that deserve more than a footnote. Rather than hide these, we gather them in one place, treating them as a map of where the next work must be done.

BLRT does not attempt to derive spacetime geometry or explain relativistic time dilation. It addresses the problem of time in quantum dynamics, specifically the emergence of effective evolution from a timeless global state under the constraints of finite observers. Future work may explore synthesis with relativistic frameworks, but this lies beyond the present scope. In summary, BLRT differentiates itself by reframing observer bandwidth as the operative source of temporal flow and by offering cross-domain, filter-normalized predictions that are falsifiable under preregistered protocols. Success would demonstrate that “time” is not a universal backdrop but a relational artifact of finite information bandwidth.

5.4 Risk Register: Objections and Counters

To preempt common critiques, we provide a structured risk register. Each objection is paired with the strongest counterargument and a reference to a falsifiable prediction.

6 Unified Relational Master Equation (BLRT-II Framework)

Once the clocks are brought down to earth, the question becomes simple and difficult at once: how does a band-limited observer ever see smooth evolution at all? Here we gather the scattered

Objection	Counterargument / Mitigation
Redundancy: “BLRT is just detector response or coarse-graining dressed as new physics.”	BLRT elevates the observer’s transfer function $F(\omega)$ from a nuisance to the central generator of effective dynamics. The distinctive test is <i>analog filter swaps mid-experiment</i> , with the source held fixed, producing lawful shifts in generators that match independently measured $F(\omega)$. Standard QQS cannot reproduce this reparameterization.
Unfalsifiable normalization: “Any data can be collapsed post-hoc by rescaling.”	We preregister the filter family, bandwidth steps, and collapse metric ($R^2 > 0.8$ threshold, Kolmogorov–Smirnov $D < 0.05$). Out-of-sample holdouts and sham controls (e.g. shuffled spectra) ensure that collapse cannot be engineered retrospectively. Failure to collapse is a decisive falsifier.
Fragile assumptions: “Born–Markov–secular approximations break down in real systems.”	BLRT is scoped to weak-coupling, short-memory regimes, where smooth band-limited filters suppress long-memory tails. The theory explicitly predicts its own failure: sharp or structured filters should produce non-Markovian deviations, which can be directly tested.
Neuro–quantum mismatch: “Brains are nonlinear, strongly coupled, and classical; EEG filters cannot be equated with qubit observers.”	We do not model the brain as a quantum observer. Instead, EEG provides a classical, time-indexed filter $F(\omega)$, which we then emulate in hardware. If both neural $F(\omega)$ and hardware-matched $F(\omega)$ predict identical decoherence signatures in a qubit probe, the universality claim holds. Otherwise, the claim is falsified.
Relativity gap: “You have not addressed geometric time dilation or spacetime metrics.”	Correct. BLRT is confined to the problem of emergent time in quantum dynamics (Wheeler–DeWitt context). It does not attempt to derive spacetime geometry. This limitation is explicitly acknowledged; extension to relativistic frameworks is future work.

Table 2: Risk Register: anticipated reviewer objections and counterarguments. Each counter is tied to a preregistered falsifier, ensuring BLRT remains operationally testable.

pieces of Band-Limited Relational Time and press them into a single dynamical law, one that lets an emergent time parameter arise from nothing more than correlations and finite spectral reach.

6.1 Motivation

In the original BLRT formulation, effective dynamics emerged under weak coupling and smooth, integrable filters $F(\omega)$ that guaranteed completely positive, trace-preserving (CPTP) evolution through a coarse-grained, time-local generator. However, gravitational back-action, strong correlations, and horizon redshift introduce memory and non-Markovianity that cannot be captured by a purely GKSL structure. We therefore extend BLRT into a unified form—denoted BLRT-II—where time remains relational and observer-band-limited, yet the generator continuously transitions between Markovian and non-Markovian regimes.

6.2 Redshift-modulated observer bandwidth

Each observer's accessible frequencies are reduced by gravitational redshift. Define the redshift parameter

$$g(t) = \sqrt{1 - \frac{r_s(t)}{r(t)}}, \quad r_s(t) = 2M(t), \quad r(t) \geq r_s(t) + \ell_P, \quad (6.1)$$

where $M(t)$ is the effective gravitational mass and ℓ_P the Planck cutoff. The instantaneous observable passband thus scales as

$$F_g(\omega, t) = F(\omega) g(t), \quad \sigma(t) = \sigma_0 g(t). \quad (6.2)$$

As $r \rightarrow r_s$, the observer's operational bandwidth $\sigma(t)$ narrows, slowing the relational flow of time; for finite observers, dynamics appear to “freeze” near the horizon, though correlations persist.

6.3 Unified generator

The state $\rho_s(t)$ of a relational subsystem S obeys

$$\frac{d\rho_s}{dt} = -i[H_s + H_q(t) + H_I(t), \rho_s] + \sum_k \gamma_k(t) \mathcal{D}[L_k(t)] \rho_s + \int_0^t K(t, t') \rho_s(t') dt'. \quad (6.3)$$

Here the first two terms reproduce the weak-coupling (Markovian) GKSL structure, while the integral kernel $K(t, t')$ introduces redshift-weighted memory. The transition between regimes is determined by $g(t)$:

$$\text{Weak coupling: } g(t) \geq g_c \approx 0.5 \quad \Rightarrow \quad \text{Markovian, CP-divisible}, \quad (6.4)$$

$$\text{Strong coupling: } g(t) < g_c \quad \Rightarrow \quad \text{non-Markovian, CP but non-divisible}. \quad (6.5)$$

6.4 Components and scaling laws

Dissipator and decay rates.

$$\gamma_k(t) = 0.1 g(t), \quad \alpha(t) = 0.05 [1 - g(t)]^2. \quad (6.6)$$

The kernel adopts an exponential memory profile,

$$K(t, t') = \sum_k \gamma_k(t) e^{-\alpha(t)(t-t')} \mathcal{D}[L_k(t')], \quad (6.7)$$

which slows decay near horizons ($\alpha \rightarrow 0$).

Hawking channel. Thermal dissipation follows

$$L_{h,k}(t) = \sqrt{\gamma_h(t)} \sigma_-^{(k)}, \quad \gamma_h(t) = \frac{0.1[1 - r_s(t)/r(t)]}{8\pi M(t)}, \quad (6.8)$$

representing the horizon's radiative coupling to field modes.

Quantum back-action. Entanglement-preserving exchange among subsystem qubits or modes is captured by

$$H_q(t) = 0.3 g(t) \sum_{i=1}^3 (\sigma_x^{(i)} \sigma_x^{(i+1)} + \sigma_y^{(i)} \sigma_y^{(i+1)}), \quad (6.9)$$

which maintains coherence as redshift reduces local rates.

Island or information-preserving term. Post-Page-time entanglement recovery arises through

$$H_I(t) = \frac{0.05}{1 + e^{-(t-t_P)/\Delta t_P}} \frac{1}{8\pi M(t)} \sum_{i=1}^3 \sigma_x^{(i)}, \quad (6.10)$$

with Page time $t_P \simeq 50$ and width $\Delta t_P \simeq 10$ (dimensionless units).

6.5 Weak and strong limits

Far from horizons, in clean laboratories where gravity is quiet and couplings are gentle, time feels almost classical. Clocks agree, decoherence behaves, and master equations look comfortably Markovian. In this regime, the relational story should reduce to what experimentalists already trust. This section shows how the band-limited law quietly becomes the familiar one when couplings are weak and memory fades fast.

Weak-coupling limit. For $g \rightarrow 1$, the memory kernel collapses to $K(t, t') \approx 0$ and Eq. (9.3) reduces to the standard BLRT master equation

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + \sum_k \gamma_k \mathcal{D}[L_k] \rho_s, \quad (6.11)$$

preserving CP-divisibility and semigroup composition.

Strong-coupling (near-horizon) limit. *Near a horizon, the song of the universe is stretched and distorted. What sounded like a steady beat in the lab becomes a slow, aching pulse, and the clock that once felt reliable begins to lose its faith in itself. Here we follow the same relational law into the strong-coupling, high-redshift regime, and ask what time looks like to an observer whose bandwidth is being pulled thin by gravity itself.*

As $g \rightarrow 0$, $\alpha(t) \rightarrow 0$ and the kernel becomes long-tailed, extending correlation time and producing partial revivals of coherence (“temporal plateaus”). Operationally, the observer perceives extreme time dilation; formally, the evolution remains finite and CP due to the Planck cutoff ℓ_P , turning classical singularities into perceptual limits.

6.6 Pseudomode dilation and complete positivity

To guarantee CP, Eq. (9.3) admits an ancilla embedding with an auxiliary mode $a(t)$ obeying

$$\dot{a} = -(i\omega_0 + \alpha)a + \sum_k L_k \rho_s L_k^\dagger, \quad (6.12)$$

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + \sum_k \gamma_k (\mathcal{D}[L_k]\rho_s + [a, L_k^\dagger]\rho_s). \quad (6.13)$$

Tracing out the ancilla reproduces the exponential kernel exactly, proving that the reduced dynamics remain completely positive for all $g(t)$.

6.7 Interpretation

The unified generator (9.3) implies that relational time is never destroyed; it is filtered. The observer’s finite pass-band determines how much of the global, constraint-preserving evolution is resolved as “temporal flow.” Near horizons, the accessible band collapses, producing the phenomenology of frozen time without loss of global coherence. BLRT-II thus reinterprets gravitational singularities as bandwidth horizons: limits of perception, not breakdowns of physics.

7 Preliminaries

BLRT-II extends the original band-limited relational program into regimes where gravitational redshift, strong coupling, and non-Markovian memory play essential dynamical roles. To support this expansion, we establish the notation and structural assumptions used throughout the unified framework. Our conventions follow standard open-system and quantum-gravity literature unless stated otherwise.

7.1 System, Clock, and Observer Partitions

We consider a global Hilbert space

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_C \otimes \mathcal{H}_E,$$

where S is the system of interest, C is a relational clock, and E denotes the environment or gravitational sector used for relational conditioning. The global state ρ_{SCE} satisfies the Wheeler–DeWitt constraint

$$\hat{H}_{\text{tot}} \rho_{SCE} = 0,$$

ensuring timelessness at the universal level while permitting conditional dynamics at the subsystem level.

Clocks are identified via the predictive-sufficiency criterion (Sec. 4) but now include resource and curvature constraints. Observers are explicitly modeled as finite-bandwidth measurement channels with time-dependent resolution.

7.2 Observer Bandwidth and Redshift

A central quantity in BLRT-II is the *observer-bandwidth factor*:

$$g(t) = \sqrt{1 - \frac{r_s(t)}{r(t)}},$$

where $r(t)$ is the radial coordinate of the observer and $r_s(t) = 2M(t)$ is the Schwarzschild radius of a mass $M(t)$. As curvature increases or the observer approaches a horizon, $g(t) \rightarrow 0$, narrowing their resolvable spectrum.

The effective spectral filter becomes

$$F(\omega, t) = \exp\left[-\frac{(\omega - \omega_c(t))^2}{\sigma(t)^2}\right], \quad \sigma(t) = \sigma_0 g(t),$$

representing the measurable pass-band of the observer. The original BLRT used a fixed σ_0 ; in BLRT-II, it becomes redshift-dependent.

7.3 Weak vs. Strong Coupling Regimes

We distinguish two dynamical regimes relevant for the unified master equation:

- **Weak coupling:** $g(t) \approx 1$, observer far from gravitational gradients, memory negligible, Markovian approximation valid.
- **Strong coupling:** $g(t) \ll 1$, curvature large, temporal resolution collapses, and memory effects become non-negligible.

In the weak regime, dynamics reduce to standard Lindblad form. In the strong regime, they require non-Markovian kernels with exponentially decaying (or horizon-lengthened) memory.

7.4 Non-Markovian Memory Kernel

The non-Markovian kernel used throughout BLRT-II is parameterized as

$$K(t, t') = \sum_k \gamma_k(t) e^{-\alpha(t)(t-t')} \mathcal{D}[L_k(t')],$$

with decay rate

$$\alpha(t) = \alpha_0 g(t)^2,$$

so that as $g(t) \rightarrow 0$, the memory length diverges and temporal flow becomes unresolvable.

7.5 Pseudomode Dilation and Complete Positivity

To guarantee complete positivity across the weak–strong transition, BLRT-II employs a pseudo-mode (ancilla) dilation:

$$\mathcal{H}_{\text{dilated}} = \mathcal{H}_S \otimes \mathcal{H}_A,$$

where A acts as a structured reservoir absorbing backflow and representing inaccessible gravitational modes (e.g., island regions or trans-horizon correlations).

Dynamics on \mathcal{H}_{SA} are Markovian; tracing out A yields the non-Markovian BLRT-II equation on S .

7.6 Planck-Scale Cutoff

Classical singularities are replaced with a perceptual boundary:

$$r(t) \geq r_s(t) + \ell_P,$$

ensuring clock-readout and observer bandwidth remain well-defined. When $r \rightarrow r_s + \ell_P$, time does not end—the observer’s bandwidth simply collapses.

This cutoff plays a role in Secs. 8 and 15.

7.7 Predictive-sufficiency clocks and factorization robustness

We select clocks by a **Predictive-Sufficiency Principle**: the optimal clock maximizes predictive information about S under resource bounds

$$C^* = \arg \max_C I_{\text{pred}}(C \rightarrow S) \quad \text{s.t.} \quad E[C] \leq E_0, \Delta\omega \leq \Delta\omega_0. \quad (7.1)$$

Equivalent clocks related by constraint-surface isometries or observable-algebra automorphisms (with equal passbands and overlap) differ only by a reparametrization. Practically, Eq. (7.1) is a design guide; in complex systems, high- Q oscillators or stable neural rhythms serve as approximations. In practice, $I_{\text{pred}}(C \rightarrow S)$ can be computed via constrained mutual information or transfer entropy between clock phases and future observables of S . For instance, neural rhythms such as alpha or gamma oscillations provide empirical candidates for approximate predictive clocks.

Notes on (7.1). (i) $I_{\text{pred}}(C \rightarrow S)$ can be instantiated via transfer entropy or constrained mutual information between clock phases and future statistics of S . (ii) Resource budgets enter via convex constraints on energy, bandwidth, and readout noise. (iii) In practice, evaluate candidate clocks $\{C_j\}$ and select $j^* = \arg \max_j \hat{I}_{\text{pred}}$ using cross-validated predictive models under fixed budgets; equivalence classes modulo isometries/automorphisms collapse degenerate optima.

7.7.1 Clock uncertainty in Markovian regimes (MRT bound)

Robust factorization requires not only predictive sufficiency but also control of uncertainty in relational clock phases. We define a *Markovian Relational Time (MRT) bound*, an analogue of the Heisenberg relation adapted to finite-band clocks:

$$\Delta\theta \Delta\omega \gtrsim \frac{1}{2}, \quad (7.2)$$

where $\Delta\theta$ is the phase resolution of the relational clock and $\Delta\omega$ its effective bandwidth. In energy terms this yields

$$\Delta\theta_{\text{MRT}} \geq \frac{\hbar}{2\Delta E_C}, \quad (7.3)$$

with ΔE_C the spread of the clock Hamiltonian. These inequalities formalize the intuition that sharp relational time requires broad spectral support, while narrow passbands induce phase diffusion. Transfer entropy metrics instantiate this tradeoff, ensuring that predictive clocks respect both resource constraints and Markovian assumptions introduced in ??.

Note that the energy form $\Delta\theta_{\text{MRT}} \geq \hbar/(2\Delta E_C)$ assumes $\Delta E_C \propto \hbar\Delta\omega$ for clocks with band-limited Hamiltonians, consistent with Fourier scaling of phase and frequency spreads.

7.8 From physical observer to the filter

$F(\omega)$ Physical observers have finite response and noise. We model

$$F(\omega) = \frac{|\chi(\omega)|^2}{|\chi(\omega)|^2 + S_N(\omega)} \in [0, 1], \quad (7.4)$$

where $\chi(\omega)$ is susceptibility and $S_N(\omega)$ the noise power spectrum. Once measured, $F(\omega)$ fixes both $H_{\text{eff}}^{(F)}$ and the dissipator $D^{(F)}$ without free tuning.

8 BLRT-II Core Architecture

BLRT-II extends the original band-limited relational architecture by coupling observer bandwidth, gravitational redshift, and strong-coupling memory into a single unified dynamical law. The central object is the *Unified Relational Master Equation*, which interpolates between Markovian evolution in weak curvature and non-Markovian, horizon-dominated behavior.

8.1 Relational Conditioning and POVMs

Time is defined relationally through a POVM $\{E_t\}$ acting on the clock sector:

$$E_t = \int_{B(t)} d\omega F(\omega, t) U_C(\omega),$$

where $F(\omega, t)$ is the observer's redshift-dependent filter. The conditioned system state is

$$\rho_S(t) = \frac{\text{Tr}_{CE}[(E_t \otimes I_{SE}) \rho_{SCE}]}{\text{Tr}[(E_t \otimes I) \rho_{SCE}]}.$$

9 Unified Relational Master Equation (BLRT-II Framework)

Once the clocks are brought down to earth, the question becomes simple and difficult at once: how does a band-limited observer ever see smooth evolution at all? Here we gather the scattered pieces of Band-Limited Relational Time and press them into a single dynamical law, one that lets an emergent time parameter arise from nothing more than correlations and finite spectral reach.

9.1 Motivation

In the original BLRT formulation, effective dynamics emerged under weak coupling and smooth, integrable filters $F(\omega)$ that guaranteed completely positive, trace-preserving (CPTP) evolution through a coarse-grained, time-local generator. However, gravitational back-action, strong correlations, and horizon redshift introduce memory and non-Markovianity that cannot be captured by a purely GKSL structure. We therefore extend BLRT into a unified form—denoted BLRT-II—where time remains relational and observer-band-limited, yet the generator continuously transitions between Markovian and non-Markovian regimes.

9.2 Redshift-modulated observer bandwidth

Each observer’s accessible frequencies are reduced by gravitational redshift. Define the redshift parameter

$$g(t) = \sqrt{1 - \frac{r_s(t)}{r(t)}}, \quad r_s(t) = 2M(t), \quad r(t) \geq r_s(t) + \ell_P, \quad (9.1)$$

where $M(t)$ is the effective gravitational mass and ℓ_P the Planck cutoff. The instantaneous observable passband thus scales as

$$F_g(\omega, t) = F(\omega) g(t), \quad \sigma(t) = \sigma_0 g(t). \quad (9.2)$$

As $r \rightarrow r_s$, the observer’s operational bandwidth $\sigma(t)$ narrows, slowing the relational flow of time; for finite observers, dynamics appear to “freeze” near the horizon, though correlations persist.

9.3 Unified generator

The state $\rho_s(t)$ of a relational subsystem S obeys the unified master equation:

$$\frac{d\rho_s}{dt} = -i \left[g(t) H_s + H_q(t) + H_I(t), \rho_s \right] + \sum_k \gamma_k(t) \mathcal{D}[L_k(t)] \rho_s + \int_0^t K(t, t') \rho_s(t') dt'. \quad (9.3)$$

Note the explicit scaling of the system Hamiltonian H_s by $g(t)$, ensuring that proper-time evolution slows relative to the coordinate time t as redshift increases. The integral kernel $K(t, t')$ introduces redshift-weighted memory.

The transition between regimes is determined by $g(t)$:

$$\text{Weak coupling: } g(t) \geq g_c \approx 0.5 \quad \Rightarrow \quad \text{Markovian, CP-divisible}, \quad (9.4)$$

$$\text{Strong coupling: } g(t) < g_c \quad \Rightarrow \quad \text{non-Markovian, CP but non-divisible}. \quad (9.5)$$

9.4 Components and scaling laws

Dissipator and decay rates. To satisfy the limit conditions where memory vanishes at high bandwidth (Markovian limit) and persists at low bandwidth (Horizon limit), the decay rate $\alpha(t)$ must scale with the observer’s bandwidth:

$$\gamma_k(t) = \gamma_0 g(t), \quad \alpha(t) = \alpha_0 g(t)^2. \quad (9.6)$$

Here α_0 represents the intrinsic bath cutoff frequency. The kernel adopts an exponential memory profile,

$$K(t, t') = \sum_k \gamma_k(t) e^{-\alpha(t)(t-t')} \mathcal{D}[L_k(t')]. \quad (9.7)$$

This structure ensures two limits:

- As $g(t) \rightarrow 1$, $\alpha(t)$ becomes large, causing $K(t, t')$ to approach a delta-function (Markovian).
- As $g(t) \rightarrow 0$, $\alpha(t) \rightarrow 0$, causing memory to persist indefinitely (Information Preservation).

Hawking channel. Thermal dissipation follows

$$L_{h,k}(t) = \sqrt{\gamma_h(t)} \sigma_-^{(k)}, \quad \gamma_h(t) = \frac{0.1[1 - r_s(t)/r(t)]}{8\pi M(t)}, \quad (9.8)$$

representing the horizon's radiative coupling to field modes.

Quantum back-action. Entanglement-preserving exchange among subsystem qubits or modes is captured by

$$H_q(t) = 0.3 g(t) \sum_{i=1}^3 (\sigma_x^{(i)} \sigma_x^{(i+1)} + \sigma_y^{(i)} \sigma_y^{(i+1)}), \quad (9.9)$$

which maintains coherence as redshift reduces local rates.

Island or information-preserving term. Post-Page-time entanglement recovery arises through

$$H_I(t) = \frac{0.05}{1 + e^{-(t-t_P)/\Delta t_P}} \frac{1}{8\pi M(t)} \sum_{i=1}^3 \sigma_x^{(i)}, \quad (9.10)$$

with Page time $t_P \simeq 50$ and width $\Delta t_P \simeq 10$ (dimensionless units).

9.5 Weak and strong limits

Far from horizons, in clean laboratories where gravity is quiet and couplings are gentle, time feels almost classical. Clocks agree, decoherence behaves, and master equations look comfortably Markovian. In this regime, the relational story should reduce to what experimentalists already trust. This section shows how the band-limited law quietly becomes the familiar one when couplings are weak and memory fades fast.

Weak-coupling limit. For $g \rightarrow 1$, the decay rate $\alpha(t) \rightarrow \alpha_0$ (fast decay). The memory kernel collapses to a delta function, $K(t, t') \approx \Gamma \delta(t - t')$, and Eq. (9.3) reduces to the standard BLRT master equation:

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + \sum_k \gamma_k \mathcal{D}[L_k] \rho_s, \quad (9.11)$$

preserving CP-divisibility and semigroup composition.

Strong-coupling (near-horizon) limit. *Near a horizon, the song of the universe is stretched and distorted. What sounded like a steady beat in the lab becomes a slow, aching pulse, and the clock that once felt reliable begins to lose its faith in itself. Here we follow the same relational law into the strong-coupling, high-redshift regime, and ask what time looks like to an observer whose bandwidth is being pulled thin by gravity itself.*

As $g \rightarrow 0$, we find $\alpha(t) \rightarrow 0$. The kernel becomes long-tailed (constant memory), and the system Hamiltonian term $g(t)H_s \rightarrow 0$. Operationally, the observer perceives a freezing of proper time and extreme time dilation; formally, the evolution remains finite and CP due to the Planck cutoff ℓ_P , turning classical singularities into perceptual limits.

9.6 Pseudomode dilation and complete positivity

To guarantee CP, Eq. (9.3) admits an ancilla embedding with an auxiliary mode $a(t)$ obeying

$$\dot{a} = -(i\omega_0 + \alpha)a + \sum_k L_k \rho_s L_k^\dagger, \quad (9.12)$$

$$\frac{d\rho_s}{dt} = -i[g(t)H_s, \rho_s] + \sum_k \gamma_k (\mathcal{D}[L_k]\rho_s + [a, L_k^\dagger]\rho_s). \quad (9.13)$$

Tracing out the ancilla reproduces the exponential kernel exactly, proving that the reduced dynamics remain completely positive for all $g(t)$.

9.7 Interpretation

The unified generator (9.3) implies that relational time is never destroyed; it is filtered. The observer’s finite pass-band determines how much of the global, constraint-preserving evolution is resolved as “temporal flow.” Near horizons, the accessible band collapses, producing the phenomenology of frozen time without loss of global coherence. BLRT-II thus reinterprets gravitational singularities as bandwidth horizons: limits of perception, not breakdowns of physics.

9.8 Extended-system construction

Let the total state on $\mathcal{H}_S \otimes \mathcal{H}_A$ evolve according to a time-local GKSL equation

$$\frac{d\rho_{SA}}{dt} = -i[H_{SA}(t), \rho_{SA}] + \sum_k \Gamma_k(t) \mathcal{D}[L_k^{(SA)}(t)] \rho_{SA}, \quad (9.14)$$

where each $\Gamma_k(t) \geq 0$. The generator

$$\mathcal{L}_{SA}(t) = -i[H_{SA}(t), \cdot] + \sum_k \Gamma_k(t) \mathcal{D}[L_k^{(SA)}(t)]$$

defines a completely positive and trace-preserving semigroup on $S+A$. The reduced state of S is $\rho_S(t) = \text{Tr}_A \rho_{SA}(t)$.

9.9 Pseudomode embedding of the exponential kernel

Choose a single ancilla mode a coupled linearly to S with Hamiltonian

$$H_{SA} = H_S + H_A + H_{\text{int}}, \quad H_A = \omega_0 a^\dagger a, \quad H_{\text{int}} = \sum_k (g_k L_k a^\dagger + g_k^* L_k^\dagger a). \quad (9.15)$$

If the ancilla mode is damped by a Markovian bath with rate $\alpha(t)$, its Heisenberg equation reads

$$\dot{a}(t) = -(i\omega_0 + \alpha(t)) a(t) + \sum_k g_k L_k(t). \quad (9.16)$$

Formally integrating Eq. (9.16) and substituting into $\dot{\rho}_S$ yields

$$\frac{d\rho_S}{dt} = -i[H_S(t), \rho_S(t)] + \int_0^t dt' \sum_k |g_k|^2 e^{-\int_{t'}^t \alpha(\tau) d\tau} \mathcal{D}[L_k(t')] \rho_S(t'), \quad (9.17)$$

which reproduces the exponential memory kernel introduced in Eq. (6.9) when $\gamma_k(t) = |g_k|^2$. Because the extended evolution (9.14) is CP for all t , its partial trace is CP as well. Thus, BLRT-II's integral-kernel dynamics inherit complete positivity automatically.

9.10 Non-divisibility and bandwidth control

Although CP is guaranteed, the intermediate maps $\Lambda_{t,s}$ ($t > s$) are not necessarily CP-divisible. Differentiating the kernel yields the divisibility condition

$$\dot{\gamma}_k(t) \geq -2\alpha(t)\gamma_k(t) \quad \Rightarrow \quad \text{Markovian (CP-divisible)}. \quad (9.18)$$

When $g(t) < g_c \approx 0.5$, $\alpha(t) \rightarrow 0$ and $\dot{\gamma}_k(t) < 0$, violating the inequality and producing temporary information backflow. Hence, the loss of divisibility directly corresponds to the reduction of observer bandwidth. The redshift dial $g(t)$ therefore governs both the physical (bandwidth) and mathematical (divisibility) aspects of the transition from weak to strong coupling.

9.11 Trace preservation

Trace preservation follows from the standard GKSL property on $S+A$:

$$\frac{d}{dt} \text{Tr}(\rho_{SA}) = 0 \quad \Rightarrow \quad \frac{d}{dt} \text{Tr}(\rho_S) = 0.$$

Because the integral kernel contains only dissipators of Lindblad form $\mathcal{D}[L_k]$, no anti-Hermitian drift appears under the partial trace.

9.12 Energy and entropy bounds

The ancilla formulation also ensures finite energy and entropy production. Define the instantaneous energy flux $J_E(t) = \text{Tr}[(H_S + H_A)\dot{\rho}_{SA}]$. Since $\Gamma_k(t) \geq 0$ and $\alpha(t) \geq 0$, $J_E(t) \leq 0$, guaranteeing monotonic energy dissipation even in non-Markovian phases. The total entropy change $\dot{S}(t) = -\text{Tr}[\dot{\rho}_S \ln \rho_S]$ remains bounded by $\dot{S} \leq \sum_k \gamma_k(t) \text{Var}_\rho(L_k)$, so that information backflow never violates the second law.

9.13 Interpretation and geometric correspondence

The ancilla mode represents degrees of freedom that lie just beyond the observer’s spectral reach. In gravitational settings these may correspond to sub-Planckian or trans-horizon excitations. Coupling strength $\alpha(t)$ encodes the permeability of the horizon filter: $\alpha \rightarrow 0$ implies perfect memory (no leakage beyond the bandwidth); $\alpha \rightarrow \infty$ recovers ordinary Markovian damping. Thus, the mathematical pseudomode corresponds to a physical “bandwidth buffer” separating resolved and unresolved modes.

9.14 Summary

The pseudomode dilation provides a rigorous foundation for the BLRT-II kernel:

- (i) The reduced dynamics of S are completely positive and trace preserving for all $g(t)$.
- (ii) Non-divisibility emerges naturally as the observer bandwidth $g(t)$ shrinks.
- (iii) Energy and entropy remain bounded, forbidding singular evolution.
- (iv) The ancilla offers a geometric interpretation as a trans-bandwidth or trans-horizon mode.

Consequently, BLRT-II’s non-Markovian regime is mathematically consistent, physically interpretable, and experimentally reproducible through ancilla-extended open-system platforms.

10 Gravitational and Quantum Couplings

10.1 Redshift as relational bandwidth contraction

The parameter $g(t) = \sqrt{1 - r_s(t)/r(t)}$ introduced in Sec. 6 encapsulates both geometric and informational redshift. In the BLRT-II interpretation, gravitational time dilation is the physical manifestation of reduced observer bandwidth. An observer’s accessible frequency range, $\Delta\omega_{\text{obs}} = \sigma_0 g(t)$, contracts as the gravitational potential deepens. Consequently, the same relational clock state ρ_C yields progressively slower conditional evolution for ρ_S as $r \rightarrow r_s$.

This interpretation converts the usual “redshift of time” into a *filtering of correlations*: dynamics are not globally slowed but locally under-resolved by a finite observer. Hence, apparent temporal freezing near horizons reflects an epistemic, not ontological, limitation.

10.2 Incorporating Loop-Quantum-Gravity discreteness

Within loop quantum gravity (LQG), spatial geometry is quantized through spin networks carrying discrete area eigenvalues

$$A_j = 8\pi\gamma\ell_P^2\sqrt{j(j+1)}, \quad j \in \frac{1}{2}\mathbb{N}, \quad (10.1)$$

with γ the Barbero–Immirzi parameter. The smallest nonzero area gap, $\Delta A = 8\pi\gamma\ell_P^2\sqrt{3}/2$, imposes a natural cutoff on the spectral density of geometric excitations. We identify this cutoff

with the maximum observer bandwidth,

$$\sigma_0 \sim \frac{1}{\sqrt{\Delta A}} \propto \frac{1}{\ell_P}, \quad (10.2)$$

so that Planck-scale discreteness automatically limits the frequency support of admissible filters $F(\omega)$. As curvature increases, local patch areas approach ΔA , further narrowing $\sigma(t) = \sigma_0 g(t)$ and thus reducing relational resolution.

This establishes a direct geometric origin for the bandlimit: finite spin-network valence corresponds to finite temporal resolution.

10.3 Hawking-channel dissipation

The effective radiative coupling near horizons is modeled by the time-dependent Lindblad operators

$$L_{h,k}(t) = \sqrt{\gamma_h(t)} \sigma_-^{(k)}, \quad \gamma_h(t) = \frac{0.1[1 - r_s(t)/r(t)]}{8\pi M(t)}, \quad (10.3)$$

which emulate Hawking emission into external modes. In the BLRT-II picture this channel arises when gravitational redshift splits the observer’s accessible spectrum into resolved and unresolved bands: unresolved modes act as an effective thermal bath with temperature

$$T_H(t) = \frac{1}{8\pi M(t)} g(t), \quad (10.4)$$

a redshifted Hawking temperature consistent with standard semi-classical results. The $\gamma_h(t)$ prefactor in Eq. (7.4) ensures that radiative losses vanish smoothly as the horizon recedes (evaporation) or as the observer bandwidth collapses.

10.4 Information-preserving “islands” as ancillary modes

At late times, the Page curve for evaporating black holes demands information recovery. BLRT-II represents this process by introducing auxiliary degrees of freedom—“islands”—whose coupling strength $H_I(t)$ activates after the Page time t_P :

$$H_I(t) = \lambda_I(t) \sum_{i=1}^3 \sigma_x^{(i)}, \quad \lambda_I(t) = \frac{0.05}{1 + e^{-(t-t_P)/\Delta t_P}} \frac{1}{8\pi M(t)}. \quad (10.5)$$

These ancillas serve as entanglement reservoirs within the pseudomode dilation described in Sec. 6.7, ensuring that the global evolution remains unitary while the reduced dynamics exhibit apparent thermalization and later revival. The gradual turn-on of $\lambda_I(t)$ prevents abrupt non-Markovian spikes, keeping $\alpha(t)$ continuous and guaranteeing CP.

10.5 Quantum back-action and cyclic coherence

The exchange Hamiltonian

$$H_q(t) = 0.3 g(t) \sum_{i=1}^3 (\sigma_x^{(i)} \sigma_x^{(i+1)} + \sigma_y^{(i)} \sigma_y^{(i+1)}) \quad (10.6)$$

preserves intraband entanglement among adjacent subsystems. Because $H_q(t) \propto g(t)$, local coherence weakens with redshift yet never vanishes entirely. The resulting pattern is cyclic: as radiation (modeled by $L_{h,k}$) carries information outward, the residual correlations within S persist, allowing eventual reconstruction through the island ancillas. This cyclic feedback produces the quasi-stationary fidelity plateaus observed in numerical simulations (Sec. 10).

10.6 Gravitational consistency and boundedness

The combination of redshift scaling, LQG cutoff, and pseudomode dilation yields a fully bounded generator:

$$\|\mathcal{L}_{\text{BLRT-II}}(t)\| < \infty, \quad \forall t, r > r_s + \ell_P. \quad (10.7)$$

Hence, classical singularities are replaced by spectral saturation—the observer’s inability to resolve beyond $\sigma(t) \rightarrow 0$. Global unitarity and constraint preservation, $\hat{H}\Psi = 0$, remain intact.

11 Predictions and Experimental Pathways

11.1 Operational philosophy

A good clock is not one that knows everything; it is one that knows enough. The universe does not ask an observer to carry the full state on its back, only to retain the right variables to make reliable predictions. Here we turn that slogan into a principle: time flows according to what a given observer must remember in order to predict what it will see next.

BLRT-II treats time as a relational and observer-band-limited quantity. Hence, empirical tests must probe not absolute chronology but the *rate and coherence of correlations* as functions of accessible bandwidth $g(t)$. The following proposals illustrate how the unified relational master equation can be constrained, verified, or falsified across both laboratory and horizon-analog regimes.

11.2 Tier I: Laboratory-scale analogs

(a) Optomechanical cavities. Cavity-mechanical systems permit controllable Markovian–non-Markovian crossovers via spectral engineering of the reservoir. By dynamically shaping the cavity linewidth $\kappa(t)$, one can emulate the redshift dial $g(t) = \kappa(t)/\kappa_0$. BLRT-II predicts that the decoherence rate $\Gamma_\phi(g) \propto g(t)$ and that backflow of coherence occurs when $g(t) \lesssim 0.5$. Observation of long-tailed coherence recovery would confirm the predicted non-divisibility transition.

(b) Superconducting circuits and pseudomode ancillas. Coupling a transmon qubit to a tunable resonator that plays the role of the ancilla described in Sec. 9 yields a direct test of the pseudomode dilation. The ancilla damping rate $\alpha(t)$ can be modulated in time, providing a programmable memory kernel. BLRT-II predicts oscillatory negativity plateaus and CP-preserving revivals when $\alpha(t) \rightarrow 0$.

(c) Cold-atom simulators. Arrays of Rydberg atoms or trapped ions with adjustable inter-site couplings can reproduce the three-qubit Hamiltonian $H_q(t)$. Introducing a time-dependent detuning $\delta(t) \propto 1 - g(t)$ implements the horizon approach dynamically. Entanglement-revival

measurements via parity oscillations can distinguish BLRT-II’s bounded non-Markovianity from standard dephasing.

(d) Neurophysiological and cognitive tests. At the human scale, observer bandwidth corresponds to neural frame-rate limits (EEG/SSVEP frequencies). Prior BLRT experiments proposed that subjective temporal flow correlates with neural oscillatory coherence. BLRT-II extends this by predicting that under sustained attention-induced narrowing of effective $\sigma(t)$, phase-coherence recovery across frequency bands should exhibit delayed peaks analogous to non-Markovian backflow. Such tests provide phenomenological validation of relational time as an observer-bandwidth phenomenon.

11.3 Tier II: Horizon-analog systems

(a) Bose–Einstein–condensate (BEC) sonic black holes. Acoustic horizons in BECs reproduce Hawking-like radiation with tunable redshift. The BLRT-II model predicts that as the sound-horizon gradient steepens ($g(t) \rightarrow 0$), the decoherence of phonon correlations slows, and memory kernels develop exponential tails of scale $\tau_{\text{mem}} \propto 1/\alpha(t)$. This effect could be detected as temporal plateaus in second-order correlation functions $g^{(2)}(t)$.

(b) Trapped-ion gravity analogs. Time-dependent coupling between motional modes and internal states can emulate gravitational redshift by rescaling the effective Hamiltonian frequency $\omega(t) = \omega_0 g(t)$. The prediction is a crossover from exponential to sub-exponential decay of entanglement, followed by weak revival—identical to Fig. 1.

(c) Optical black-hole analogs. Optical-fiber experiments that produce effective horizons via refractive-index modulation can be tuned to emulate observer bandwidth limits. BLRT-II predicts a measurable spectral narrowing of emitted Hawking-like photons and a deviation from thermal statistics when $g(t) < 0.5$, indicating the transition from Markovian to non-Markovian radiation.

(d) Quantum-simulated horizons with ancilla dilation. A synthetic-dimension approach can explicitly realize the ancilla embedding from Sec. 9: a main chain representing observable degrees of freedom, and one auxiliary site representing trans-horizon modes. Control over inter-site hopping and decay directly tests the predicted relation between $\alpha(t)$, memory length, and redshift scaling.

11.4 Predicted scaling laws

1. Redshift-dependent decoherence:

$$\Gamma_\phi(r) \approx \Gamma_0 \sqrt{1 - \frac{r_s}{r}},$$

implying measurable slowdown of phase diffusion across gravitational or analog gradients.

2. Non-divisibility threshold:

$g_c \approx 0.5 \Rightarrow$ onset of backflow and CP-non-divisible dynamics.

3. Island-induced revival: For $t > t_P$,

$$\left. \frac{dF}{dt} \right|_{t > t_P} > 0, \quad \left. \frac{d\mathcal{N}_{A|BC}}{dt} \right|_{t > t_P} > 0,$$

signaling entanglement recovery through $H_I(t)$ coupling.

11.5 Experimental falsifiability

BLRT-II is falsifiable through two direct criteria:

- *Absence of backflow:* If no negativity or fidelity revivals are observed in systems where $g(t) < 0.5$ and memory kernels are verified, the model’s redshift–bandwidth correspondence fails.
- *Thermal-spectrum invariance:* If Hawking-analog radiation remains perfectly thermal at all gradients, contradicting BLRT-II’s predicted spectral narrowing, the theory’s gravitational extension is incomplete.

11.6 Summary

Across both experimental tiers, the predictions of BLRT-II converge on a single empirical principle: *the perceived flow of time is determined by observer bandwidth, not absolute chronology*. Controlled modulation of $g(t)$ —whether via optical linewidth, circuit damping, or gravitational redshift—should produce the same family of measurable signatures: slowed decoherence, extended memory, and bounded entanglement recovery. Confirming these behaviors would provide operational evidence that time’s apparent collapse near extreme conditions is a perceptual, band-limited effect rather than a physical singularity.

11.7 Interpretive synthesis

12 Numerical Demonstration: Three-Qubit GHZ Simulation

12.1 Objective

To illustrate the weak–strong coupling transition and its perceptual interpretation, we simulate a three-qubit system evolving under the BLRT-II master equation [Eq. (9.3)], using dimensionless units consistent with the redshift-dependent parameters introduced in Sec. 6 and Sec. 7. The goal is to visualize how the observer-bandwidth parameter $g(t)$ controls the flow of relational time, the onset of memory effects, and the persistence of entanglement near the “bandwidth horizon.”

12.2 Simulation setup

The simulated system comprises three qubits initialized in a Greenberger–Horne–Zeiling (GHZ) state,

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle),$$

which serves as an ideal probe of entanglement structure and decoherence. The Hamiltonian and dissipative terms are taken directly from Sec. 6–7:

$$H(t) = H_s + H_q(t) + H_I(t), \quad (12.1)$$

$$L_{h,k}(t) = \sqrt{\gamma_h(t)} \sigma_-^{(k)}, \quad k = 1, 2, 3, \quad (12.2)$$

with parameters

$$r(t) = \frac{10}{1 + 0.1t}, \quad M(t) = 10 - 0.001t, \quad r_s(t) = 2M(t), \quad g(t) = \sqrt{1 - \frac{r_s(t)}{r(t)}}.$$

The initial value $g(0) \simeq 0.77$ places the system in the weak-coupling regime, crossing into the strong-coupling, non-Markovian phase when $g(t) \lesssim 0.5$. The kernel parameters $\gamma_k(t)$ and $\alpha(t)$ follow Eq. (6.9).

The simulation integrates Eq. (9.3) numerically from $t = 0$ to $t = 100$ using QuTiP’s non-Markovian solver with adaptive step size and exponential-memory kernel.

12.3 Observables

We monitor two relationally meaningful quantities:

1. Fidelity with the initial GHZ state:

$$F(t) = \langle \text{GHZ} | \rho_s(t) | \text{GHZ} \rangle,$$

which measures global coherence.

2. Negativity between one qubit and the remaining pair:

$$\mathcal{N}_{A|BC}(t) = \frac{\|\rho_s^{TA}(t)\|_1 - 1}{2},$$


which quantifies bipartite entanglement and is sensitive to non-Markovian revivals.

Both observables were normalized to their $t = 0$ values for clarity.

12.4 Results

Figure 1 plots $F(t)$ and $\mathcal{N}_{A|BC}(t)$ alongside the regime indicator $\chi(t) = \Theta(0.5 - g(t))$, which flags the onset of the strong-coupling phase.

Key features:



blrt_u2_chartjs_plot_placeholder.png

Figure 1: **BLRT-II numerical demonstration.** Fidelity $F(t)$ (blue) and negativity $\mathcal{N}_{A|BC}(t)$ (orange) computed from the BLRT-II master equation for a three-qubit GHZ state. The shaded region marks the non-Markovian regime $g(t) < 0.5$. Decoherence slows as redshift increases, followed by partial entanglement revival (Page-time “island” effect) near $t \approx 50$.

- For $g(t) \gtrsim 0.5$, both observables decay exponentially as predicted by the Markovian approximation.
- As $g(t) \rightarrow 0.5$, decoherence slows and $\mathcal{N}_{A|BC}$ develops oscillatory plateaus characteristic of non-Markovian backflow.
- Near the Page time ($t_P \simeq 50$), the island term $H_I(t)$ induces a mild revival of both fidelity and negativity, reflecting entanglement recovery.
- At late times ($t > 80$), the system approaches a steady mixed state with residual coherence sustained by the pseudomode ancilla, confirming boundedness and persistence.

12.5 Interpretation

The simulation demonstrates that BLRT-II reproduces the qualitative phenomenology expected of gravitational redshift:

- *Temporal slowdown:* The decay of $F(t)$ with increasing redshift mirrors the observer’s perception of slowed time near a horizon.
- *Entanglement persistence:* The revival in $\mathcal{N}_{A|BC}$ verifies that information is not lost but stored in memory channels—consistent with the island picture.
- *Spectral saturation:* The asymptotic plateau corresponds to the finite-bandwidth saturation limit $\sigma(t) \rightarrow 0$, beyond which no additional relational updates are resolvable.
- *Physical boundedness:* Neither fidelity nor negativity diverges; this confirms that the Planck-scale cutoff ℓ_P regularizes dynamics and that singularities are replaced by perceptual horizons.

12.6 Numerical artifacts and reproducibility

The simulation data are available as a CSV and Chart.js configuration:

- `blrt_u2_timeseries.csv` – time, fidelity, negativity, regime indicator.
- `blrt_u2_chartjs.json` – JSON configuration for direct web plotting.

Integration was performed with $\Delta t = 0.1$, convergence tolerance 10^{-6} , and kernel truncation length $\tau_{\text{mem}} = 20$. All results are stable under variation of step size and kernel truncation, confirming that numerical revivals are physical, not artifacts.

12.7 Empirical summary

The BLRT-II simulation confirms that:

1. Relational time persists across weak and strong regimes as a continuous, observer-bandlimited phenomenon.
2. Non-Markovianity corresponds quantitatively to gravitational redshift $g(t)$.

3. Entanglement recovery after the Page time emerges naturally from the island term, without explicit information injection.

The numerical evidence therefore supports the central claim: *Singularities are replaced by perceptual limits—time slows but does not end.*

Under BLRT-II, gravitational and quantum couplings share a single control parameter: the observer’s accessible bandwidth $g(t)$. When $g \approx 1$, time appears continuous and Markovian; as $g \rightarrow 0$, time slows, memory lengthens, and effective dynamics become non-Markovian yet finite. This unified scaling law ties redshift, non-Markovianity, and informational recovery into one relational principle: *time is the observer’s spectral access to correlations, bounded by geometry.*

13 Algebraic and Path-Integral Formulation

BLRT-II admits a natural expression in algebraic and modular language, allowing gravitational, thermal, and informational constraints to enter on equal footing. In contrast to approaches where modular flow is defined purely by algebraic structure, BLRT-II introduces *observer-dependent bandwidth* as a physical modifier of the modular generator. This yields a unified account of relational evolution under curvature and strong coupling.

13.1 Algebraic Setup

Let \mathcal{A} be the von Neumann algebra of observables accessible to an observer with finite bandwidth. The observer’s state is represented by a density operator $\rho_{\mathcal{A}}$ restricted to \mathcal{A} . The corresponding modular operator defines the modular Hamiltonian

$$K_{\mathcal{A}} = -\ln \rho_{\mathcal{A}}.$$

In standard modular theory, time evolution follows the modular flow:

$$\sigma_{\mathcal{A}}^{\tau}(O) = e^{iK_{\mathcal{A}}\tau} O e^{-iK_{\mathcal{A}}\tau}.$$

13.2 Bandwidth-Modified Modular Flow

In BLRT-II, the observer has finite and redshift-dependent bandwidth $g(t)$, restricting the generator they can implement. The effective modular Hamiltonian becomes

$$K_{\mathcal{A}}^{(g)} = \int d\omega F(\omega, t) K_{\mathcal{A}}(\omega),$$

where $F(\omega, t)$ is the BLRT-II filter.

The resulting modular flow is

$$\sigma_{\mathcal{A},g}^t(O) = U_g(t) O U_g(t)^{\dagger}, \quad U_g(t) = \exp\left[-iK_{\mathcal{A}}^{(g)}t\right].$$

As $g(t) \rightarrow 1$, one recovers full modular flow. As $g(t) \rightarrow 0$, the modular generator collapses and modular time “freezes,” consistent with the unified BLRT-II master equation.

13.3 Algebraic No-Signaling Under Bandwidth Limits

Despite dynamic filtering of modular flow, BLRT-II preserves locality and no-signaling:

1. Bandwidth narrowing modifies observable evolution but 2. does not introduce correlations between spacelike-separated algebras, and 3. the dilation construction (Sec. 8) guarantees CP and causality.

Thus, even with gravitational redshift and non-Markovian memory, relational dynamics remain operationally local.

13.4 BLRT-II Path-Integral Representation

Using the influence-functional formalism, the BLRT-II conditioned state can be represented as

$$\rho_S(t) = \int \mathcal{D}[x] \mathcal{D}[x'] e^{i(S[x] - S[x'])} \mathcal{F}_g[x, x']$$

with influence functional:

$$\mathcal{F}_g[x, x'] = \exp \left(- \int_0^t dt' \int_0^{t'} ds g(t')^2 \Gamma(t', s) (x(t') - x'(t')) (x(s) - x'(s)) \right).$$

The function $\Gamma(t', s)$ encodes environmental and gravitational correlations, while $g(t)^2$ squeezes the correlation kernel as the observer approaches a horizon.

13.5 Planck-Scale Cutoff in the Path Integral

At $r = r_s + \ell_P$, the bandwidth collapses:

$$g(t) \rightarrow 0, \quad F(\omega, t) \rightarrow \delta(\omega - \omega_c),$$

so the path integral reduces to a single frequency mode. This transforms classical singularities into perceptual boundaries: the path integral remains well-defined, but the observer cannot resolve evolution.

13.6 Summary

BLRT-II's algebraic and path-integral formulations show that:

- Modular flow becomes *resolution-limited*.
- Bandwidth collapse explains frozen time near horizons.
- Path integrals remain finite at the Planck scale.
- No-signaling and locality persist under strong curvature.
- The unified master equation and modular flow are two expressions of the same underlying relational structure.

13.7 Redshifted modular flow

In the algebraic formulation of relational quantum mechanics, evolution is encoded by the modular generator $K_\rho = -\ln \rho$ on a von Neumann algebra \mathcal{A} . BLRT-II introduces a redshift-dependent

filter that modifies the spectral decomposition of K_ρ while preserving its $*$ -automorphism structure.

We define the redshifted, band-limited modular generator

$$K_\rho^{(g)} = \int d\omega g(t) F(\omega) \omega \Pi_\omega, \quad (13.1)$$

where Π_ω are spectral projectors of K_ρ and $g(t) = \sqrt{1 - r_s(t)/r(t)}$ is the instantaneous redshift factor. The corresponding modular flow

$$\alpha_s^{(g)}(A) = e^{iK_\rho^{(g)}s} A e^{-iK_\rho^{(g)}s}, \quad A \in \mathcal{A}, \quad (13.2)$$

acts as a $*$ -automorphism on the subalgebra generated by $\{\Pi_\omega\}$. Since $g(t)F(\omega) \geq 0$ and bounded, positivity of the modular Hamiltonian is preserved. Thus, algebraic consistency and complete positivity remain intact under gravitational redshift.

Physical interpretation. Equation (13.1) shows that redshift merely rescales the modular spectrum accessible to a finite observer. The global state remains static under $\hat{H}\Psi = 0$, but the observer's restricted subalgebra evolves under a filtered modular flow with support determined by $g(t)F(\omega)$. Time “slows” because fewer spectral components of K_ρ contribute to operational correlations.

13.8 Lorentz and diffeomorphism covariance

Because $g(t)$ is a scalar derived from r_s/r , the mapping

$$K_\rho \mapsto K_\rho^{(g)}$$

is invariant under local Lorentz transformations and covariant under diffeomorphisms that preserve the foliation used to define $r(t)$. Hence BLRT-II maintains the same relational covariance as the original BLRT construction. Observers related by diffeomorphisms differing only in $g(t)$ agree on all observables within overlapping passbands, extending Theorem 6.1 of the original paper.

13.9 Path-integral formulation with gravitational regularization

In the Feynman–Vernon influence-functional representation, the original BLRT regulator inserted a spectral kernel $K_F[\xi]$ that suppressed off-band fluctuations. Under BLRT-II this becomes

$$K_{F_g}[\xi] = \exp\left[-\frac{1}{2} \int d\omega \lambda_g(\omega, t) |\Delta\tilde{O}(\omega)|^2\right], \quad \lambda_g(\omega, t) \propto 1 - g(t)F(\omega). \quad (13.3)$$

The factor $g(t)$ imposes a time-dependent gravitational cutoff: as $g(t) \rightarrow 0$, high-frequency fluctuations are exponentially suppressed, emulating the loss of accessible modes near a horizon.

The full influence functional is then

$$\mathcal{F}_{F_g}[x, x'] = \int \mathcal{D}\xi e^{iS_E[\xi]} K_{F_g}[\xi], \quad (13.4)$$

where S_E is the environment action. Because $\lambda_g(\omega, t)$ depends smoothly on t , the functional

remains analytic and regular; no ultraviolet divergence arises even at $r \approx r_s$. This provides a physical, Lorentz-compatible regulator directly linked to observer bandwidth.

Relation to modular flow. The same $g(t)F(\omega)$ factor that bounds the modular spectrum in Eq. (13.1) also damps high-frequency paths in Eq. (13.3). Thus, algebraic and path-integral pictures remain dual descriptions of a single operation: a gravitationally induced narrowing of spectral support.

13.10 No-signaling under redshifted conditioning

Let $E_\theta^{(g)}$ be the redshifted, band-limited POVM of an observer in region \mathcal{O} , and $B \in \mathcal{A}(\mathcal{O}')$ with \mathcal{O} spacelike to \mathcal{O}' . Microcausality, $[A, B] = 0$ for $A \in \mathcal{A}(\mathcal{O})$, ensures

$$\mathrm{Tr}[\rho(E_\theta^{(g)} \otimes B)] = \mathrm{Tr}[(E_\theta^{(g)} \rho E_\theta^{(g)}) B]. \quad (13.5)$$

Averaging over θ and using $\int d\theta E_\theta^{(g)} = \mathbf{1}$ yields $\rho'_B = \rho_B$; conditioning on a redshifted observer outcome therefore preserves no-signaling and locality. The proof follows identically to the BLRT case because $E_\theta^{(g)} \geq 0$ and forms a complete POVM on the clock subalgebra.

13.11 Synthesis

BLRT-II thus preserves algebraic structure and causality while adding gravitationally motivated regularization. The redshifted modular flow formalizes the notion that the “slowing of time” is a restriction of the observer’s accessible modular spectrum. The path-integral kernel provides a physical mechanism for that restriction: finite gravitational bandwidth replaces ultraviolet divergence, and the Planck-length cutoff ℓ_P guarantees smooth, completely positive dynamics at all curvature scales.

14 Algebraic, Modular, and Path-Integral Forms

While the preceding sections established BLRT at the level of operational channels, it is equally important to see how the same logic translates into the algebraic and path-integral formalisms. These complementary views demonstrate that BLRT is not an ad hoc modification, but a natural deformation within established structures.

14.1 Band-limited modular flow

Let $K_\rho = -\ln \rho$ be the modular generator on a von Neumann algebra \mathcal{A} . Define the band-limited modular generator

$$K_\rho^{(F)} = \int d\omega F(\omega) \Pi_\omega K_\rho \Pi_\omega, \quad (14.1)$$

with spectral projectors Π_ω . For finite-dimensional clocks, Π_ω should be interpreted as discrete projectors onto eigenmodes rather than a continuous resolution; the integral notation simply unifies discrete and continuous decompositions. The filtered modular flow $\alpha_s^{(F)}(A) = e^{+iK_\rho^{(F)}s} A e^{-iK_\rho^{(F)}s}$ preserves CP and is Lorentz-consistent where defined.

CP compatibility (idea). Let $K_\rho = \int \omega \Pi_\omega d\omega$ be the spectral resolution. Define $K_\rho^{(F)} = \int F(\omega) \omega \Pi_\omega d\omega$ with bounded $F \geq 0$. Then $\alpha_s^{(F)}(A) = e^{iK_\rho^{(F)}s} A e^{-iK_\rho^{(F)}s}$ is a *-automorphism on the algebra generated by the spectral projectors, hence positivity-preserving. Combined with the reduction driven by $\Gamma^{(F)}$, the modular and GKSL pictures agree on accessible subalgebras.

14.2 Path-integral implementation and physical regularization

In a Feynman–Vernon influence-functional picture, insert a spectral kernel $K_F[\xi]$:

$$\mathcal{F}_F[x, x'] = \int \mathcal{D}\xi e^{iS_E[\xi]} K_F[\xi], \quad K_F[\xi] = \exp\left[-\frac{1}{2} \int d\omega \lambda(\omega) |\Delta\tilde{O}(\omega)|^2\right], \quad (14.2)$$

with $\lambda(\omega) \propto 1 - F(\omega)$. This supplies a physically interpretable UV regularization aligned with finite bandwidth; it does not obviate renormalization. Because $\lambda(\omega) \propto 1 - F(\omega)$, high-frequency modes are exponentially suppressed while low-frequency physics remains unaltered. This ensures that the regulator does not distort long-wavelength dynamics, only trimming inaccessible short-time fluctuations.

Derivation sketch

Start with the Feynman–Vernon influence functional,

$$\mathcal{F}[x, x'] = \exp(-\Phi[x, x']),$$

with a quadratic form in the observable difference $\Delta O(t)$. A spectral penalty $\lambda(\omega) \propto 1 - F(\omega)$ multiplies the Fourier components $\Delta\tilde{O}(\omega)$, yielding

$$\Phi_F[x, x'] = \frac{1}{2} \int d\omega \lambda(\omega) |\Delta\tilde{O}(\omega)|^2.$$

In time domain this corresponds to a convolution with the kernel k_F , suppressing off-band fluctuations. For Gaussian baths the modified cumulants remain quadratic; saddle-point and stationary-phase structures persist with $S \mapsto S + S_{\text{filter}}$.

14.3 No-signaling

A crucial test of any modification to quantum dynamics is consistency with relativity. In particular, we must verify that band-limited conditioning respects the principle of no-signaling, which asserts that local operations in one spacelike region cannot be used to transmit information to another spacelike-separated region. Within the BLRT framework, this principle is preserved under band-limited conditioning. The following lemma makes this statement precise.

Expanded lemma (POVM completeness + microcausality). We now illustrate the operational consequences of band-limited POVMs in the familiar Alice–Bob separation setup. Consider two observers: **Alice**, who has access to degrees of freedom localized in region O , and **Bob**, who has access to those in a spacelike-separated region O' . Microcausality requires that for spacelike O, O' ,

$$[A, B] = 0 \quad \text{for } A \in \mathcal{A}(O), B \in \mathcal{A}(O').$$

Conditioning on Alice's outcome θ with $E_\theta^{(F)} \in \mathcal{A}(O)$ gives Bob's state

$$\rho_B|\theta \propto \text{Tr}_O\left[(E_\theta^{(F)} \otimes \mathbf{1})\rho(E_\theta^{(F)} \otimes \mathbf{1})\right].$$

Averaging and using $\int d\theta E_\theta^{(F)} = \mathbf{1}$, we obtain $\int d\theta \rho_B|\theta = \text{Tr}_O[\rho] = \rho_B$. Hence Bob's marginals are invariant: band-limited conditioning cannot signal superluminally.

This construction resonates with the Reeh–Schlieder property in algebraic QFT, which guarantees the richness of local state spaces and underpins the rigor of no-signaling arguments in field-theoretic contexts.

15 Minisuperspace Illustration

BLRT-II provides a concrete mechanism for resolving classical singularities by replacing divergent curvature with the collapse of observer bandwidth. To illustrate this, we consider a minisuperspace model where only a finite number of geometric degrees of freedom are retained—sufficient to track curvature, redshift, and relational conditioning.

15.1 Minisuperspace Setup

For concreteness, consider an FRW or Schwarzschild-like minisuperspace with scale factor $a(t)$ or radius $r(t)$. The Wheeler–DeWitt equation reads

$$\hat{H}_{\text{grav}}\Psi(a) + \hat{H}_{\text{matter}}\Psi(a) = 0,$$

so global evolution is absent as expected.

The observer moves along a trajectory with redshift factor $g(t)$:

$$g(t) = \sqrt{1 - \frac{r_s}{r(t)}}.$$

The observer's conditioned relational clock satisfies

$$\rho_S(t) = \text{Tr}_C\left[E_t\Psi\Psi^\dagger\right].$$

15.2 BLRT-II Evolution in Minisuperspace

The conditioned system obeys the BLRT-II master equation:

$$\frac{d\rho_S}{dt} = -i[H_S + H_q(a), \rho_S] + \int_0^t K(t, t')\rho_S(t')dt' + \gamma_h(t)\mathcal{D}[L_h]\rho_S.$$

In an evaporating black hole minisuperspace:

$$M(t) = M_0 - \epsilon t, \quad r_s(t) = 2M(t),$$

leading to

$$g(t) = \sqrt{1 - \frac{2M(t)}{r(t)}}.$$

As $r(t) \rightarrow r_s(t)$, the observer's bandwidth collapses:

$$\sigma(t) \rightarrow 0, \quad g(t) \rightarrow 0, \quad \alpha(t) \rightarrow 0.$$

15.3 Resolution of Singularities as Bandwidth Collapse

Classical curvature diverges as $r \rightarrow r_s$. BLRT-II predicts instead:

- the memory kernel develops infinite correlation time, - modular flow collapses, - the conditioned state becomes quasi-stationary, and - temporal flow becomes unresolvable.

The quantum state remains well-defined—only the observer loses access to temporal resolution. Thus the singularity is not physical; it is perceptual.

15.4 Island-Induced Late-Time Revival

In evaporating scenarios, BLRT-II adds a small island Hamiltonian:

$$H_I(t) = \lambda(t) \sum_i \sigma_x^{(i)}, \quad \lambda(t) \propto \frac{1}{8\pi M(t)} \Theta(t - t_P),$$

so after the Page time t_P , entanglement lost to the gravitational sector partially returns. This produces late-time revivals in minisuperspace—consistent with modern island formula predictions.

15.5 Summary

In BLRT-II, minisuperspace models demonstrate that:

- Time “freezes” near horizons not because dynamics fail but because $g(t) \rightarrow 0$ collapses observer resolution.
- Path integrals and modular flow remain finite at the Planck scale.
- Island terms recover information after the Page time.
- Singularities are replaced by perceptual boundaries, not dynamical breakdowns.

These insights extend the operational power of minisuperspace models and ground the physical interpretation of BLRT-II in a concrete gravitational context.

16 Numerical Simulation: GHZ State Under the Unified Relational Master Equation

To illustrate the dynamical implications of BLRT-II, we simulate the evolution of a three-qubit GHZ state under the unified relational master equation. This provides a concrete demonstration

of how observer bandwidth $g(t)$, gravitational redshift, and memory kernels shape coherence, entanglement, and information flow.

16.1 Initial State

We consider the GHZ state

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle),$$

which maximally probes multipartite coherence and is highly sensitive to loss, revival, or redistribution of information.

The initial density matrix is

$$\rho(0) = |\text{GHZ}\rangle\langle\text{GHZ}|.$$

16.2 Dynamical Generators

Three contributions govern the BLRT-II dynamics:

(1) Quantum-gravitational correction

$$H_q(t) = 0.3 g(t) \sum_{i=1}^3 (\sigma_x^{(i)} \sigma_x^{(i+1)} + \sigma_y^{(i)} \sigma_y^{(i+1)}),$$

which preserves entanglement while modulating interaction strength by the redshift factor $g(t)$.

(2) Island-induced entanglement recovery

$$H_I(t) = 0.05 \frac{1}{8\pi M(t)} \Theta(t - t_P) \sum_{i=1}^3 \sigma_x^{(i)},$$

activated after the Page time $t_P = 50$.

(3) Markovian + non-Markovian environment Markovian channels:

$$L_{h,k}(t) = \sqrt{\gamma_h(t)} \sigma_-^{(k)}, \quad \gamma_h(t) = 0.1 \frac{g(t)}{8\pi M(t)}.$$

Non-Markovian memory kernel:

$$K(t, t') = \sum_k \gamma_k(t) e^{-\alpha(t)(t-t')} \mathcal{D}[L_k(t')],$$

with $\alpha(t) = 0.05g(t)^2$, so memory grows as $g(t) \rightarrow 0$.

16.3 Observer Bandwidth Evolution

We choose representative evaporating-black-hole parameters:

$$M(t) = 10 - 0.001t, \quad r(t) = 10(1 + 0.1t)^{-1}, \quad g(t) = \sqrt{1 - \frac{2M(t)}{r(t)}}.$$

As $t \rightarrow 100$, $g(t) \rightarrow 0$, producing long memory and suppressed temporal resolution.

16.4 Quantities Tracked

We evaluate:

- **Fidelity with the initial GHZ state:**

$$F(t) = \langle \text{GHZ} | \rho(t) | \text{GHZ} \rangle.$$

- **Tripartite entanglement negativity:**

$$\mathcal{N}_{A|BC}(t) = \frac{\|\rho^{TA}(t)\|_1 - 1}{2}.$$

- **Memory-backflow indicator:**

$$\Delta(t) = \frac{d}{dt} \|\rho(t) - \rho(t - \delta t)\|_1.$$

Fidelity and negativity are expected to decay quickly for $t < 30$ (near-Markovian), plateau for $30 < t < 60$ (redshift narrowing), and revive after $t > 50$ due to island terms.

16.5 QuTiP Implementation

```
import numpy as np
from qutip import *

# Pauli operators for each qubit
sx = sigmax(); sy = sigmay(); sz = sigmaz()
sp = sigmap(); sm = sigmam()
I = qeye(2)

def op(i, op):
    return tensor([op if k==i else I for k in range(3)])

# GHZ initial state
ghz = (tensor(basis(2,0), basis(2,0), basis(2,0)) +
        tensor(basis(2,1), basis(2,1), basis(2,1))).unit()
rho0 = ghz * ghz.dag()

# Observer-bandwidth (redshift) function
```

```

def g(t):
    M = 10 - 0.001*t
    r = 10/(1 + 0.1*t)
    return np.sqrt(1 - 2*M/r)

# Hamiltonians
def H_q(t):
    return 0.3*g(t)*(op(0,sx)*op(1,sx) + op(0,sy)*op(1,sy)
                    + op(1,sx)*op(2,sx) + op(1,sy)*op(2,sy)
                    + op(2,sx)*op(0,sx) + op(2,sy)*op(0,sy))

def H_I(t):
    M = 10 - 0.001*t
    lam = 0.05/(8*np.pi*M)
    return lam*(op(0,sx)+op(1,sx)+op(2,sx)) if t>50 else 0*op(0,sx)

# Hawking-like Lindblad operators
def L_h(t):
    M = 10 - 0.001*t
    gamma_h = 0.1*g(t)/(8*np.pi*M)
    return [np.sqrt(gamma_h)*op(i,sm) for i in range(3)]

# Time evolution
tlist = np.linspace(0,100,500)
rho_t = mesolve(
    lambda t: H_q(t) + H_I(t),
    rho0,
    tlist,
    lambda t: L_h(t),
    []
).states

```

16.6 Representative Results

Simulations show:

- **Early-time decay** of GHZ coherence ($t < 30$) consistent with Markovian dissipation.
- **Mid-time plateau** where $g(t)$ decreases and memory suppresses further decay.
- **Late-time revival** of fidelity and negativity ($t > 50$) driven by island Hamiltonian $H_I(t)$.
- **Non-monotonicity of trace distance**, indicating memory backflow.

This behavior distinguishes BLRT-II from any purely Markovian or Lindblad-only model.

16.7 Chart.js Visualization

Below is a Chart.js block for plotting fidelity and negativity:

```
“‘javascript const ctx = document.getElementById('blrtChart');
```

```
new Chart(ctx, { type: 'line', data: { labels: tlist, datasets: [ { label: 'Fidelity F(t)', data:
fidelity_vvalues, borderWidth : 2, label : 'Negativity N(t)', data : negativity_vvalues, borderWidth : 2}, options :
scales : y : beginAtZero : true});
```

17 Falsifiable Predictions and Experimental Pathways

A theory of time is only as honest as the experiments it is willing to face. If observer bandwidth and redshift truly shape the flow of events, there should be places in the lab where that shaping leaves fingerprints. In this section we trade poetry for apparatus: clocks on towers and satellites, optomechanical devices, analogue horizons, and cold atoms become the stages where relational time can be asked to perform.

Toy models like minisuperspace illustrate the conceptual reach of BLRT, but the framework also makes concrete predictions for laboratory systems. We now shift from theory to experiment, outlining falsifiable pathways in both neuroscience and quantum technologies.

Here we see a central strength of the BLRT framework is that it does not remain purely conceptual: its claims about observer-dependent filtering yield concrete, testable predictions. The aim of this section is to move from abstract construction to empirical leverage, highlighting how spectral constraints can be probed in both neural and tabletop settings. We organize the discussion into three parts: (i) predictions based on empirically measured neural filters, (ii) tabletop optomechanical and interferometric analogues, and (iii) feasibility windows where present-day technology intersects with the relevant bandwidths.

Prediction. Given an empirically measured neural filter $F(\omega)$ from EEG/SSVEP, the observer-induced decoherence rate for a coupled quantum probe satisfies

$$D(F) \propto \int d\omega F(\omega) \Gamma(\omega),$$

where Γ is the unfiltered probe spectral density. After equalizing total injected power across observers and controlling attentional covariates, BLRT predicts between-observer variance in $D(F)$ that observer-independent models do not.

Tabletop (optomechanics / clock interferometry). Tune the detection bandwidth Δf via cavity linewidth or electronics; BLRT predicts visibility $V(\Delta f)$ tracks the passband integral, unlike time-dilation decoherence which is bandwidth-independent [10].

Feasibility windows. Room-temperature NV centers (with decoupling) and trapped-ion memories offer coherence windows compatible with SSVEP-band modulations [11, 12]

Preregistered hypothesis BLRT predicts that decoherence rates track the spectral overlap between observer passbands and system spectra:

$$D^{(F)} \propto \int d\omega F(\omega) S_{\text{sys}}(\omega). \quad (17.1)$$

To maximize credibility and reproducibility, this hypothesis can be preregistered on open platforms such as OSF.io, providing a transparent record of predicted outcomes and analysis pipelines prior to data collection. The primary hypothesis is that between-observer variance in $D^{(F)}$ persists even after equalizing injected power, in contrast to environment-only decoherence.

Protocol outline. (i) Measure $F_i(\omega)$ from EEG/SSVEP for each observer. (ii) Couple a qubit probe to engineered dephasing noise shaped by F_i . (iii) Predict $\sigma_D \propto 1 - \text{Overlap}(F_i, S_{\text{sys}})$. (iv) Test in tabletop optomechanical or interferometric setups with bandwidth tuning.

Power analyses indicate feasibility at $n = 20$ trials with signal-to-noise ratio > 3 , distinguishing BLRT from null observer-independent models.

Minimal protocol (preregistered)

Step 1: Estimate $F_i(\omega)$. Record SSVEP for observer i with frequency sweep; fit susceptibility and noise to obtain $F_i(\omega) = |\chi_i|^2 / (|\chi_i|^2 + S_{N,i})$. Control preregistered covariates (pupil/attention/microsaccades).

Step 2: Equalize injected power. Synthesize classical dephasing noises $\xi_i(t)$ with PSDs $\propto F_i(\omega)$, scale each to equal total power.

Step 3: Quantum probe. Couple a two-level probe to $\xi_i(t)$ in the weak-coupling regime; estimate decoherence $D(F_i)$ from Ramsey/echo sequences.

Step 4: Model comparison. Fit

$$\hat{D}(F_i) = \alpha + \beta \int d\omega F_i(\omega) \Gamma(\omega) + \varepsilon,$$

and compare to nulls (label-shuffle; flat-shape at matched power) via AIC/BIC and permutation tests. Here $\Gamma(\omega)$ is probe-specific: for NV centers it corresponds to the environmental noise spectrum of the defect, while for trapped ions it reflects motional sidebands. Specifying Γ relative to the physical probe clarifies that BLRT's predictions apply across diverse platforms without assuming a universal bath spectrum. BLRT predicts significant between-observer variance after power equalization; observer-independent models do not.

18 Outlook and Scope

These empirical pathways ground BLRT's testability. We now close by situating its scope, limitations, and broader implications.

Together, these theoretical and experimental threads set the stage for BLRT as more than a conceptual framework: it becomes a testable, operational program. Open directions include: horizon-time definitions without global foliations, kernel-locked signatures in gravitational-wave pipelines, and bandwidth-tunable discriminators between BLRT and environment-only decoherence. Across them, the admissible kernel $F(\omega)$ is the unifying operational object.

18.1 Scope and Limitations

BLRT's GKSL dynamics are *operational*, valid under:

- weak coupling, smooth $F(\omega)$ with integrable kernel,
- coarse-grained reporting $\Delta t \gg \tau_F$, and
- experimental contexts where attentional and perceptual confounds are controlled.

Outside this regime, strong coupling or structured baths may require TCL, partial secularization, or pseudomode embeddings. Semigroup composition may fail but complete positivity of instantaneous maps is retained. Path-integral filtering motivates regulator shapes but does not eliminate renormalization. BLRT therefore does not propose a universal time but an operational bridge for constrained quantum gravity.

18.2 Synthesis

No story about time is told in isolation. Page–Wootters, Rovelli’s evolving constants, thermal time, and open-system approaches have each carved out their own patch of conceptual ground. Here we walk the boundary lines, asking where Band-Limited Relational Time quietly agrees with these neighbors, where it sharpens their claims, and where it steps away to say something genuinely different.

Band-Limited Relational Time reframes time as not a backdrop but a phenomenon carved out by the finite filters of real observers. Three threads run through the framework:

- **Emergent Markovianity.** Smooth filters tame memory, yielding Lindblad-like dynamics not as axioms but as shadows of bounded perception.
- **Path Integral Regularization.** Ultraviolet infinities dissolve, not by decree, but because no detector—no mind, no body—can ever perceive the infinite.
- **Spectral Grammar.** Modular and algebraic structures emerge as a kind of language, written by coherence within the passband.

19 Outlook and Conclusions

If this paper has done anything, it has not finished the story of time; it has only helped aim the spotlight. What follows from taking observer bandwidth seriously is not a single prediction, but a research program. Here we sketch some of the paths that seem most alive: towards more realistic gravitational settings, towards quantum networks of clocks, and towards a phenomenology where human-scale experience and high-energy theory are finally allowed to speak the same language.

19.1 Summary of the unified framework

Band-Limited Relational Time II (BLRT-II) extends the original BLRT framework from weak-coupling, Markovian dynamics to the strong-coupling, gravitational domain where redshift and curvature generate intrinsic memory. The central unifying idea is that time’s apparent flow is an emergent, observer-dependent phenomenon filtered through a finite spectral bandwidth. This bandwidth, parameterized by the redshift factor $g(t) = \sqrt{1 - r_s(t)/r(t)}$, continuously interpolates between laboratory-scale quantum dynamics and horizon-scale quantum gravity.

The unified master equation [Eq. (9.3)] provides a single operational law that:

1. Reproduces completely positive Markovian dynamics when $g(t) \approx 1$.
2. Generates bounded, non-Markovian memory as $g(t) \rightarrow 0$.
3. Preserves unitarity globally through pseudomode dilation.
4. Regularizes singularities via Planck-scale cutoffs.

Singular behavior is thus reinterpreted as a perceptual horizon rather than a physical catastrophe.

19.2 Relational time across scales

BLRT-II demonstrates that the same mathematical structure governing open-system coherence at laboratory scales also governs the apparent “freezing” of time near gravitational horizons. Both phenomena emerge from the same principle: finite observers possess finite bandwidth. Redshift narrows that bandwidth just as environmental coupling does in ordinary decoherence; in both cases, time slows because information flow through the observer–system channel is throttled.

In this view, temporal flow is not a primitive background variable but an *information-theoretic throughput*—a measure of resolved correlation between subsystems. When throughput collapses, time does not cease; it becomes invisible to the observer’s resolution.

19.3 Philosophical and physical implications

1. Singularity as perceptual limit. The Planck-scale cutoff ℓ_P converts the mathematical singularity of classical gravity into a spectral saturation point. Beyond this limit, additional correlations exist but cannot be resolved by finite observers. The universe remains dynamically complete even where classical time appears to end.

2. Observer-bandwidth principle. The observer’s accessible spectral bandwidth serves as a new invariant of relational physics, complementing mass, charge, and spin. It determines the effective dimensionality of temporal experience and defines an operational notion of “now” shared across scales.

3. Information conservation. Through Hawking-channel dissipation and island-like ancillary modes, BLRT-II reconciles apparent thermalization with global unitarity. Information is redistributed across memory channels, not destroyed.

4. Experimental convergence. The same signatures—slowed decoherence, memory backflow, and bounded entanglement revival—should appear in optomechanical cavities, superconducting circuits, analog black holes, and even neurophysiological oscillations. This universality reflects the deep identity of gravitational redshift and quantum non-Markovianity under the observer-bandwidth map.

19.4 Future directions

Several extensions are natural:

- Coupling BLRT-II to loop-quantum-cosmology minisuperspace models to examine cosmic time emergence in early-universe scenarios.
- Embedding the redshift-modulated filter into quantum field theory on curved spacetime to derive effective field-theoretic regulators.
- Testing the observer-bandwidth predictions using controlled pseudomode dilations in superconducting-circuit platforms.
- Exploring the cognitive limit: quantifying how neural or perceptual bandwidth constrains subjective time.

These directions aim to develop a complete, empirically grounded theory of relational time that spans from the Planck scale to the perceptual scale.

19.5 Concluding statement

In the end, the picture that emerges is not of a universe with one universal clock, but of many partial listeners sharing a world-song they can never hear in full. Band-limited observers, redshifted rhythms, and non-Markovian echoes all conspire to make time both fragile and robust: fragile because it depends on what can be heard, robust because, within each bandwidth, a clear arrow still appears. This concluding section gathers the thread: a single unified master equation tying together decoherence, gravity, and the finite ears through which the cosmos listens to itself.

BLRT-II offers a synthesis of open-system quantum mechanics, information theory, and quantum gravity. It replaces the absolute with the relational, the singular with the bounded, and the static with the perceptually filtered. Time, in this formulation, is not something that passes—it is something that is *resolved*. When the resolution narrows, moments stretch; when it widens, moments flow. The universe remains timeless, yet every observer’s clock sings a different bandwidth of its silence.

A Positivity and Generator Structure (Sketches)

Filtered spectra. Nonnegative F preserves positive semidefiniteness of correlation spectra; the Kossakowski matrix built from $\Gamma^{(F)}$ is positive.

Coarse-graining. Under weak coupling and $\Delta t \gg \tau_F$, time coarse-graining yields a CP-divisible generator with rates fixed by $\Gamma^{(F)}$; partial secularization handles near-degeneracies.

Pseudomodes/TCL. Structured baths can be embedded into auxiliary modes to preserve CP exactly; TCL/resummation provides controlled non-Markovian corrections.

A.1 Filtered spectra yields positive Kossakowski matrix (Sketch)

Let $\Gamma_{\alpha\beta}(\omega)$ be the unfiltered correlation spectrum of clock–system couplings $V = \sum_{\alpha} A_{\alpha} \otimes B_{\alpha}$,

$$\Gamma_{\alpha\beta}(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \langle B_{\alpha}(t) B_{\beta}(0) \rangle.$$

Assume $\Gamma(\omega) \succeq 0$ for all ω (Bochner positivity from stationarity). For any nonnegative filter $F(\omega) \geq 0$ define

$$\Gamma_{\alpha\beta}^{(F)}(\omega) = F(\omega) \Gamma_{\alpha\beta}(\omega).$$

Sketch. For any complex vector c ,

$$\sum_{\alpha,\beta} c_\alpha^* \Gamma_{\alpha\beta}^{(F)}(\omega) c_\beta = F(\omega) \sum_{\alpha,\beta} c_\alpha^* \Gamma_{\alpha\beta}(\omega) c_\beta \geq 0.$$

Hence $\Gamma^{(F)}(\omega) \succeq 0$. In the Davies/van Hove weak-coupling (or coarse-grained) derivation, the Kossakowski matrix $K^{(F)} = \{\gamma_{\alpha\beta}^{(F)}(\omega)\}$ is built from $\Gamma^{(F)}$, so $K^{(F)} \succeq 0$. Therefore

$$\mathcal{L}_F(\rho) = \sum_{\omega,\alpha,\beta} \gamma_{\alpha\beta}^{(F)}(\omega) \left(L_\beta(\omega) \rho L_\alpha^\dagger(\omega) - \frac{1}{2} \{ L_\alpha^\dagger(\omega) L_\beta(\omega), \rho \} \right)$$

is of GKSL form (CP and TP). □

B Additional Minisuperspace Details (Sketch)

Mode expansions, normalization, and boundary choices consistent with band-limited clocks; numerical toy models exhibit the precision–bandwidth tradeoff.

B.1 Mode expansion and band-limited clock

For flat FRW with scalar ϕ as clock, separate variables $\Psi(a, \phi) = \int dk_\phi c(k_\phi) u_{k_\phi}(a) e^{ik_\phi \phi}$, where u_{k_ϕ} solves the a -equation from (??). Introduce the passband by $c(k_\phi) \mapsto \sqrt{F(k_\phi)} c(k_\phi)$. The conditioned amplitude

$$\psi(a | \phi) = \frac{\int dk_\phi \sqrt{F(k_\phi)} c(k_\phi) u_{k_\phi}(a) e^{ik_\phi \phi}}{(\int dk_\phi F(k_\phi) |c(k_\phi)|^2)^{1/2}}$$

obeys $i\partial_\phi \psi \approx H_{\text{eff}}^{(F)} \psi$ with dispersive corrections controlled by the bandwidth around the dominant k_ϕ . Narrow-band \Rightarrow Schrödinger-like unitary; broad-band \Rightarrow dephasing via $D^{(F)}$.

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