Planck–ϕ Relaxation and the Origin of Fundamental Constants: A ϕ-Substrate Model of Gravity, Resonance, and the Factor of Three

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Abstract

We construct a dynamical vacuum model in which all fundamental interactions arise from a single scalar substrate field whose preferred equilibrium is governed by the golden ratio $\phi = (1 + \sqrt{5})/2$. The vacuum has a "bare" Planck configuration, characterized by the usual Planck units, and a ϕ-stabilized equilibrium that is dynamically selected by non-resonant (KAM/Hurwitz) stability criteria. Gravity emerges as the small hydrodynamic residual between these two states, explaining both the weakness and universality of the gravitational interaction. We show that the Einstein perihelion precession factor of three can be rewritten in a form naturally involving $\phi^2 + \phi^{-2} = 3$, linking the relativistic correction to a ϕ-stability identity. Within this framework, dimensionless constants such as the fine-structure constant α, the weak mixing angle $\sin^2\theta_W$, and mass ratios are expressed as response coefficients of the substrate, depending only on ϕ and a finite set of dimensionless couplings. We provide explicit formulas illustrating how these emerge from the Planck–ϕ hierarchy, and we relate this construction to existing ϕ-based approaches such as the Golden Function model of the fine-structure constant.[1]

1 Introduction

A persistent tension in fundamental physics is the status of "constants." Newton's G, the fine-structure constant α, the weak mixing angle θ_W, and the hierarchy of particle masses are usually treated as empirical inputs. General relativity (GR) and the Standard Model (SM) explain how these constants operate, but not why they take the values they do.

Historically, however, the giants of physics did *not* view the vacuum as empty bookkeeping. Einstein spoke frankly of a medium whose stress-energy is encoded in the metric; Dirac and Heisenberg sought algebraic and spectral structures behind observed numbers; Tesla

[1]See, e.g., Pellis, *Golden Function Model of the Origin of the Fine-Structure Constant*, SSRN preprint 5387893, 2025.[1]

treated fields as the primary reality. To them, constants hinted at an underlying medium, not a bag of arbitrary parameters.

Modern high-energy theory often moves in the opposite direction. String theory, loop quantum gravity, and other programs frequently discard the physical vacuum entirely in favor of abstract manifolds, extended objects in 10 or 11 dimensions, or combinatorial graphs of "pure geometry." In doing so, they implicitly demote Newton, Einstein, Dirac, Heisenberg, and even Kepler and Tesla to a kind of historical approximation: clever but fundamentally wrong, waiting to be overwritten by more elaborate mathematics.

The ϕ-substrate framework developed here takes the opposite stance:

> The great theories of the 20th century are not mere approximations; they are essentially correct descriptions of a structured vacuum, missing only a single organizing principle: a stability condition that selects a golden-ratio equilibrium.

We ask a concrete question: if the vacuum is a dynamical medium whose bare configuration is set by Planck units and whose stable configuration is selected by ϕ-based non-resonant stability, can we:

1. Understand why GR carries a factor of 3 in the perihelion precession?

2. Model gravity as the hydrodynamic residual between two nearly identical vacuum states?

3. Express fundamental dimensionless constants as response coefficients of this medium, determined by the Planck–ϕ hierarchy?

We will not claim to have a final, experimentally confirmed theory. But we will show that, within a consistent substrate model, one can *derive* the constants in the precise sense: they appear as calculated functions of underlying vacuum parameters, rather than as independent inputs.

2 Vacuum as a Substrate: Planck and ϕ Scales

2.1 Planck units as the bare configuration

We begin with the usual Planck units:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}}, \tag{1}$$

$$t_P = \sqrt{\frac{\hbar G}{c^5}}, \tag{2}$$

$$m_P = \sqrt{\frac{\hbar c}{G}}, \tag{3}$$

$$E_P = m_P c^2 = \sqrt{\frac{\hbar c^5}{G}}. \tag{4}$$

These define a natural UV scale of the vacuum. In conventional thinking, they are mathematical combinations of $\hbar, c, G$. In the substrate view, they are properties of a *bare* configuration of the underlying medium:

$$\Phi(x) = \Phi_{\text{bare}} + \delta\Phi(x), \tag{5}$$

where Φ_{bare} encodes a Planck-scale correlation length ℓ_P and characteristic frequency $\omega_P = 1/t_P$.

2.2 ϕ as the equilibrium ratio

We posit that the medium relaxes from this bare configuration to a dynamically selected equilibrium characterized by the golden ratio

$$\phi = \frac{1+\sqrt{5}}{2} = [1; 1, 1, 1, \dots], \tag{6}$$

whose infinite continued fraction expansion makes it the *most irrational* number in the usual Diophantine sense.

We introduce an equilibrium field value:

$$\Phi_0 = \phi\, \Phi_{\text{bare}}, \tag{7}$$

and an associated equilibrium length scale

$$\ell_\phi = \phi^k \ell_P, \tag{8}$$

where k is a rational exponent determined by the substrate potential and the way Φ couples to the metric. The key assumption is:

> The vacuum chooses a configuration in which the relevant dynamical frequency ratios are maximally non-resonant, which is achieved when they are organized around powers of ϕ.

2.3 A minimal substrate Lagrangian

A simple scalar-field substrate Lagrangian capturing these ideas is:

$$\mathcal{L}_\Phi = \frac{1}{2} Z(\Phi)\partial_\mu\Phi\partial^\mu\Phi - V(\Phi), \tag{9}$$

where $Z(\Phi)$ is an effective wavefunction renormalization (possibly field-dependent) and $V(\Phi)$ is a potential with two distinguished field values:

$$V'(\Phi_{\text{bare}}) \approx 0, \quad \text{Planck saddle (unstable/less stable)}, \tag{10}$$

$$V'(\Phi_0) = 0, \quad \Phi_0 = \phi\Phi_{\text{bare}}, \quad \text{dynamically stable minimum.} \tag{11}$$

The vacuum relaxation $\Phi_{\text{bare}} \to \Phi_0$ defines a small dimensionless parameter

$$\epsilon \equiv \frac{\Phi_0 - \Phi_{\text{bare}}}{\Phi_{\text{bare}}} = \phi - 1 = \frac{1}{\phi} \approx 0.618\dots, \tag{12}$$

and higher powers $\epsilon^n \sim \phi^{-n}$ provide naturally small expansion parameters. The actual smallness relevant for gravity and other couplings will come from *combinations* of these powers, as in ϕ^{-n} with large n.

3 Why ϕ? KAM and Hurwitz Stability

3.1 Most non-resonant irrational

Consider a set of coupled oscillators with frequencies $\{\omega_i\}$. Resonances occur when there exist integer relations

$$\sum_i n_i\omega_i = 0, \quad n_i \in \mathbb{Z}, \quad \text{not all zero.} \tag{13}$$

KAM (Kolmogorov–Arnold–Moser) theory tells us that quasiperiodic tori survive small perturbations when such integer combinations are sufficiently "badly approximated." For two modes, the key quantity is the frequency ratio

$$\theta = \frac{\omega_2}{\omega_1}, \tag{14}$$

and its Diophantine properties.

Define the *measure of irrationality*:

$$\mu(\theta) = \inf_{p,q\in\mathbb{Z},q\neq 0} q^2 \left|\theta - \frac{p}{q}\right|. \tag{15}$$

It is a classical result in number theory that $\mu(\theta)$ is maximal when θ has the *slowest* converging continued fraction, which occurs for

$$\theta = [1; 1, 1, 1, \dots] = \phi. \tag{16}$$

In other words, the golden ratio is the most difficult number to approximate by rationals with small denominators; it is the most robust to low-order resonances.

3.2 Stability principle

We now translate that into a vacuum-selection principle:

> Among all possible substrate configurations, the vacuum selects those in which the dominant mode ratios are closest to ϕ (and its powers), thereby minimizing resonant instabilities. This is the "non-resonant stability" axiom.

Mathematically, we can encode this as an effective stabilizing term in the vacuum free energy:

$$\mathcal{S}_{\text{stab}}[\{\omega_i\}] = \lambda_{\text{stab}} \sum_{i<j} f\left(\frac{\omega_i}{\omega_j}\right), \tag{17}$$

with

$$f(\theta) \propto \frac{1}{\mu(\theta)}, \tag{18}$$

so that configurations with θ close to ϕ^n minimize $\mathcal{S}_{\text{stab}}$.

The upshot is simple: the spectrum of substrate excitations wants to organize itself into ϕ-scaled bands. This is exactly the kind of structure explored in the Golden Function framework for α^{-1}.[1]

4 General Relativity and the Factor of Three

4.1 Einstein precession

In GR, the advance of perihelion per orbit for a planet of semi-major axis a, eccentricity e, orbiting a mass M is

$$\Delta\varpi_{\mathrm{GR}} = \frac{6\pi GM}{a(1-e^2)c^2}. \tag{19}$$

The corresponding Newtonian (no-precession) reference is a closed ellipse with angle 2π per orbit.

We can rewrite (19) as

$$\Delta\varpi_{\mathrm{GR}} = 3 \times \frac{2\pi GM}{a(1-e^2)c^2}. \tag{20}$$

The factor of three is often described as arising from the relativistic nature of the gravitational field (time dilation plus spatial curvature plus spatial anisotropy). Here we point out that, inside the ϕ-substrate picture, this factor can be naturally expressed via a ϕ-identity.

4.2 The $\phi^2 + \phi^{-2} = 3$ identity

The golden ratio satisfies

$$\phi^2 = \phi + 1, \tag{21}$$

and therefore

$$\begin{aligned}
\phi^2 + \phi^{-2} &= \phi^2 + \frac{1}{\phi^2} && (22)\\
&= \phi^2 + \frac{1}{\phi+1} && (23)\\
&= \phi^2 + (2-\phi) \quad \left(\text{using } \phi(\phi-1) = 1 \Rightarrow \tfrac{1}{\phi+1} = 2-\phi\right) && (24)\\
&= (\phi^2 - \phi + 1) + 2 && (25)\\
&= 1 + 2 = 3. && (26)
\end{aligned}$$

That is,

$$3 = \phi^2 + \phi^{-2}. \tag{27}$$

4.3 Embedding the factor of three into the substrate

In the substrate picture, the GR correction can be interpreted as

$$\Delta\varpi_{\mathrm{GR}} = (\phi^2 + \phi^{-2})\, \Delta\varpi_{\mathrm{base}}, \tag{28}$$

where

$$\Delta\varpi_{\mathrm{base}} \equiv \frac{2\pi GM}{a(1-e^2)c^2} \tag{29}$$

is the precession one would obtain from a simpler geometric correction in a medium without the full ϕ-stabilized elasticity. The ϕ^2 and ϕ^{-2} pieces can be associated with, respectively, compressive and shear components of vacuum response in the hydrodynamical limit of the substrate.

The point is not that GR "secretly uses ϕ" in its equations, but that within a ϕ-organized substrate, the observed factor of three arises naturally as a sum over two ϕ-based response channels, each corresponding to a different deformation mode of the vacuum.

5 Emergent Constants from Planck–ϕ Hierarchy

5.1 Substrate parameters and dimensionless ratios

We now introduce a set of substrate parameters:

- Bare scale $\Lambda_P \sim 1/\ell_P$, the Planck UV cutoff.
- Equilibrium scale $\Lambda_\phi \sim 1/\ell_\phi = \phi^{-k}\Lambda_P$.
- Dimensionless substrate couplings $\{\lambda_i\}$ appearing in $V(\Phi)$ and $Z(\Phi)$.

All physical constants must be expressible as dimensionless functions of ϕ and the λ_i. For example, a generic dimensionless constant C (such as α, $\sin^2\theta_W$, or a mass ratio) will have the form

$$C = F_C(\phi; \lambda_1, \lambda_2, \dots), \tag{30}$$

with F_C determined by solving the substrate field equations and matching to low-energy observables.

The key structural assumption is that F_C admits a convergent series in powers of ϕ^{-1}:

$$F_C(\phi; \{\lambda\}) = \sum_{n=-N}^{N} c_n(\{\lambda\})\phi^{-n}, \tag{31}$$

with a small number of non-zero c_n enforced by stability and symmetry (e.g., reflection or duality between ϕ and ϕ^{-1}).

5.2 Fine-structure constant

In QED, the fine-structure constant is

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}. \tag{32}$$

In the substrate picture, e^2 and ε_0 are not fundamental; they emerge from the vacuum's response to excitations of Φ.

We postulate that α^{-1} is given by a substrate spectral sum over ϕ-scaled modes:

$$\alpha^{-1} = \mathcal{N}_\alpha \sum_{n=n_{\min}}^{n_{\max}} w_n(\{\lambda\})\,\phi^n, \tag{33}$$

where:

- $n \in \mathbb{Z}$ indexes discrete substrate modes (e.g., topological sectors or KAM tori),
- w_n are dimensionless weights determined by the couplings $\{\lambda\}$ and topological constraints,
- $\mathcal{N}_\alpha$ is a normalization fixed by a matching condition (e.g., requiring the low-energy photon propagator to be canonically normalized).

This is structurally similar to the Golden Function approach, where α^{-1} is approximated by a ϕ-scaled summation of rational components.[1] The difference here is ontological: the sum (33) emerges from a specific substrate Lagrangian and its mode structure, rather than being posited directly.

In principle, given a concrete $V(\Phi)$ and $Z(\Phi)$, one could compute the mode spectrum and the w_n explicitly, yielding a model-dependent but genuine *derivation* of α^{-1} from ϕ and $\{\lambda\}$.

5.3 Weak mixing angle

Similarly, the weak mixing angle θ_W encodes the ratio of electroweak gauge couplings:

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2}. \tag{34}$$

In the substrate framework, g and g' arise from the effective stiffness of vacuum modes that couple to $SU(2)_L$ and $U(1)_Y$. Denote the corresponding substrate response coefficients as κ_2 and κ_1, respectively. We posit

$$g^{-2} \propto \kappa_2(\phi; \{\lambda\}) = \sum_n a_n \phi^n, \tag{35}$$

$$g'^{-2} \propto \kappa_1(\phi; \{\lambda\}) = \sum_n b_n \phi^n, \tag{36}$$

so that

$$\sin^2\theta_W = \frac{\kappa_1}{\kappa_1 + \kappa_2} = \frac{\sum_n b_n \phi^n}{\sum_n (a_n + b_n)\phi^n}. \tag{37}$$

Again, this is a *structural derivation*: once κ_1 and κ_2 are computed from the substrate dynamics, $\sin^2\theta_W$ is not free; it is determined by ϕ and $\{\lambda\}$ via (37).

5.4 Mass hierarchies and Yukawa couplings

Let m_f denote a fermion mass, arising from Yukawa coupling to a substrate-induced vacuum expectation value (VEV) v_ϕ:

$$m_f = y_f v_\phi. \tag{38}$$

We define

$$v_\phi = \eta\, m_P \phi^{-p}, \tag{39}$$

where p encodes how far the electroweak scale lies below the Planck scale in the ϕ-hierarchy, and η is a dimensionless factor of order unity.

We then posit that the Yukawa couplings y_f are themselves organized by ϕ:

$$y_f = y_0 \phi^{-q_f}, \tag{40}$$

with $q_f \in \mathbb{Q}$ encoding a discrete set of hierarchies. Then

$$\frac{m_f}{m_P} = \eta y_0 \phi^{-(p+q_f)}. \tag{41}$$

Ratios of masses become simple powers of ϕ:

$$\frac{m_{f_1}}{m_{f_2}} = \phi^{-(q_{f_1} - q_{f_2})}. \tag{42}$$

Fitting the observed mass spectrum would amount to choosing rational exponents q_f (subject to constraints from the substrate field equations) so that experimental mass ratios are matched within uncertainties. In that sense, the *pattern* of masses is derived from *discrete* choices of q_f rather than a continuum of arbitrary Yukawa couplings.

5.5 Gravitational coupling and smallness

Finally, consider the dimensionless gravitational coupling between two particles of mass m:

$$\alpha_G(m) = \frac{Gm^2}{\hbar c}. \tag{43}$$

At the Planck mass m_P, we have

$$\alpha_G(m_P) = 1. \tag{44}$$

At lower scales, we use the mass relation above:

$$\alpha_G(m_f) = \left(\frac{m_f}{m_P}\right)^2 = \eta^2 y_0^2 \phi^{-2(p+q_f)}. \tag{45}$$

Thus, the extreme smallness of gravity at ordinary scales is simply the statement that $p + q_f$ is large and positive: the vacuum has relaxed through many steps in the ϕ-hierarchy from the Planck bare state to the observed low-energy configuration. The *same* ϕ-stability that organizes mode frequencies and masses also suppresses gravitational strength.

6 Discussion

We have built a single, coherent narrative:

- The vacuum is a real medium, not an abstract book-keeping device.
- Its bare configuration is characterized by the Planck scale.
- Non-resonant stability selects a golden-ratio equilibrium, as ϕ is the most irrational number in the KAM/Hurwitz sense.

- General relativity, viewed as the hydrodynamic limit of this medium, naturally carries a factor of three in perihelion precession, which fits neatly into the $\phi^2 + \phi^{-2} = 3$ identity.

- Fundamental constants are response coefficients of the substrate: their values are functions of ϕ and a finite set of dimensionless couplings.

- Within this model, "deriving" a constant means: given $V(\Phi)$, $Z(\Phi)$, and the mode structure, the constant is calculated, not chosen.

This is, of course, an incomplete theory. We have not written down a full renormalizable action, nor performed explicit loop calculations. But the architecture is tightly constrained: once ϕ is chosen as the organizing principle, and once the Planck–ϕ hierarchy is fixed, there is very little freedom left.

Crucially, this framework does *not* discard Newton, Einstein, Dirac, or Heisenberg. It takes their work as the hydrodynamic and spectral limits of a deeper medium. The arrogance does not lie in proposing a structured vacuum; it lies in discarding the giants as "approximations" in favor of ever more elaborate abstract scaffolding that never touches the ground.

7 Conclusion

We have shown how a ϕ-substrate model can, in principle, generate:

1. The Einstein factor of three in perihelion precession as a vacuum response organized by the identity $\phi^2 + \phi^{-2} = 3$.

2. A hierarchy of scales from the Planck bare state to low-energy physics via powers of ϕ.

3. Dimensionless constants as explicit functions of ϕ and a small number of substrate couplings.

Whether Nature actually uses this mechanism is an empirical question. But logically and mathematically, the route is clear: start from a concrete substrate Lagrangian, compute the mode structure, and derive the constants as we derive elastic moduli in condensed matter.

If this picture is even approximately right, then the fine-structure constant, the weak mixing angle, and the gravitational coupling are not arbitrary decorations on a manifold. They are the fingerprints of a golden-ratio-stabilized vacuum, whispering that the giants were right about the medium all along.

References

[1] S. Pellis, *Golden Function Model of the Origin of the Fine-Structure Constant*, SSRN Preprint 5387893 (2025), available at `https://ssrn.com/abstract=5387893`.