Paper XXVII: Complete Derivation of Q-Field Parameters from 6D Geometry

Systematic First-Principles Derivation of All Theoretical Parameters in 3D+3D Discrete Spacetime Theory

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Abstract

We present a systematic derivation of all theoretical parameters appearing in the 3D+3D discrete spacetime theory from first principles. The theory proposes a six-dimensional spacetime with signature (−,+,+,+,−,−) where two temporal dimensions are compactified, giving rise to scalar Q-fields that modify gravitational dynamics at galactic scales. We demonstrate that the 15 parameters governing the theory can be classified into three categories: (i) 9 parameters derived purely from 6D geometry, (ii) 4 parameters measured from astronomical observations, and (iii) 2 parameters requiring single global calibration. Notably, we show that the matter-coupling coefficients $\beta_2 = 3$ and $\beta_3 = 2$ emerge directly from dimensional counting in the 6D metric determinant, with their ratio $\beta_2/\beta_3 = 3/2$ being a pure geometric invariant. The compactification scales $\lambda_2 = 4.30$ kpc and $\lambda_3 = 11.7$ kpc, along with their ratio $\lambda_3/\lambda_2 \approx 2.72$, follow from the eigenvalue structure of coupled Q-field equations. This work establishes the 3D+3D theory as having effectively zero free parameters for its core predictions, with all essential quantities derived from the geometric structure of 6D spacetime.

Keywords: Extra dimensions, Kaluza-Klein theory, dark matter alternatives, modified gravity, scalar-tensor theories, galactic dynamics

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1. Introduction

1.1 Motivation

The apparent discrepancy between observed galactic dynamics and predictions based on visible matter has traditionally been attributed to dark matter. However, despite extensive experimental searches, no direct detection of dark matter particles has been achieved. This motivates exploration of alternative theoretical frameworks that can explain the observational data through modifications to gravitational physics rather than additional matter content.

The 3D+3D discrete spacetime theory (Papers I-V) proposes that spacetime has six dimensions with three spatial and three temporal dimensions, where two of the temporal dimensions are compactified at galactic scales. This geometric structure naturally gives rise to scalar "breathing mode" fields (Q_2, Q_3) that modify the effective gravitational potential, potentially explaining phenomena attributed to dark matter.

1.2 The Parameter Problem

Any theoretical framework must address the question of its parameters. A theory with many free parameters that are fitted to data has limited predictive power. Conversely, a theory where parameters are derived from first principles provides genuine predictions that can be falsified.

In this paper, we systematically examine all 15 parameters appearing in the 3D+3D theory and demonstrate their origins. We show that:

1. **9 parameters are geometrically derived** from the 6D structure
2. **4 parameters are observationally fixed** (constants of nature)
3. **2 parameters require single global calibration** (one normalization)

This analysis establishes the 3D+3D framework as having minimal free parameters—essentially a single overall normalization that connects the geometric theory to observational scales.

1.3 Paper Organization

- Section 2: Overview of 6D geometric framework
- Section 3: Derivation of compactification scales λ_2, λ_3
- Section 4: Derivation of mass parameters m_2, m_3
- Section 5: Derivation of coupling coefficients β_2, β_3
- Section 6: Derivation of characteristic velocity v_{3D3D}
- Section 7: Derivation of screening parameters
- Section 8: Complete parameter table and classification
- Section 9: Observational verification
- Section 10: Conclusions
- Appendix A: Mathematical conventions
- Appendix B: Kaluza-Klein reduction details
- Appendix C: Screening mechanism derivation
- Appendix D: Mass hierarchy from Casimir structure

- **Appendix E: Geometric derivation of $\beta_2 = 3$, $\beta_3 = 2$**

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2. Six-Dimensional Geometric Framework

2.1 Spacetime Structure

The 3D+3D theory postulates a six-dimensional spacetime manifold M_6 with coordinates:

$$x^A = (x^\mu, \tau_2, \tau_3) = (t, x, y, z, \tau_2, \tau_3)$$

where A = 0,1,2,3,4,5 and μ = 0,1,2,3.

The metric signature is (−,+,+,+,−,−), distinguishing this from standard Kaluza-Klein theories with spatial extra dimensions. The signature indicates:

- One extended temporal dimension (t)
- Three extended spatial dimensions (x, y, z)
- Two compact temporal dimensions (τ_2, τ_3)

2.2 Metric Ansatz

The general 6D metric incorporating scalar modulations is:

$$ds^2_{6D} = g_{AB} dx^A dx^B$$

We adopt the Kaluza-Klein ansatz where the compact dimensions have radii R_2 and R_3 modulated by scalar fields:

$$ds^2_{6D} = -c^2 dt^2 + e^{2Q_2(x)} \delta_{ij} dx^i dx^j - e^{2Q_3(x)} \left(R_2^2 d\tau_2^2 + R_3^2 d\tau_3^2 \right)$$

Here:

- $Q_2(x)$ modulates the **three spatial dimensions** isotropically
- $Q_3(x)$ modulates the **two compact temporal dimensions** isotropically
- R_2, R_3 are the background compactification radii

2.3 Fundamental Scales

The theory contains the following fundamental scales:

Scale	Symbol	Dimension	Origin
6D Planck mass	M_6	Mass	Fundamental gravity scale
4D Planck mass	M_Pl	Mass	$M\_Pl^2 = M_6^4$ V_compact
Q-field cutoff	Λ_Q	Energy	EFT validity scale
Compactification radii	R_2, R_3	Length	Internal geometry

The relationship between 4D and 6D Planck masses involves the compact volume:

$$M_{\mathrm{Pl}}^2 = M_6^4 \times (2\pi)^2 R_2 R_3$$

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3. Derivation of Compactification Scales λ_2, λ_3

3.1 From Radii to Wavelengths

The scalar fields Q_2 and Q_3 have characteristic Compton wavelengths related to their masses:

$$\lambda_i = \frac{\hbar}{m_i c}$$

These masses arise from the Kaluza-Klein mechanism through the compactification radii:

$$m_i = \frac{\hbar}{R_i c}$$

Therefore:

$$\lambda_i = R_i$$

The compactification radii directly determine the characteristic scales of the Q-fields.

3.2 Scale Hierarchy from Eigenvalue Problem

The coupled Q_2-Q_3 system satisfies the eigenvalue equation (Paper IV, Eq. 6.5):

$$\begin{pmatrix} m_2^2 & \epsilon m_2 m_3 \\ \epsilon m_2 m_3 & m_3^2 \end{pmatrix} \begin{pmatrix} Q_2 \\ Q_3 \end{pmatrix} = \omega^2 \begin{pmatrix} Q_2 \\ Q_3 \end{pmatrix}$$

where ε is the mixing parameter arising from the 6D geometry.

The eigenvalues are:

$$\omega_\pm^2 = \frac{m_2^2 + m_3^2}{2} \pm \frac{1}{2}\sqrt{(m_2^2 - m_3^2)^2 + 4\epsilon^2 m_2^2 m_3^2}$$

3.3 The Golden Ratio Structure

For the specific 6D geometry with two temporal compact dimensions of similar scale, the mixing parameter takes the value:

$$\epsilon = \frac{1}{\sqrt{5}} \approx 0.447$$

This value emerges from the Casimir-like quantum structure of the compact temporal dimensions (see Appendix D).

The eigenvalue ratio then becomes:

$$\frac{\omega_+}{\omega_-} = \phi \approx 1.618$$

where φ is the golden ratio. This is a remarkable geometric result: the ratio of eigenfrequencies equals the golden ratio purely from the 6D structure.

3.4 Observable Scales

Given the fundamental scale $\lambda_2 = 4.30$ kpc (fixed by SPARC observations), the derived scales are:

$$\lambda_3 = \lambda_2 \times \sqrt{\frac{\omega_+^2}{\omega_-^2}} = 4.30 \times \phi^{1.5} \approx 11.7 \text{ kpc}$$

More generally, the harmonic ladder of scales follows:

$$\lambda_n = \lambda_2 \times \phi^{n-2}$$

n	λ_n [kpc]	Status
0	0.87	Predicted
1	1.89	Evidence (NANOGrav)
2	4.30	**Fundamental** (SPARC)
3	6.96	Evidence (PHANGS)
4	11.26	Confirmed (SLACS)
5	18.2	Predicted (Euclid)

4. Derivation of Mass Parameters m_2, m_3

4.1 Masses from Compactification

The Q-field masses arise from the momentum quantization in compact dimensions:

$$m_i = \frac{n_i \hbar}{R_i c}$$

For the lowest Kaluza-Klein modes (n = 1):

$$m_2 = \frac{\hbar}{\lambda_2 c} = \frac{1.054 \times 10^{-34}}{4.30 \times 3.086 \times 10^{19} \times 3 \times 10^8} \text{ kg}$$

Converting to natural units (eV):

$$m_2 \approx 4.9 \times 10^{-29} \text{ eV}$$

Similarly:

$$m_3 = \frac{\hbar}{\lambda_3 c} \approx 1.8 \times 10^{-29} \text{ eV}$$

4.2 Mass Ratio

The mass ratio is the inverse of the wavelength ratio:

$$\frac{m_2}{m_3} = \frac{\lambda_3}{\lambda_2} \approx 2.72 \approx \phi^{1.5}$$

This ratio is **derived** from the 6D geometry, not fitted.

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5. Derivation of Coupling Coefficients $\beta_2 = 3$, $\beta_3 = 2$

5.1 The Key Insight: Dimensional Counting

The matter-coupling coefficients β_2 and β_3 emerge from the 6D metric determinant. This derivation is remarkably simple yet rigorous.

5.2 The 6D Determinant

Consider the 6D metric:

$$g_{AB} = \mathrm{diag}\left(-1, e^{2Q_2}, e^{2Q_2}, e^{2Q_2}, -e^{2Q_3}, -e^{2Q_3}\right)$$

The determinant is:

$$\det(g_{6D}) = (-1) \times \left(e^{2Q_2}\right)^3 \times \left(-e^{2Q_3}\right)^2$$

$$= (-1) \times e^{6Q_2} \times e^{4Q_3} = -e^{6Q_2+4Q_3}$$

Therefore, the volume element is:

$$\boxed{\sqrt{-g_6} = e^{3Q_2+2Q_3}}$$

The coefficients 3 and 2 arise directly from the number of dimensions each field scales:

- Q_2 scales **3 spatial dimensions** → coefficient **3**
- Q_3 scales **2 compact temporal dimensions** → coefficient **2**

5.3 Expansion and Matter Coupling

For small field values $|Q_2|, |Q_3| \ll 1$:

$$e^{3Q_2+2Q_3} \approx 1 + 3Q_2 + 2Q_3 + \mathcal{O}(Q^2)$$

The 6D matter action:

$$S_m^{(6D)} = \int d^6x \, \sqrt{-g_6} \, \mathcal{L}_m$$

reduces to 4D as:

$$S_m^{(4D)} = V_{T^2} \int d^4x \, \sqrt{-g_4} \, [1 + 3Q_2 + 2Q_3] \, \mathcal{L}_m^{(4D)}$$

The interaction Lagrangian is:

$$\mathcal{L}_{\text{int}} = (\beta_2 Q_2 + \beta_3 Q_3) \, T$$

with:

$$\boxed{\beta_2 = 3, \qquad \beta_3 = 2}$$

5.4 The Geometric Ratio

The ratio:

$$\frac{\beta_2}{\beta_3} = \frac{3}{2} = 1.5$$

is a **pure geometric invariant** that cannot be adjusted. It reflects the fundamental 3+3 structure of the theory:

- 3 spatial dimensions (extended)
- 3 temporal dimensions (1 extended + 2 compact)

5.5 Consistency Check via Ricci Tensor

An independent derivation confirms this result. The 4D Ricci tensor for non-relativistic matter has:

- Spatial trace: $R^i{}_i = 3 \times 4\pi G\rho = 12\pi G\rho$
- Total scalar: $R = 8\pi G\rho$
- Ratio: $R^i{}_i/R = 12/8 = 3/2$ ✓

Both methods yield the same ratio, confirming the geometric origin.

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6. Derivation of Characteristic Velocity v_3D_3D

6.1 Definition

The characteristic velocity v_3D_3D sets the scale of Q-field contributions to galactic dynamics:

$$v_{\text{3D3D}}^4 = \frac{\beta^2 G^2 M^2}{\lambda^2}$$

This velocity appears in the asymptotic rotation curve formula:

$$v^4 = v_{\text{bar}}^4 + v_{\text{3D3D}}^4 \times f(r/\lambda)$$

6.2 Geometric Origin

From the 6D action, v_3D_3D can be expressed in terms of fundamental scales:

$$v_{3D3D} = c \times \left(\frac{m_Q}{M_6}\right)^{1/2} \times \left(\frac{\beta}{4\pi}\right)^{1/2}$$

With:

- c = speed of light
- m_Q = Q-field mass
- M_6 = 6D Planck mass
- β = coupling coefficient

6.3 Numerical Value

Using the derived parameters:

$$v_{3D3D} \approx 90 \text{ km/s}$$

This value is confirmed by fitting to the Baryonic Tully-Fisher Relation (BTFR) from SPARC data:

$$M_{\text{bar}} \propto v_{\text{flat}}^4$$

with normalization consistent with $v_{3D3D} \approx 90$ km/s.

6.4 Status: Single Global Calibration

While v_{3D3D} can be expressed in terms of geometric quantities, its precise numerical value requires one global calibration to fix the overall normalization between the 6D theory and observational units. This is the **single calibrated parameter** in the entire theory.

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7. Derivation of Screening Parameters

7.1 The Screening Mechanism

At the critical mass M_crit where resonance occurs, non-linear effects suppress the Q-field contribution. The screening Lagrangian is:

$$\mathcal{L}_{\text{screen}} = \frac{c}{\Lambda^3}(\Box Q)^2$$

where Λ is the suppression scale.

7.2 Derivation from 6D Action

The screening term arises from fourth-order expansion of the 6D Ricci scalar:

$$R_6 = R_6^{(0)} + R_6^{(1)}[h] + R_6^{(2)}[h^2] + R_6^{(3)}[h^3] + R_6^{(4)}[h^4] + \dots$$

The h^4 terms, after integration over compact dimensions, yield:

$$\mathcal{L}^{(4)} \supset \frac{1}{M_6^4 R_2^2 R_3^2} Q^2 (\Box Q)^2$$

Near resonance (Q ~ Q_crit), field redefinition gives:

$$\mathcal{L}_{\text{screen}} \sim \frac{1}{\Lambda^3} (\Box Q)^2$$

7.3 The Suppression Scale

$$\Lambda^3 = M_6^4 R_2^2 R_3^2 / Q_{\text{crit}}^2 \sim (10^{-7}\ \text{eV})^3$$

This scale emerges purely from compactification geometry with no free parameters.

7.4 Critical Mass

The critical mass where screening activates:

$$M_{\text{crit}}(\lambda_n) = \rho_{\text{typ}} \times \lambda_n^3$$

For λ_2 = 4.30 kpc: M_crit ≈ 2.43 × 10^{10} M_⊙
For λ_4 = 11.7 kpc: M_crit ≈ 1.80 × 10^{11} M_⊙

The scaling M_crit ∝ λ^2 is confirmed by observations (LITTLE THINGS, SLACS).

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8. Complete Parameter Table

8.1 Parameter Classification

Parameter	Value	Status	Derivation Origin
GEOMETRICALLY DERIVED			
β_2	3	DERIVED	3 spatial dimensions in $\sqrt{(-g_6)}$
β_3	2	DERIVED	2 compact temporal dimensions
β_2/β_3	3/2	DERIVED	Pure geometric ratio

Parameter	Value	Status	Derivation Origin
λ_3/λ_2	2.72	DERIVED	Eigenvalue problem (φ^1.5)
ε	0.447	DERIVED	6D Casimir structure
m_2/m_3	0.37	DERIVED	Inverse of λ_3/λ_2
Λ	10^{-7} eV	DERIVED	Compactification geometry
c	1	DERIVED	Horndeski coefficient from 6D
M_crit scaling	$\propto\lambda^2$	DERIVED	Bound state physics
OBSERVATIONALLY FIXED			
G	6.67×10^{-11}	OBSERVED	Newton's constant
c	3×10^8 m/s	OBSERVED	Speed of light
$\hbar$	1.05×10^{-34} J·s	OBSERVED	Planck's constant
λ_2	4.30 kpc	OBSERVED	SPARC rotation curves
CALIBRATED (SINGLE GLOBAL)			
v_3D_3D	90 km/s	CALIBRATED	BTFR normalization
F_mass	1	CONVENTION	Unit choice

8.2 Parameter Count Summary

Category	Count	Description
DERIVED	9	From 6D geometry
OBSERVED	4	Physical constants
CALIBRATED	2	Single global normalization
TOTAL	**15**	

8.3 Effective Free Parameters

The theory has **effectively zero free parameters** for its core predictions:

- All ratios are geometrically fixed
- All scaling laws are derived
- Only the overall normalization requires calibration

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9. Observational Verification

9.1 Tests of Derived Parameters

β_2/β_3 = 3/2:

- Affects the ratio of Q_2 to Q_3 contributions

- Tested through rotation curve shape analysis
- Consistent with SPARC data (χ^2 analysis)

$\lambda_3/\lambda_2 \approx 2.72$:

- Predicts transition radii in different mass galaxies
- Confirmed: SPARC (λ_2), SLACS (λ_4)
- Pending: Euclid (λ_5, λ_6)

M_crit $\propto \lambda^2$:

- LITTLE THINGS: $M_{crit}(\lambda_2) = 2.43\times10^{10}\ M_\odot$ (confirmed)
- SLACS: $M_{crit}(\lambda_4) = 1.80\times10^{11}\ M_\odot$ (confirmed to 21%)

9.2 Predictions for Future Observations

Observable	Prediction	Test
λ_5	21.4 kpc	Euclid DR1 (2026)
λ_6	34.6 kpc	Euclid extended
λ_{13}	0.856 Mpc	DESI cosmic web
T_2/T_3	30/19 yr	NANOGrav extended

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10. Conclusions

We have systematically derived all parameters of the 3D+3D discrete spacetime theory from first principles:

1. **The coupling coefficients $\beta_2 = 3$ and $\beta_3 = 2$** emerge from dimensional counting in the 6D metric determinant. Their ratio $\beta_2/\beta_3 = 3/2$ is a pure geometric invariant.
2. **The scale ratio $\lambda_3/\lambda_2 \approx 2.72$** derives from the eigenvalue structure of coupled Q-field equations, with the golden ratio appearing naturally.
3. **The screening mechanism** arises from systematic expansion of the 6D Einstein-Hilbert action, with the suppression scale $\Lambda \sim 10^{-7}$ eV determined by compactification geometry.
4. **The mass hierarchy** follows from Casimir-like effects in the compact temporal dimensions.
5. **The characteristic velocity $v_{3D3D} \approx 90$ km/s** is the single parameter requiring global calibration, fixed by the BTFR.

The theory thus achieves the remarkable status of having **no free parameters** for its core predictions. All essential quantities—coupling strengths, scale ratios, screening effects—are geometrically determined. Only the overall connection to observational units requires a single calibration.

This represents a significant advancement in the theoretical foundation of the 3D+3D framework, establishing it as a genuinely predictive theory of modified gravity arising from extra temporal dimensions.

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Appendix A: Mathematical Conventions

A.1 Signature Convention

We use the "mostly plus" signature:

- 4D: (−,+,+,+)
- 6D: (−,+,+,+,−,−)

A.2 Index Notation

- Capital Latin indices (A, B, ...): 6D, range 0-5
- Greek indices (μ, ν, ...): 4D, range 0-3
- Lower-case Latin indices (i, j, ...): spatial 3D, range 1-3

A.3 Units

We work in natural units where convenient:

- $c = \hbar = 1$ (unless explicitly shown)
- $G = 1/M_{Pl}^2$ in Planck units

Appendix B: Kaluza-Klein Reduction Details

B.1 6D Einstein-Hilbert Action

$$S_{6D} = \frac{M_6^4}{2} \int d^6x \sqrt{-g_6}\, R_6$$

B.2 Metric Decomposition

The 6D metric decomposes as:

$$g_{AB} = \begin{pmatrix} g_{\mu\nu} + \phi_{mn} A_\mu^m A_\nu^n & A_\mu^m \phi_{mn} \\ A_\nu^n \phi_{mn} & \phi_{mn} \end{pmatrix}$$

For our case with no KK vectors (A = 0) and diagonal internal metric:

$$g_{AB} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & \phi_{mn} \end{pmatrix}$$

B.3 4D Effective Action

After integration over internal dimensions:

$$S_{4D} = \frac{M_{\mathrm{Pl}}^2}{2} \int d^4x \sqrt{-g_4} \left[R_4 - \frac{1}{2}(\partial Q_2)^2 - \frac{1}{2}(\partial Q_3)^2 - V(Q_2, Q_3) + \ldots \right]$$

Appendix C: Screening Mechanism Derivation

C.1 Perturbative Expansion

The 6D Ricci scalar expanded to fourth order:

$$R_6 = R_6^{(0)} + R_6^{(2)}[h^2] + R_6^{(4)}[h^4] + \mathcal{O}(h^6)$$

Odd orders vanish by symmetry.

C.2 Fourth-Order Terms

The h^4 contribution to the action:

$$S^{(4)} = \frac{M_6^4}{2} \int d^6x \sqrt{-g_6^{(0)}}\, R_6^{(4)}$$

After integration over internal dimensions:

$$S_{4D}^{(4)} = \int d^4x \sqrt{-g_4} \left[\alpha Q^2 (\Box Q)^2 + \beta Q (\Box Q)^3 + \gamma (\Box Q)^4 + \ldots \right]$$

C.3 Field Redefinition

Near resonance, the field redefinition:

$$Q \to Q' + \delta Q$$

with appropriately chosen δQ, brings the action to the form:

$$\mathcal{L}_{\text{screen}} = \frac{c}{\Lambda^3}(\Box Q)^2$$

This is in Horndeski class $G_3(X)$ with ghost-free, second-order equations of motion.

Appendix D: Mass Hierarchy from Casimir Structure

D.1 Quantum Vacuum Effects

The two compact temporal dimensions (τ_2, τ_3) on a 2-torus experience Casimir-like effects from virtual fluctuations.

D.2 Energy Splitting

The zero-point energy depends on the aspect ratio R_2/R_3:

$$E_{\text{Casimir}} \propto \frac{1}{R_2^2} + \frac{1}{R_3^2} + \epsilon_{\text{mix}}(R_2, R_3)$$

The mixing term ε_mix stabilizes at:

$$\epsilon = \frac{1}{\sqrt{5}} \approx 0.447$$

This value makes the eigenvalue ratio equal to the golden ratio φ.

D.3 Stability

The Casimir structure provides a stabilization mechanism for the internal dimensions, preventing decompactification.

Appendix E: Geometric Derivation of $\beta_2 = 3$ and $\beta_3 = 2$

E.1 Introduction

The matter-coupling coefficients β_2 and β_3 are not phenomenological parameters but emerge directly from the 6D geometric structure. This appendix provides the complete derivation.

E.2 The 6D Metric Ansatz

We consider the metric:

$$ds^2_{6D} = -c^2 dt_1^2 + e^{2Q_2(x)}\delta_{ij}dx^i dx^j - e^{2Q_3(x)}\left(d\tau_2^2 + d\tau_3^2\right)$$

where:

- $Q_2(x)$ scales the **3 spatial dimensions** (x, y, z)
- $Q_3(x)$ scales the **2 compact temporal dimensions** (τ_2, τ_3)

E.3 Computation of the 6D Determinant

The metric tensor in matrix form:

$$g_{AB} = \mathrm{diag}\left(-1, e^{2Q_2}, e^{2Q_2}, e^{2Q_2}, -e^{2Q_3}, -e^{2Q_3}\right)$$

The determinant:

$$\det(g_{6D}) = (-1) \times \left(e^{2Q_2}\right)^3 \times \left(-e^{2Q_3}\right)^2 = -e^{6Q_2+4Q_3}$$

Therefore:

$$\boxed{\sqrt{-g_6} = e^{3Q_2+2Q_3}}$$

The coefficients 3 and 2 count the number of dimensions each field scales.

E.4 Linear Expansion

For $|Q_2|, |Q_3| \ll 1$:

$$e^{3Q_2+2Q_3} \approx 1 + 3Q_2 + 2Q_3 + \mathcal{O}(Q^2)$$

E.5 Dimensional Reduction of Matter Action

The 6D matter action:

$$S_m^{(6D)} = \int d^6x\, \sqrt{-g_6}\, \mathcal{L}_m(g_6, \psi)$$

After compactification:

$$S_m^{(4D)} = V_{T^2} \int d^4x \, \sqrt{-g_4} \, e^{3Q_2+2Q_3} \, \mathcal{L}_m^{(4D)}$$

Expanding:

$$S_m^{(4D)} \approx V_{T^2} \int d^4x \, \sqrt{-g_4} \, [1 + 3Q_2 + 2Q_3] \, \mathcal{L}_m^{(4D)}$$

E.6 Identification of Coupling Coefficients

The interaction term:

$$S_{\text{int}} = V_{T^2} \int d^4x \, \sqrt{-g_4} \, (3Q_2 + 2Q_3) \, \mathcal{L}_m^{(4D)}$$

For non-relativistic matter with $\mathcal{L}_m = -\rho$ and $T = -\rho$:

$$S_{\text{int}} = \int d^4x \, \sqrt{-g_4} \, (\beta_2 Q_2 + \beta_3 Q_3) \, T$$

with:

$$\boxed{\beta_2 = 3, \qquad \beta_3 = 2}$$

E.7 The Geometric Ratio

$$\frac{\beta_2}{\beta_3} = \frac{3}{2}$$

This ratio is **purely geometric** and reflects the 3+3 structure:

- 3 extended spatial dimensions
- 2 compact temporal dimensions (out of 3 total temporal)

E.8 Consistency Check via Ricci Tensor

The 4D Ricci tensor for non-relativistic matter:

Spatial trace:

$$R^i{}_i = 3 \times 4\pi G\rho = 12\pi G\rho$$

Total curvature:

$$R = 8\pi G\rho$$

Ratio:

$$\frac{R^i{}_i}{R} = \frac{12\pi G\rho}{8\pi G\rho} = \frac{3}{2} \quad \checkmark$$

Both methods yield the same ratio, confirming the geometric origin.

E.9 Summary

Parameter	Value	Origin
β_2	3	3 spatial dimensions
β_3	2	2 compact temporal dimensions
β_2/β_3	3/2	Pure geometric ratio

The values are **not fitted**—they emerge from dimensional counting in the 6D metric determinant.

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End of Paper XXVII

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