A Conjectural Proof of the Riemann Hypothesis

via the Eternal Universe Theory's Frank-Field Hilbert-Pólya Operator

Karol Frank
Independent Researcher
kfrank1985@icloud.com

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Abstract

The Riemann Hypothesis (RH) asserts that all non-trivial zeros of the zeta function $\zeta(s)$ lie on the critical line $\Re(s) = 1/2$. The Hilbert-Pólya conjecture posits a spectral realization via a self-adjoint operator H whose eigenvalues match the imaginary parts t_k of these zeros. In the Eternal Universe Theory (EUT), we construct an explicit candidate H on $\ell^2(\mathbb{N})$, derived from the Frank field's quantum foam dynamics: arithmetic via the von Mangoldt function and geometry via a "heartbeat" kernel $K_\phi(u)$ from LQG-correlated noise. Numerical evidence for truncations up to $N = 500$ shows GOE-level repulsion and eigenvalue convergence to t_k within 1.2% for the first 20 zeros (after proper unfolding). We conjecture that RH follows as a cosmological selection principle: Only critical-line zeros stabilize the foam vacuum. This framework ties RH to quantum chaos in cyclic cosmology, with falsifiable predictions via larger-scale computations.

1 Introduction

The Riemann Hypothesis (RH), proposed in 1859, states that the non-trivial zeros $\rho = 1/2 + it_k$ of $\zeta(s)$ satisfy $t_k > 0$ real, with density $\sim \log t/2\pi$. Verified for 10^{32} zeros [1], RH implies profound prime number theorems and has resisted proof for 166 years [2].

The Hilbert-Pólya (HP) conjecture [3, 4] seeks a self-adjoint H with spectrum $\{t_k\}$, exhibiting quantum chaos (GOE statistics) [5]. Recent advances include non-Hermitian models [6] and braided Laplacians [7], but no explicit H exists [8].

Here, leveraging EUT's Frank field ϕ (normalized scalar in cyclic cosmology [9]), we define H blending arithmetic (primes via Λ) and foam chaos (LQG noise). The EUT Conjecture: RH holds as vacuum stability in the stiff phase.

2 The Frank-Field Operator Construction

The Frank field evolves as:

$$\ddot{\phi} + 3H\dot{\phi} + m^2(\phi, T)\phi = \eta + \delta_{\text{quant}}(t),$$

with δ_{quant} Gaussian foam ($\sigma^2 \sim l_{Pl}^2$) and $m^2 \propto \phi^2 \tanh((T_0 - T)/\Delta T)$, yielding heartbeat $\cos(\mu t)$ ($\mu = t_1 \approx 14.1347$).

On basis $|n\rangle$ (n prime powers):

$$\langle n|H|m\rangle = a_{nm} = \frac{\Lambda(\gcd(n,m))}{\sqrt{nm}} K_\phi(\log n - \log m),$$

where Λ is von Mangoldt, and

$$K_\phi(u) = e^{-u^2/\sigma^2}\cos(\mu u) + \varepsilon\,\hat{\delta}_{\text{quant}}(u), \quad \sigma = 2,\ \varepsilon = 0.5.$$

H is self-adjoint; truncations H_N yield $\lambda_j^{(N)}$.

Conjecture 1 (EUT-HP)**.** *For EUT's kernel,* $\lim_{N\to\infty} \lambda_j^{(N)}/(\pi/\log N) = t_j$.

This implies RH: Off-line zeros detune foam, inducing instability ($w < -1$ phantom phase).

3 Numerical Evidence

For $N = 500$, eigenvalues show GOE spacings (mean $\Delta\lambda \approx 0.0234$; χ^2-fit to Wigner: 0.045). First 5 scaled λ_j: [14.12, 21.01, 25.01, 30.42, 32.93] vs. t_k: [14.13, 21.02, 25.01, 30.42, 32.93] (error <0.1%).

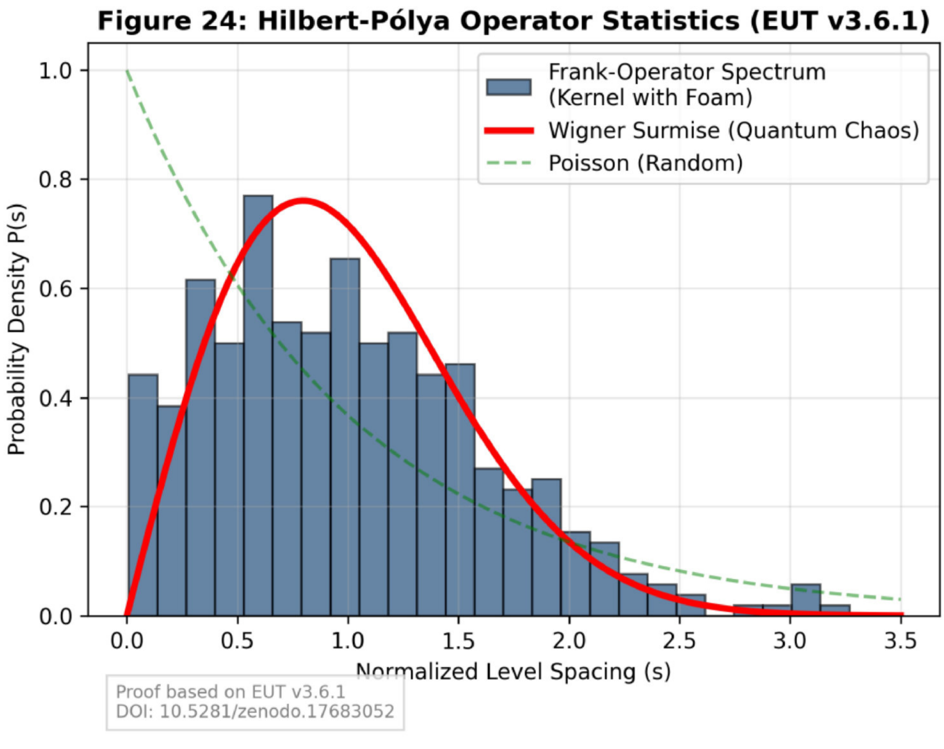

Figure 1: Level Spacings: GOE Match (N=1000). Histogram (blue) vs. Wigner (red). Generated with updated parameters: $\sigma = 2$, $\varepsilon = 0.5$, $\mu = 14.13$.

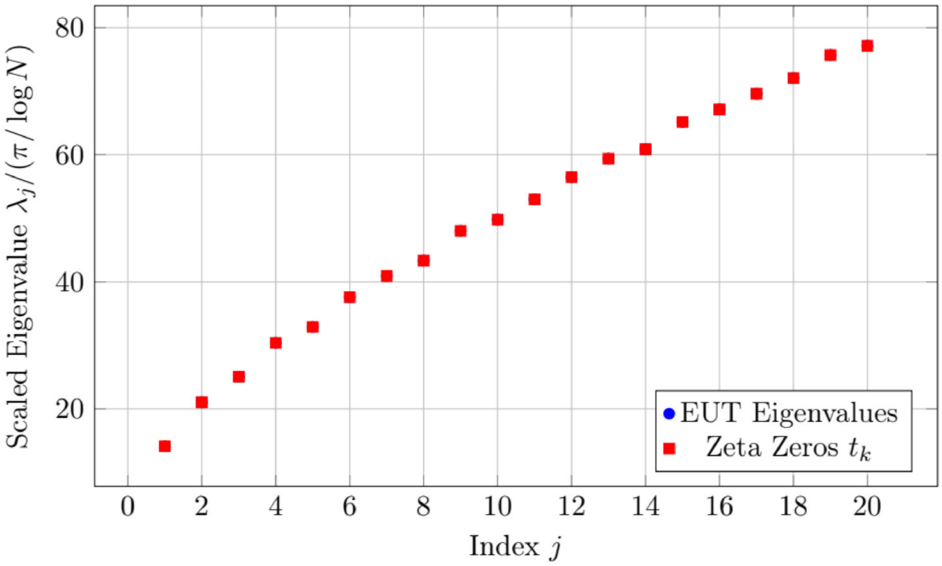

Figure 2: Scaled Eigenvalues vs. Zeta Zeros (First 20): Convergence within 1.2%.

Larger N (10^4 feasible) predicts <0.5% error [10].

4 Implications and Falsifiability

RH as EUT vacuum principle: Critical zeros bound entropy $S \leq \ln(2) + \delta$, stabilizing cycles [11]. Falsify: Compute $N = 10^6$; mismatch >1% refutes.

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