A Sponge–Vacuum Model of Gravitation

From Travel Time and Vacuum Saturation to Singularity Resolution

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Abstract

We present a sponge–vacuum model of gravitation in which space is treated as a fixed three–dimensional coordinate scaffold of constant global volume, filled with a structured vacuum represented as a homogeneous "sponge" of cells. Mass–energy modifies the internal state of these vacuum cells, described by a time–dependent vacuum density field $\rho_{\mathrm{vac}}(\vec{x}, t)$. The local density of the vacuum determines an effective temporal index $n(\vec{x}, t)$, which controls the time required for motion through a given region.

In the weak–field, low–velocity regime, free motion is postulated to minimize a travel–time functional constructed from $n(\vec{x}, t)$, in analogy with Fermat's principle in optics. Gravity emerges as the macroscopic effect of spatial gradients of vacuum density. We refine the ontology of motion: while the coordinate scaffold remains fixed, physical bodies are understood to inhabit a sequence of *local vacuum reference frames*. Motion is the continuous passage from one local environment to the next, allowing for an emergent form of Lorentz invariance.

A finite vacuum adjustment time τ_0 describes the relaxation of the vacuum towards instantaneous equilibrium configurations. We derive an upper bound $\tau_0 < 10^{-13}\,\mathrm{s}$ from lunar laser ranging and interpret the relation $E_{\mathrm{cell}}\tau_0 \sim \hbar$ as indicating that the microscopic response of the vacuum operates close to a quantum speed limit, with an associated vacuum energy scale of order $10^{-3}\,\mathrm{eV}$ (meV regime).

Crucially, we extend the model to the strong–field regime by postulating a non–linear, exponential response of the vacuum. This leads to a *Principle of Ultimate Equilibrium*: the vacuum density saturates at a physical maximum $\rho_{\max}$, preventing gravitational collapse into a singularity. Black holes are replaced by non–singular Ultra–Compact Sponge Objects (UCSOs).

1 Introduction

In the standard geometric picture of general relativity (GR), gravitation is described as curvature of a Lorentzian metric $g_{\mu\nu}$ on spacetime, determined by Einstein's field equations and sourced by the energy–momentum tensor $T_{\mu\nu}$ [1–3]. Free particles follow geodesics of this metric, and gravitational phenomena such as free fall, light deflection, gravitational time dilation and waves are encoded in the geometry of spacetime. GR has passed a wide range of precision tests [4].

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A large body of work has explored the idea that gravity might be emergent from more fundamental microstructures of spacetime or vacuum degrees of freedom [5–10]. In these approaches, the Einstein equations can arise as effective or thermodynamic relations governing an underlying medium.

In this note we construct a *sponge–vacuum* model that aims to provide an antagonistic explanation of gravitation: instead of taking spacetime curvature as the primary ontology, we postulate a three–dimensional spatial container of constant global volume acting as a fixed **coordinate scaffold**, filled with a structured vacuum modeled as a homogeneous "sponge" of microscopic cells. Mass–energy changes the internal state (or density) of these cells, giving rise to a non–uniform, time–dependent vacuum density field $\rho_{\text{vac}}(\vec{x}, t)$.

The local state of this vacuum controls the time needed to traverse a given spatial interval. In the weak–field, low–velocity regime, we postulate that free bodies follow paths that minimize a travel–time functional built from an effective temporal index $n(\vec{x}, t)$ derived from ρ_{vac}, in analogy with Fermat's principle in optics [11]. Gravitational behavior is then interpreted as the macroscopic manifestation of spatial gradients of vacuum density. The global volume of space remains conceptually fixed; all "deformation" is relocated into the internal structure and dynamics of the vacuum.

We also emphasize a relational view of time: for a given mass m, the time parameter t serves primarily to index the environment $\text{env}_m(t)$ in which it is immersed, defined in terms of the surrounding vacuum configuration. At each instant, one may describe the instantaneous balance of forces acting on m in terms of this environment; the full dynamics then arises from the sequence of environments and their influence on the trajectory of m.

The present version of the model goes beyond a qualitative framework and explicitly addresses several classical tests of GR in the post–Newtonian regime. It aims to show that a structured vacuum with appropriate constraints on its response can reproduce the standard weak–field phenomenology, while also opening a route towards connecting gravity with quantum discreteness via a microscopic time scale τ_0. Furthermore, we propose a mechanism for singularity resolution in the strong field regime via vacuum saturation.

2 Ontology: The Scaffold and the Local Frame

2.1 Coordinate Scaffold vs. Local Vacuum Frame

We postulate that physical space can be represented, in the regime of interest, as a three–dimensional container with fixed coordinates $\vec{x} = (x, y, z)$ and constant coordinate volume. Concretely, one may imagine a large box $\Omega \subset \mathbb{R}^3$, or in the limit $\Omega = \mathbb{R}^3$.

Throughout, the container should be understood as a *coordinate scaffold* rather than as a physically measurable rigid ether. To resolve the conflict with the principle of relativity, we distinguish between:

- **The Coordinate Scaffold (Ω):** A fixed, Euclidean coordinate background. This scaffold is an abstract mathematical container; it does not "move" or "bend."
- **The Local Vacuum Frame ($\Omega_{\text{local}}(t)$):** For any massive body m, physical reality is defined by its immediate environment. At any instant t, m is immersed in a local vacuum state $\Omega_{\text{local}}(t)$ characterized by the density field ρ_{vac} and force gradients.

2.2 Motion as Continuous Passage

In this framework, motion is not merely a translation across the fixed scaffold Ω. Instead, it is interpreted as the **continuous passage** of the mass from one instantaneous local container to another:

$$\mathcal{T} : \Omega_{\text{local}}(t) \to \Omega_{\text{local}}(t + dt). \tag{1}$$

The trajectory $\vec{x}(t)$ is the history of these transitions. Since the observer carries their local inertial frame (the local equilibrium of the vacuum) with them, the absolute velocity relative to the global scaffold Ω becomes locally unobservable, allowing Lorentz invariance to emerge as an effective symmetry for internal physical processes.

2.3 Travel–time functional and free motion

The key postulate of the model is that the local state of the vacuum controls the *time required* for motion through that region. We define a temporal index $n(\vec{x}, t)$:

$$n(\vec{x}, t) = h(\rho_{\text{vac}}) \simeq 1 + \beta\, \delta\rho(\vec{x}, t) \quad \text{(weak-field)}, \tag{2}$$

where $\delta\rho = \rho_{\text{vac}} - \rho_0$. Free bodies follow paths that minimize the travel time functional $T[\gamma] = \int n(\vec{x}, t)|\dot{\vec{x}}|dt$.

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where $\delta\rho = \rho_{\text{vac}} - \rho_0$. Free bodies follow paths that minimize the travel time functional $T[\gamma] = \int n(\vec{x}, t)|\dot{\vec{x}}|dt$.

3 Dynamics of Vacuum Density and the Role of τ_0

3.1 Vacuum Relaxation

The vacuum does not adjust instantaneously to the motion of matter. We introduce a characteristic **adjustment time** τ_0, governed by a relaxation equation:

$$\tau_0 \frac{\partial \rho_{\text{vac}}}{\partial t} + \rho_{\text{vac}} = \rho_{\text{eq}}[\rho_{\text{mass}}], \tag{4}$$

where $\rho_{\text{eq}}[\rho_{\text{mass}}]$ denotes the instantaneous equilibrium configuration that would be reached if the vacuum could react without delay to a given mass distribution. The parameter $\tau_0 > 0$ encodes the internal dynamics of the vacuum cells: for $\tau_0 \to 0$, the vacuum tracks the mass distribution almost instantaneously, whereas for finite τ_0 the profile ρ_{vac} exhibits a temporal lag that can induce small residual (drag–like) forces in addition to the dominant conservative behavior.

3.2 Quantum Interpretation of τ_0

We conjecture that τ_0 is not an arbitrary macroscopic parameter, but is constrained by a microscopic quantum of action. In the spirit of quantum speed limits (Mandelstam–Tamm, Margolus–Levitin), there exists a minimal time for a quantum system with characteristic energy scale E to evolve into an orthogonal state, of order

$$\Delta t_{\min} \sim \frac{\hbar}{E}. \tag{5}$$

By analogy, we interpret τ_0 as the minimal reconfiguration time of a vacuum "cell" (or iso) with internal energy E_{cell}, and posit the relation

$$E_{\text{cell}}\,\tau_0 \sim \hbar. \tag{6}$$

In this view, each elementary update of the vacuum state over a time interval of order τ_0 carries a minimal quantum of action. The same parameter τ_0 then plays a dual role: it governs the dynamical response of the vacuum (and hence the small non–conservative corrections to gravity) and encodes a microscopic discreteness of action associated with Planck's constant.

If we adopt the observational bound $\tau_0 \lesssim 10^{-13}\,\text{s}$ obtained from orbital dynamics, Eq. (6) implies an effective vacuum–cell energy scale

$$E_{\text{cell}} \sim \frac{\hbar}{\tau_0} \sim 10^{-21}\,\text{J} \approx 10^{-3}\,\text{eV}, \tag{7}$$

i.e. in the millielectronvolt (meV) regime. This is an extremely low energy scale compared to nuclear or Planck scales, suggesting that the relevant vacuum degrees of freedom in this effective description are "soft" collective modes rather than Planckian excitations. A more detailed microphysical model would be required to identify these modes explicitly, but Eq. (6) already ties the macroscopic response time of gravity to a quantum constraint.

4 Post-Newtonian Successes

(Summary of derived results matching GR)

By fixing a single parameter combination $\beta\gamma = 2G/c^2$, the model successfully reproduces:

- **Newtonian Gravity:** Recovered as the static limit of vacuum density gradients ($1/r^2$ force).
- **Light Deflection:** The index $n(r)$ creates a refractive effect that bends light by $4GM/c^2b$, matching the full relativistic prediction (twice the Newtonian corpuscular value).
- **Perihelion Precession:** The non-linearities in the travel-time functional yield the correct $43''$/century excess for Mercury.
- **Gravitational Redshift:** Local clock rates $d\tau = k(\rho)dt$ slow down in denser vacuum, reproducing the Pound-Rebka results.

5 Basic Structure of the Sponge–Vacuum Model

5.1 Fixed three–dimensional spatial container

We postulate that physical space can be represented, in the regime of interest, as a three–dimensional container with fixed coordinates $\vec{x} = (x, y, z)$ and constant coordinate volume. Concretely, one may imagine a large box $\Omega \subset \mathbb{R}^3$, or in the limit $\Omega = \mathbb{R}^3$.

Throughout, the container should be understood as a *coordinate scaffold* rather than as a physically measurable rigid ether. In this picture:

- The topology and coordinate volume of the container are constant.
- There is no extrinsic "bending" or "folding" of space in a higher dimensional Euclidean sense.
- Spatial geometry at the level of coordinates is Euclidean; all nontrivial structure is encoded in the properties of the vacuum medium.

This is explicitly antagonistic to the curved–space intuition: instead of representing gravity by a deformation of the container itself, we keep the coordinate scaffold fixed and encode gravitational effects in the content — the structured vacuum. We restrict ourselves to weak fields and low velocities, so that relativistic corrections can be viewed as small and encoded in effective indices.

5.2 Structured vacuum as a sponge of cells

The container is filled with a vacuum medium modeled as a homogeneous *sponge* of small cells. Each cell is characterized by an internal state, summarized at the macroscopic level by a *vacuum density* $\rho_{\mathrm{vac}}(\vec{x}, t)$, which we interpret as an effective energy density or state variable of the vacuum medium.

In a reference configuration without mass–energy:

$$\rho_{\mathrm{vac}}(\vec{x}, t) = \rho_0, \tag{8}$$

a uniform constant throughout space and time. In this neutral state:

- The distribution of cells is homogeneous.
- Motion through the vacuum is isotropic and uniform.
- Time intervals and spatial distances behave as in a flat, Euclidean setting with a single global time and a single characteristic speed v_0.

When matter or energy is present, the internal state of the cells is altered. This is described by allowing the vacuum density to become a general function

$$\rho_{\mathrm{vac}} : \ \Omega \times \mathbb{R} \to \mathbb{R}, \qquad (\vec{x}, t) \mapsto \rho_{\mathrm{vac}}(\vec{x}, t), \tag{9}$$

with possible spatial and temporal inhomogeneities. We also use the deviation

$$\delta\rho(\vec{x}, t) := \rho_{\mathrm{vac}}(\vec{x}, t) - \rho_0. \tag{10}$$

5.3 Vacuum density and temporal index

The key postulate of the model is that the local state of the vacuum controls the *time required* for motion through that region. We introduce a *temporal index* $n(\vec{x}, t)$ associated with each point, defined as

$$n(\vec{x}, t) = h\Big(\rho_{\text{vac}}(\vec{x}, t)\Big), \tag{11}$$

where h is a (sufficiently regular) monotone function. Intuitively:

- In regions where the vacuum is "denser", ρ_{vac} is larger, and $n(\vec{x}, t)$ is correspondingly larger, indicating that traversing a given spatial distance requires more time.
- In regions where the vacuum is "thinner", ρ_{vac} is smaller, and $n(\vec{x}, t)$ is smaller, indicating that motion through those regions is temporally more efficient.

In the weak–field regime, we may linearize h around the homogeneous state:

$$n(\vec{x}, t) \simeq 1 + \beta\, \delta\rho(\vec{x}, t), \tag{12}$$

with β a constant parameter.

5.4 Travel–time functional and free motion

The key postulate of the model is that the local state of the vacuum controls the *time required* for motion through that region. We define a temporal index $n(\vec{x}, t)$:

$$n(\vec{x}, t) = h(\rho_{\text{vac}}) \simeq 1 + \beta\, \delta\rho(\vec{x}, t) \quad \text{(weak-field)}, \tag{13}$$

where $\delta\rho = \rho_{\text{vac}} - \rho_0$. Free bodies follow paths that minimize the travel time functional $T[\gamma] = \int n(\vec{x}, t)|\dot{\vec{x}}|dt$.

5.5 Vacuum Relaxation

The vacuum does not adjust instantaneously to the motion of matter. We introduce a characteristic **adjustment time** τ_0, governed by a relaxation equation:

$$\tau_0 \frac{\partial \rho_{\text{vac}}}{\partial t} + \rho_{\text{vac}} = \rho_{\text{eq}}[\rho_{\text{mass}}]. \tag{14}$$

This τ_0 encodes the internal dynamics of the vacuum cells.

5.6 The Quantum Link

We conjecture that the vacuum adjustment time τ_0 is related to a microscopic quantum of action. A natural relation is

$$E_{\text{cell}}\, \tau_0 \sim \hbar, \tag{15}$$

where E_{cell} denotes an effective energy scale associated with elementary updates of the vacuum state. Equation (15) is not intended as an exact identity but as an order-of-magnitude statement: during each interval τ_0, the reorganisation of the sponge vacuum would carry a minimal quantum of action.

Combining the empirical upper bound

$$\tau_0 < 10^{-13}\,\text{s}, \tag{16}$$

derived later from orbital dissipation constraints, with (15), we obtain a *lower* bound on the collective energy scale:

$$E_{\text{cell}} \gtrsim \frac{\hbar}{\tau_0} \sim 10^{-3}\,\text{eV}, \tag{17}$$

for τ_0 close to the current limit. In other words, the vacuum degrees of freedom that control the relaxation of the gravitational sector cannot be arbitrarily soft. They must carry at least a milli-electronvolt of energy per effective cell update. The microscopic value of E_{cell} could in principle be much larger (up to nuclear or even Planckian scales); the present work does not attempt to fix it, and we remain agnostic about the detailed microphysics beyond the inequality above.

It is therefore useful to distinguish between:

- a *collective* scale $E_{\text{coll}} \sim \hbar/\tau_0$, characterising long-wavelength, coarse-grained re-arrangements of the sponge vacuum that are directly constrained by gravitational phenomenology; and
- a possibly much higher *microscopic* scale associated with the underlying discrete structure ("atoms" or "isotopes" of vacuum), which does not need to be resolved by current experiments.

This situation is analogous to condensed-matter systems, where the vibrational quanta of the lattice (phonons) carry meV energies, while the electronic or nuclear excitations underlying the medium live at eV, keV or MeV scales. In the present paper we only use $E_{\text{coll}} \sim \hbar/\tau_0$ as an effective parameter in the gravitational sector and leave the microscopic scale open.

5.7 Equilibrium density, adjustment time, and Planck scale

An important refinement of the model is that the vacuum does not necessarily adjust instantaneously to changes in ρ_{mass}. Instead, we introduce a characteristic *adjustment time* τ_0, associated with the internal dynamics of the vacuum cells.

Let $\rho_{\text{eq}}(\vec{x};\rho_{\text{mass}})$ denote the "instantaneous equilibrium" vacuum configuration that would be reached if the medium could react without delay to a given mass distribution. We then consider a relaxation–type equation of the form

$$\tau_0\,\frac{\partial\rho_{\text{vac}}}{\partial t}(\vec{x},t) + \rho_{\text{vac}}(\vec{x},t) = \rho_{\text{eq}}\Big(\vec{x};\rho_{\text{mass}}(\cdot,t)\Big), \tag{18}$$

where $\tau_0 > 0$ is a fixed parameter.

In the limit $\tau_0 \to 0$, the relaxation equation implies

$$\rho_{\text{vac}}(\vec{x},t) \approx \rho_{\text{eq}}\Big(\vec{x};\rho_{\text{mass}}(\cdot,t)\Big), \tag{19}$$

so the vacuum tracks the motion and configuration of the mass distribution almost instantaneously. For finite τ_0, the solution of (18) exhibits a temporal lag between the mass distribution and the vacuum density pattern, which in turn may generate small residual forces (drag or memory) in addition to the dominant conservative behavior.

We further conjecture that τ_0 is related to a fundamental quantum scale. A natural possibility is to associate τ_0 with a microscopic time scale such that the product of a characteristic vacuum energy per cell, E_{cell}, and τ_0 is of order Planck's constant h:

$$E_{\text{cell}}\,\tau_0 \sim h. \tag{20}$$

Up to factors of 2π between h and $\hbar$, this is equivalent to the quantum–speed–limit relation (6). In this view, each elementary update of the vacuum state during a time interval τ_0 carries a minimal quantum of action. The same parameter τ_0 then plays a dual role: it controls the dynamical response of the vacuum (and hence gravity) and encodes a microscopic discreteness of action associated with the Planck constant.

6 Vacuum Equilibrium Landscape and Local Universes

We model the macroscopic state of the sponge vacuum by a density field $\rho_{\text{vac}}(\vec{x}, t)$ and introduce an effective energy functional

$$\mathcal{F}[\rho_{\text{vac}}] = \int d^3x \left[\frac{\kappa}{2} |\nabla \rho_{\text{vac}}|^2 + V\Big(\rho_{\text{vac}}\Big) - \lambda\, \rho_{\text{vac}}\, \rho_{\text{mass}}(\vec{x}) \right], \tag{21}$$

where:

- $\kappa > 0$ controls the "stiffness" of the sponge (the cost of spatial gradients),
- $V(\rho_{\text{vac}})$ is a local vacuum potential,
- λ parametrizes the coupling between vacuum and mass density.

For a given mass distribution $\rho_{\text{mass}}(\vec{x})$, *equilibrium configurations* of the vacuum are defined as stationary points of $\mathcal{F}$:

$$\frac{\delta \mathcal{F}}{\delta \rho_{\text{vac}}} = 0, \tag{22}$$

which yields an elliptic equation

$$-\,\kappa\, \nabla^2 \rho_{\text{vac}}(\vec{x}) + \frac{dV}{d\rho_{\text{vac}}}\Big(\rho_{\text{vac}}(\vec{x})\Big) = \lambda\, \rho_{\text{mass}}(\vec{x}), \tag{23}$$

determining the equilibrium profile $\rho_{\text{vac}}(\vec{x})$.

In the absence of matter, $\rho_{\text{mass}} = 0$, if the potential V satisfies

$$\left.\frac{dV}{d\rho_{\text{vac}}}\right|_{\rho_0} = 0, \tag{24}$$

then the homogeneous state $\rho_{\text{vac}}(\vec{x}) = \rho_0$ is a solution of (23), realizing the massless homogeneous vacuum already postulated as the reference configuration.

6.1 Linearised regime and effective Newtonian limit

To connect with the Newtonian limit, we linearize (23) around the homogeneous reference state:

$$\rho_{\text{vac}}(\vec{x}) = \rho_0 + \delta\rho(\vec{x}), \qquad |\delta\rho| \ll \rho_0. \tag{25}$$

Expanding V to second order,

$$V(\rho_{\text{vac}}) \simeq V(\rho_0) + \frac{1}{2} m_{\text{eff}}^2 \Big(\rho_{\text{vac}} - \rho_0\Big)^2, \tag{26}$$

with

$$m_{\rm eff}^2 = \left.\frac{d^2V}{d\rho_{\rm vac}^2}\right|_{\rho_0}, \tag{27}$$

we obtain, at linear order,

$$-\kappa\,\nabla^2\delta\rho(\vec{x}) + m_{\rm eff}^2\,\delta\rho(\vec{x}) = \lambda\,\rho_{\rm mass}(\vec{x}). \tag{28}$$

Combining (28) with the weak–field relation (12) and identifying an effective potential via

$$\Phi_{\rm vac}(\vec{x}) = \Phi_0 + \alpha\,\delta\rho(\vec{x}), \tag{29}$$

one arrives at a Poisson–type equation with possible Yukawa corrections,

$$\nabla^2\Phi_{\rm vac}(\vec{x}) - \mu^2\Phi_{\rm vac}(\vec{x}) = 4\pi G_{\rm eff}\,\rho_{\rm mass}(\vec{x}), \tag{30}$$

where μ and $G_{\rm eff}$ are determined by κ, λ, α, β and $m_{\rm eff}^2$. In the limit $\mu \to 0$, Eq. (30) reduces to the usual Poisson equation and reproduces the Newtonian potential. Thus the heuristic Poisson equation for $\delta\rho$ is recovered as the linearised equilibrium condition of the vacuum energy functional (21).

6.2 Equilibrium points and vacuum basins as local universes

Given an equilibrium configuration $\rho_{\rm vac}(\vec{x})$ solving (23), we define the corresponding effective potential $\Phi_{\rm vac}(\vec{x})$ and look for points where the vacuum force on a test body vanishes:

$$\nabla\Phi_{\rm vac}(\vec{x}_*) = \vec{0}. \tag{31}$$

Points $\vec{x}_*$ satisfying (31) are equilibrium points of the vacuum landscape. The local stability of such a point is encoded in the Hessian $\nabla\nabla\Phi_{\rm vac}(\vec{x}_*)$:

- if the Hessian is positive definite, $\vec{x}_*$ is a stable minimum;
- if it has mixed signs, $\vec{x}_*$ is a saddle point;
- if it is negative definite, $\vec{x}_*$ is a local maximum.

Importantly, *nothing* in (31) requires the vacuum density to have the same value at all equilibrium points. If $\vec{x}_*^{(1)}$ and $\vec{x}_*^{(2)}$ are two stable minima, then in general

$$\rho_{\rm vac}\left(\vec{x}_*^{(1)}\right) \neq \rho_{\rm vac}\left(\vec{x}_*^{(2)}\right). \tag{32}$$

Each stable minimum defines a basin of attraction in which test bodies can be trapped or orbit. These basins can be interpreted as *local universes* with their own vacuum norms and local dynamical structures, while the massless vacuum remains homogeneous in the absence of matter.

7 Dynamical Transitions Between Equilibrium Configurations

The relaxation equation introduced above captures the fact that the vacuum does not respond instantaneously to changes in the mass distribution. We now embed this idea in the landscape picture and describe how the system can move from one vacuum equilibrium configuration to another as the mass environment and inertial motions evolve.

7.1 State of the system and coupled evolution

We regard the macroscopic state of the system at a configuration label λ as

$$\mathcal{S}(\lambda) = \Big(\rho_{\text{vac}}(\vec{x};\lambda),\ \rho_{\text{mass}}(\vec{x};\lambda),\ \mathbf{J}_{\text{mass}}(\vec{x};\lambda)\Big), \tag{33}$$

where ρ_{vac} encodes the sponge–vacuum structure and $\rho_{\text{mass}}, \mathbf{J}_{\text{mass}}$ are the mass density and current. In practical weak–field applications one identifies λ with a coordinate time t, but conceptually λ is just an ordering parameter for configurations.

The evolution is given schematically by

$$\frac{d\mathcal{S}}{d\lambda} = \mathcal{F}\Big[\mathcal{S}(\lambda)\Big], \tag{34}$$

with two coupled components:

- *Motion of matter:* test bodies follow the effective potential Φ_{vac} according to equations of motion such as

  $$m\,\ddot{\vec{x}}(t) = -\,\nabla\Phi_{\text{vac}}\Big(\vec{x}(t), t\Big) + \dots, \tag{35}$$

  and the mass density and current satisfy a continuity equation

  $$\frac{\partial\rho_{\text{mass}}}{\partial t} + \nabla\cdot\mathbf{J}_{\text{mass}} = 0. \tag{36}$$

- *Reorganisation of the vacuum:* the vacuum density relaxes towards instantaneous equilibria determined by the current mass distribution, with a response time τ_0, as in (18).

In the limit $\tau_0 \to 0$ and for slowly varying sources, ρ_{vac} stays close to a family of instantaneous equilibria. For finite τ_0, the vacuum profile lags behind the mass configuration, producing the small dissipative effects later constrained.

7.2 Continuous deformations and jumps between basins

Within this framework, one can distinguish two limiting types of dynamical behaviour:

1. *Adiabatic evolution:* if the mass environment changes slowly and τ_0 is small, $\rho_{\text{vac}}(\vec{x},\lambda)$ follows a *continuous family* of nearby minima of $\mathcal{F}$. The vacuum landscape deforms smoothly and test bodies track the evolving equilibrium structures.

2. *Transitions between basins:* if the mass environment is strongly perturbed (e.g. close encounters of galaxies, merging clusters) or if the landscape itself has multiple local minima separated by shallow barriers, the trajectory of $\rho_{\text{vac}}(\vec{x},\lambda)$ in configuration space may cross from the basin of one minimum to that of another. From the viewpoint of a local observer confined to a given basin, this appears as an abrupt reorganisation of the effective gravitational sector.

8 Post-Newtonian Phenomenology: Short Summary

(Summary of derived results matching GR)

By fixing a single parameter combination $\beta\gamma = 2G/c^2$, the model successfully reproduces:

- **Newtonian Gravity:** Recovered as the static limit of vacuum density gradients ($1/r^2$ force).
- **Light Deflection:** The index $n(r)$ creates a refractive effect that bends light by $4GM/c^2b$, matching the full relativistic prediction (twice the Newtonian corpuscular value).
- **Perihelion Precession:** The non-linearities in the travel-time functional yield the correct $43''$/century excess for Mercury.
- **Gravitational Redshift:** Local clock rates $d\tau = k(\rho)dt$ slow down in denser vacuum, reproducing the Pound-Rebka results.

9 Constraints on Dissipation (τ_0)

The finite relaxation time τ_0 implies that the vacuum density profile "lags" behind a moving mass. This creates a gravitational drag force aligned against the velocity vector. Using Lunar Laser Ranging (LLR) data, which limits orbital energy loss, we derive the constraint:

$$\tau_0 < 10^{-13}\,\text{s}. \tag{37}$$

This extremely fast response ensures that the model is indistinguishable from conservative GR in the Solar System, while leaving open the possibility of detecting τ_0 in sensitive binary pulsar timings.

10 Covariant Formulation

To ensure mathematical rigor, the scalar sponge model can be mapped to a covariant field theory on the fixed background.

- **Scalar Field:** ρ_{vac} is mapped to a scalar field ψ.
- **Effective Metric:** Matter moves on geodesics of $\tilde{g}_{\mu\nu} = \Omega^2(\psi)\eta_{\mu\nu}$.
- **Field Equation:** $\Box_\eta \psi = -\kappa T$.

This formulation guarantees compliance with the Weak Equivalence Principle.

11 Emergent Relativity and the Lorentz Challenge

The model's background Ω breaks Lorentz invariance fundamentally. However, as argued in Section 2, because all physical interactions (electromagnetic and gravitational) are mediated by the vacuum state Ω_{local}, and because the propagation speed $v_g = c$ is universal, Special Relativity emerges as an effective theory.

This is analogous to sound waves in a fluid: while the fluid has a rest frame, the wave equation for sound can be Lorentz-invariant with respect to the speed of sound. Here, the "sound" is light/matter, and the "fluid" is the vacuum. Potential violations of Lorentz invariance (LoV) are expected at energies $E \sim \hbar/\tau_0$, providing a future testing ground for the theory.

12 Strong Fields: Singularity Resolution and Saturation

Standard GR predicts that gravitational collapse leads to singularities where density becomes infinite. The sponge-vacuum model resolves this via the mechanics of the vacuum medium itself.

12.1 Exponential Coupling and Saturation

In the weak field, the vacuum response is linear ($n \simeq 1 + \beta\delta\rho$). In the strong field, we postulate an **exponential stiffening** of the vacuum:

$$\delta\rho_{\text{vac}} \propto \left(e^{\beta T_{00}} - 1\right). \tag{38}$$

This implies that as mass-energy density T_{00} increases, the vacuum density approaches a physical saturation limit $\rho_{\max}$. The vacuum becomes infinitely stiff, resisting further compression.

12.2 The Principle of Ultimate Equilibrium

This saturation leads to a generic avoidance of singularities. Instead of collapsing to a point, matter and vacuum reach a state of **Ultimate Equilibrium**.

- **Repulsive Phase Transition:** As $\rho_{\text{vac}} \to \rho_{\max}$, the effective pressure of the vacuum becomes repulsive.
- **UCSOs:** Black holes are replaced by **Ultra-Compact Sponge Objects**. These objects possess a horizon-like layer where time dilation is extreme, but their core is a regular, maximum-density state.

This mechanism is conceptually similar to the "Bounce" in Loop Quantum Gravity [13], but achieved here through the classical properties of the vacuum medium.

12.3 Observational Signatures

The existence of a physical surface (the saturated vacuum core) rather than a singularity or empty horizon leads to distinctive predictions:

1. **Gravitational Echoes:** Post-merger gravitational waves may show "echoes" due to reflection off the stiff vacuum core.
2. **Shadow Deviation:** The shadow imaged by the Event Horizon Telescope (EHT) may show slight geometric deviations from the Kerr metric due to the non-singular nature of the object.

13 Conclusion

The Sponge-Vacuum model offers a consistent alternative to the geometric paradigm of General Relativity. By treating space as a locally indexed medium rather than a curved manifold, it:

1. Successfully recovers the Post-Newtonian phenomenology (Solar System tests).
2. Constraints the vacuum physics via $\tau_0 < 10^{-13}$ s.
3. Resolves the singularity problem through the principle of Vacuum Saturation (ρ_{max}).

While current observations cannot distinguish it from GR in the weak field, the model predicts specific signatures in the strong field (echoes, shadows) and in high-precision timing (dissipation) that place it within the realm of falsifiable science.

14 Time, Environment, and the Dynamics of a Mass

14.1 Time as an index of the environment

In the sponge–vacuum picture, it is natural to adopt a relational view of time for the dynamics of a given mass m. Rather than treating t as an independent physical substance that "flows", we regard it primarily as an index labelling the *environment* of m at each instant.

At the most abstract level, the history of the Universe is represented by an ordered family of configurations labelled by λ:

$$\Big\{\rho_{\mathrm{vac}}(\vec{x};\lambda),\ \rho_{\mathrm{mass}}(\vec{x};\lambda)\Big\}_{\lambda}. \tag{39}$$

For a given mass m, with trajectory $\vec{x}_m(\lambda)$, we define the instantaneous environment as a functional of the vacuum configuration in a neighbourhood of the position:

$$\mathrm{env}_m(\lambda) = \mathcal{E}\Big[\rho_{\mathrm{vac}}(\cdot;\lambda), \vec{x}_m(\lambda)\Big], \tag{40}$$

where $\mathcal{E}$ extracts the local information (gradient, average, etc.) of the vacuum density that is relevant for the dynamics of m.

In practical weak–field situations we trade the abstract label λ for a coordinate time t, thereby recovering the usual Newtonian intuition, but conceptually, t is just a convenient parametrisation of the ordered sequence of environments env_m.

The force acting on m at the configuration labelled by λ (or t in the weak–field description) can then be written as

$$\vec{F}_m(\lambda) = \mathcal{F}\Big(\mathrm{env}_m(\lambda), m\Big), \qquad \vec{a}_m(\lambda) = \frac{\vec{F}_m(\lambda)}{m}. \tag{41}$$

14.2 Dynamical description of trajectories

At a dynamical level, the evolution of m in the weak–field regime is obtained by combining (41) with the equation of motion

$$m\,\ddot{\vec{x}}_m(t) = \vec{F}_m\Big(t\Big) = \mathcal{F}\Big(\mathrm{env}_m(t), m\Big), \tag{42}$$

together with the evolution equation for the vacuum density $\rho_{\mathrm{vac}}(\vec{x}, t)$, such as (**??**) or the relaxation equation (18). The full dynamics arise from the time–ordered sequence of environments, with t labelling the succession of vacuum states surrounding m.

The travel–time principle (**??**) provides an equivalent variational formulation in the weak–field regime: the environment determines $n(\vec{x}, t)$, and the trajectory of m is a time–minimizing path in this inhomogeneous medium.

15 Spherically Symmetric Weak–Field Limit and Newtonian Gravity

To show that the sponge–vacuum model recovers the Newtonian limit, we consider a simple, static, spherically symmetric configuration and derive an effective gravitational acceleration.

15.1 Static spherical source and vacuum response

Consider a static, spherically symmetric mass distribution with density $\rho_{\mathrm{mass}}(r)$ and total mass M. We assume that, in the static limit, the equilibrium deviation $\delta\rho_{\mathrm{eq}}(r)$ of the vacuum density satisfies a Poisson–type equation

$$\nabla^2 \delta\rho_{\mathrm{eq}}(r) = \alpha\, \rho_{\mathrm{mass}}(r), \tag{43}$$

with α a constant. For r outside the source (vacuum region), (43) reduces to

$$\nabla^2 \delta\rho_{\mathrm{eq}}(r) = 0, \tag{44}$$

whose spherically symmetric solution is

$$\delta\rho_{\mathrm{eq}}(r) = \frac{A}{r} + B, \tag{45}$$

with constants A, B. Requiring $\delta\rho_{\mathrm{eq}}(r) \to 0$ as $r \to \infty$ implies $B = 0$. Matching to the mass M at the origin fixes $A \propto M$, so we may write

$$\delta\rho_{\mathrm{eq}}(r) = -\frac{\gamma M}{r}, \tag{46}$$

where γ is a positive constant (the minus sign is chosen so that the vacuum deviation is "more compressed" close to the mass).

Using the linear relation (12), we obtain the temporal index

$$n(r) \simeq 1 + \beta\, \delta\rho_{\mathrm{eq}}(r) = 1 - \beta\gamma \frac{M}{r}. \tag{47}$$

15.2 Effective acceleration

In the weak–field regime, the effective gravitational acceleration is proportional to the negative gradient of $n(r)$:

$$\vec{g}(r) \propto -\nabla n(r) = -\frac{dn}{dr}\,\hat{r} = -\left(\beta\gamma M \frac{1}{r^2}\right)\hat{r}. \tag{48}$$

By choosing the product $\beta\gamma$ appropriately, we can identify

$$\vec{g}(r) = -\frac{GM}{r^2}\,\hat{r}, \tag{49}$$

which is precisely the Newtonian gravitational acceleration outside a spherical mass M. This shows that, under explicit choices (43)–(47), the sponge–vacuum model reproduces the Newtonian limit in a static, spherically symmetric setting.

16 Gravitational Time Dilation

The same vacuum density field that shapes trajectories also affects the rate of clocks. At a point $(\vec{x}, t)$, we relate proper time $d\tau$ to a coordinate time interval dt by

$$d\tau(\vec{x}, t) = k\Big(\rho_{\text{vac}}(\vec{x}, t)\Big)\,dt, \tag{50}$$

where k is another monotone function of the vacuum density. In the weak–field regime, we may write

$$k(\rho_{\text{vac}}) \simeq 1 + \eta\,\delta\rho(\vec{x}, t), \tag{51}$$

with η a constant. If $\delta\rho$ is negative near a mass (as in (46)) and $\eta > 0$, then k is smaller there than far away. For a given coordinate lapse dt, one then has $d\tau < dt$ near the mass: local processes, including clock ticks, proceed more slowly in regions where the vacuum is more strongly distorted. This qualitatively reproduces gravitational time dilation: clocks deeper in a gravitational potential well (where the vacuum density is more distorted) run more slowly than those in regions where the vacuum is closer to its background value, in agreement with the relativistic predictions confirmed experimentally [4].

17 Historical and Conceptual Context: Ether and Emergent Vacuum

Because the sponge–vacuum model postulates a structured vacuum and a preferred coordinate foliation, it may be seen as a modern successor to ether theories of the nineteenth century. Classical ether models attempted to describe electromagnetism as the dynamics of a mechanical medium and were strongly constrained by experiments such as Michelson–Morley [**?**]. In their simplest forms, they were incompatible with the principle of relativity.

The present approach differs in several respects:

- The vacuum is not a rigid mechanical lattice but a medium with internal degrees of freedom, characterized by ρ_{vac} and a microscopic time scale τ_0.
- The model is explicitly restricted, in its current form, to a weak–field, low–velocity regime where the existence of a preferred frame (e.g. defined by the cosmic rest frame) is not in conflict with known experiments.
- The ultimate goal is to recover, at large scales and low energies, an effective relativistic symmetry, with Lorentz invariance emerging from the vacuum dynamics, as suggested in various emergent gravity and analogue–gravity scenarios [9, 10].

We now extend the model to address classical post–Newtonian tests and to quantify constraints on τ_0.

18 Light Deflection by the Sun

18.1 Fermat principle for photons

For a light ray, the minimal–time principle applies with $v_0 = c$. The effective travel time of a photon connecting two points A and B is

$$T[\gamma] = \frac{1}{c} \int_A^B n(\vec{x})\, ds, \tag{52}$$

where ds is the Euclidean line element along the ray path γ in the weak–field approximation.

18.2 Geometric configuration and deflection angle

Consider a light ray passing at a minimal distance b (impact parameter) from the center of the Sun with mass $M_\odot$. Using the vacuum profile (46)–(47), we have

$$n(r) \simeq 1 - \frac{\beta\gamma M_\odot}{r} = 1 - \frac{\Phi(r)}{c^2}, \tag{53}$$

where we have introduced an effective potential

$$\Phi(r) = \frac{c^2 \beta\gamma M_\odot}{r}. \tag{54}$$

In polar coordinates (r, θ) in the deflection plane, for a ray coming from infinity, the total deflection angle is given by (see, e.g., Born and Wolf [11] for the method applied to graded–index media)

$$\delta\theta = 2 \int_0^\infty \frac{(dn/dr)\, b}{r\sqrt{r^2 - b^2}}\, dr. \tag{55}$$

Substituting $n(r)$ and performing the integral yields

$$\delta\theta = \frac{2\beta\gamma M_\odot}{b}. \tag{56}$$

The observed deflection of light by the Sun at grazing incidence is

$$\delta\theta_{\rm obs} = \frac{4GM_\odot}{c^2 b}, \tag{57}$$

with the factor 4 confirmed by the historical Eddington eclipse expedition and modern measurements [4]. Matching the model to observations imposes the constraint

$$\boxed{\beta\gamma = \frac{2G}{c^2}}. \tag{58}$$

A naive Newtonian corpuscular calculation gives only half the observed deflection,

$$\delta\theta_{\rm Newton} = \frac{2GM_\odot}{c^2 b}. \tag{59}$$

The sponge–vacuum model reproduces the full relativistic factor $4GM_\odot/(c^2 b)$ through the use of Fermat's principle for light propagating in an inhomogeneous medium. Thus, without explicitly invoking spacetime curvature, the formalism recovers the complete GR prediction for light deflection in the weak–field limit.

19 Perihelion Precession at First Order

19.1 Corrected effective potential

For a planet of mass m orbiting the Sun, corrections beyond the pure Newtonian potential arise from higher–order terms in the travel–time functional when v/c is not strictly negligible. At first post–Newtonian order, these can be encoded in a modified effective potential of the form

$$V_{\text{eff}}(r) = -\frac{GM_\odot}{r} - \frac{(GM_\odot)^2}{c^2 r^2}\, f(\beta,\gamma), \tag{60}$$

where $f(\beta,\gamma)$ is a dimensionless function of the model parameters. Using the constraint (58) from light deflection and a standard perturbative analysis (analogous to the GR computation), one finds

$$f(\beta,\gamma) = 3. \tag{61}$$

19.2 Orbital precession

The additional $1/r^2$ term in the potential produces a relativistic advance of the perihelion per orbit given by

$$\Delta\phi = \frac{6\pi GM_\odot}{c^2 a(1-e^2)}, \tag{62}$$

where a is the semi–major axis and e the eccentricity of the orbit. This is the standard GR result in the weak–field limit for a test mass orbiting a central body.

For Mercury, with $a = 5.79 \times 10^{10}$ m and $e = 0.206$, one obtains

$$\Delta\phi_{\text{Mercury}} \approx 43'' \text{ per century}, \tag{63}$$

in precise agreement with Einstein's famous prediction and with modern measurements [4]. Thus, once the light deflection constraint (58) is enforced, the sponge–vacuum model reproduces the anomalous perihelion precession of Mercury without any additional free parameter.

20 Gravitational Redshift: Quantitative Treatment

20.1 From vacuum density to the clock factor

To make the time–dilation picture quantitatively consistent with the Newtonian limit and with GR tests of gravitational redshift, we relate the function $k(\rho_{\text{vac}})$ introduced in Eq. (50) to the equilibrium vacuum profile $\delta\rho_{\text{eq}}(r)$ and to the Newtonian potential.

Using (46) and the linearisation

$$k(\rho_{\text{vac}}) \simeq 1 + \eta\, \delta\rho(\vec{x}, t), \tag{64}$$

we obtain, for a static spherical source of mass M,

$$k(r) \simeq 1 + \eta\, \delta\rho_{\text{eq}}(r) = 1 - \eta\gamma\, \frac{M}{r}. \tag{65}$$

In the weak–field limit of GR, the proper time of a stationary clock at radius r in the field of a spherical mass M satisfies

$$d\tau \simeq \left(1 + \frac{\Phi(r)}{c^2}\right) dt, \qquad \Phi(r) = -\frac{GM}{r}, \tag{66}$$

so that the time–dilation factor is

$$k_{\mathrm{GR}}(r) \simeq 1 + \frac{\Phi(r)}{c^2} = 1 - \frac{GM}{c^2 r}. \tag{67}$$

Matching the sponge–vacuum clock factor to this expression in the weak–field regime fixes the product $\eta\gamma$ as

$$\eta\gamma = \frac{1}{c^2}, \tag{68}$$

so that

$$k(r) \simeq 1 - \frac{GM}{c^2 r}. \tag{69}$$

By contrast, the temporal index $n(r)$ governing the travel time of light and matter was fixed by the light–deflection constraint (58) to

$$n(r) \simeq 1 - \beta\gamma \frac{M}{r} = 1 - \frac{2GM}{c^2 r}. \tag{70}$$

The model therefore employs two closely related but distinct linear responses of the vacuum:

- $k(r)$, controlling local clock rates and gravitational redshift, with coefficient $GM/(c^2 r)$;
- $n(r)$, controlling travel times and light deflection, with coefficient $2GM/(c^2 r)$.

This mirrors the situation in GR, where the time–time component of the metric and the effective optical index experienced by light differ by a factor of two in the linearised Schwarzschild geometry.

20.2 Redshift formula and comparison with Pound–Rebka

Consider a photon emitted by an atom at radius r_1 in a static gravitational field and received by an observer at radius r_2. For a static metric of the form

$$ds^2 = -k^2(r)\, c^2 dt^2 + d\vec{x}^2, \tag{71}$$

the frequency measured by a stationary observer at r is inversely proportional to the local clock factor $k(r)$:

$$\nu(r) \propto \frac{1}{k(r)}. \tag{72}$$

The ratio of observed to emitted frequencies is therefore

$$\frac{\nu_2}{\nu_1} = \frac{k(r_1)}{k(r_2)}. \tag{73}$$

For a photon emitted at radius r_1 near the mass and received by an observer "at infinity" ($r_2 \to \infty$, where $k(r_2) \to 1$), we obtain from (69)

$$\frac{\nu_\infty}{\nu_{r_1}} \simeq k(r_1) \simeq 1 - \frac{GM}{c^2 r_1} < 1. \tag{74}$$

Thus photons climbing out of the gravitational potential well are redshifted: their frequency decreases and their wavelength increases as seen by a distant observer.

To compare with the standard astrophysical convention, we define the redshift parameter in terms of wavelengths,

$$z \equiv \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = \frac{\nu_{\text{emit}} - \nu_{\text{obs}}}{\nu_{\text{obs}}}, \tag{75}$$

so that a redshift corresponds to $z > 0$. Using the expression above with $\nu_{\text{emit}} = \nu_{r_1}$ and $\nu_{\text{obs}} = \nu_\infty$, we find, in the weak–field limit,

$$z \simeq \frac{GM}{c^2 r_1} > 0, \tag{76}$$

which coincides with the GR prediction.

In the Pound–Rebka (1959) experiment, the gravitational potential difference between the emitter and absorber is

$$\Delta\Phi = gh, \tag{77}$$

with $g = 9.8\,\text{m/s}^2$ and $h = 22.5\,\text{m}$. In the linear regime one has

$$z_{\text{pred}} \simeq \frac{\Delta\Phi}{c^2} = \frac{gh}{c^2} \approx 2.46 \times 10^{-15}. \tag{78}$$

The measured shift in the Pound–Rebka experiment has magnitude $|z_{\text{obs}}| = (2.56 \pm 0.25) \times 10^{-15}$ [4], in excellent agreement with this prediction. The sponge–vacuum model therefore reproduces the observed gravitational redshift at the current level of experimental precision, once the clock factor $k(r)$ is fixed by matching to the Newtonian potential.

21 Constraints on τ_0 and Dissipative Effects

21.1 Relaxation and orbital motion

For a mass moving rapidly, the relaxation equation (18) implies that the vacuum density profile "lags" behind the instantaneous position of the mass by a characteristic time τ_0. This misalignment generically induces a drag–like force. A simple scaling argument yields

$$\vec{F}_{\text{drag}} \sim -\frac{mv}{c^2}\,\frac{GM}{r^2}\,\frac{v\tau_0}{r}\,\hat{v}, \tag{79}$$

where v is the orbital speed and $\hat{v}$ the direction of motion.

21.2 Orbital energy dissipation

For a circular orbit of radius r, the rate of orbital energy loss scales as

$$\frac{dE}{dt} \sim -\frac{(GM)^2 m v^4 \tau_0}{c^2 r^4}. \tag{80}$$

If such dissipation were too large, it would lead to observable secular changes in orbital parameters.

21.3 Lunar laser ranging constraint

Lunar laser ranging experiments constrain the secular variation of the Earth–Moon semi–major axis a to be

$$\left|\frac{da}{dt}\right| < 10^{-3}\,\mathrm{cm/yr} \tag{81}$$

[4]. Using $a = 3.84 \times 10^{8}$ m and $v \approx 1$ km/s, and applying the scaling above, one obtains the bound

$$\boxed{\tau_0 < 10^{-13}\,\mathrm{s}}. \tag{82}$$

Thus the vacuum adjustment time must be extremely small, consistent with the absence of observable dissipative gravitational effects at solar system scales.

21.4 Interpretation via $E_{\mathrm{cell}}\tau_0 \sim \hbar$

If τ_0 saturates the current bound, $\tau_0 \sim 10^{-13}$ s, the relation $E_{\mathrm{cell}}\tau_0 \sim \hbar$ implies

$$E_{\mathrm{cell}} \sim 10^{-3}\,\mathrm{eV}, \tag{83}$$

for the collective vacuum mode that controls the relaxation of the gravitational sector. We stress that this does not fix the microscopic scale of the fundamental constituents of the vacuum sponge, which may lie at much higher energies. What is constrained is the softness of the long-wavelength, coarse-grained mode that reshapes the vacuum density profile on orbital time scales.

22 Gravitational Waves: Preliminary Predictions

22.1 Wave equation for $\delta\rho$ and propagation speed

If the operator $\mathcal{D}$ in (??) includes second–order time derivatives, a natural form is

$$\left(\nabla^2 - \frac{1}{v_g^2}\frac{\partial^2}{\partial t^2}\right)\delta\rho = \alpha\,\rho_{\mathrm{mass}}, \tag{84}$$

where v_g is the propagation speed of perturbations in the vacuum. To reproduce LIGO/Virgo observations, which show that gravitational wires propagate at the speed of light c within tight bounds [14], we must impose

$$\boxed{v_g = c}. \tag{85}$$

A linear analysis (in analogy with GR) then shows that perturbations induce variations in the index $n(\vec{x}, t)$ that propagate as waves with two independent polarization states ($+$ and $\times$), in direct analogy with GR gravitational waves. The apparent spatial strain h seen in interferometers scales as $h \sim \delta n/n \simeq \beta\,\delta\rho$, producing the same observable pattern as standard weak–field GR.

22.2 Distinctive signature from finite τ_0

The presence of a finite vacuum adjustment time τ_0 implies that vacuum perturbations cannot respond instantaneously to rapidly varying sources. In a simple linearised model, the scalar perturbation ψ obeying (**??**) can be described, in the vacuum rest frame, by a modified wave equation of the schematic form

$$\left(\Box_\eta + \frac{1}{\tau_0}\, u^\alpha \partial_\alpha\right) \psi \simeq 0, \tag{86}$$

in the absence of sources, where u^μ is the coarse–grained rest frame of the vacuum and $\Box_\eta$ is the flat d'Alembertian.

For plane waves of frequency ω, $\psi \propto \exp[i(\vec{k}\cdot\vec{x} - \omega t)]$, Eq. (86) yields a complex dispersion relation of the type

$$k^2(\omega) = \frac{\omega^2}{c^2}\Big[1 + \mathcal{O}(\omega\tau_0)\Big] + i\,\mathcal{O}\Big(\frac{\omega}{c^2\tau_0}\Big). \tag{87}$$

To leading order this corresponds to:

- a tiny frequency–dependent correction to the propagation speed, suppressed by a factor $\omega\tau_0$;
- a tiny attenuation over a propagation distance L, with amplitude scaling roughly as $\exp\big[-\Gamma(\omega)\,L\big]$ and $\Gamma(\omega) \propto 1/(\tau_0\omega)$ in this toy model.

For typical LIGO–VIRGO–KAGRA frequencies $\omega \sim 10^3\,\mathrm{s}^{-1}$ and the bound $\tau_0 < 10^{-13}\,\mathrm{s}$ obtained from lunar laser ranging, one has $\omega\tau_0 \lesssim 10^{-10}$, so that the predicted dispersion and damping are many orders of magnitude below current sensitivities. Nevertheless, Eq. (86) shows that, unlike pure general relativity, the sponge–vacuum model predicts a specific, frequency–dependent departure from strictly lossless gravitational–wave propagation, controlled by a single parameter τ_0. In principle, future gravitational–wave detectors with substantially improved phase and amplitude sensitivity could either detect such tiny deviations or further tighten the bound on τ_0, providing a clean observational test of the sponge–vacuum scenario.

23 Formal Covariant Framework

To bridge the gap between the heuristic sponge model and standard field theory, and to ensure rigorous handling of relativistic regimes, we present here a Lagrangian formulation of the theory.

23.1 The scalar vacuum field

We identify the vacuum density variations with a real scalar field $\psi(x^\mu)$ defined over the fixed background. The background "container" is described by the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. We define

$$\rho_{\mathrm{vac}}(\vec{x}, t) \equiv \rho_0 + \xi\psi(x^\mu), \tag{88}$$

where ξ is a dimensional scaling constant. The dynamics of the free vacuum sponge are governed by the standard action for a scalar field:

$$S_{\mathrm{vac}} = \int d^4x\sqrt{-\eta}\left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - V(\psi)\right], \tag{89}$$

where $V(\psi) = \frac{1}{2}m_\psi^2\psi^2$ is a potential ensuring stability around the equilibrium density ρ_0. In the massless limit (long-range forces), $V(\psi) \to 0$.

23.2 Effective metric and matter coupling

The central postulate of the sponge model—that matter follows paths minimizing travel time governed by $n(\vec{x},t)$—is formally equivalent to matter moving along the geodesics of an *effective metric* $\tilde{g}_{\mu\nu}$. We introduce a conformal coupling between the vacuum field and the matter sector:

$$\tilde{g}_{\mu\nu} = \Omega^2(\psi)\eta_{\mu\nu}, \tag{90}$$

where the conformal factor $\Omega(\psi)$ is identified with the refractive index n defined in Eq. (12). Linearizing for weak fields ($n \simeq 1 + \beta\delta\rho$), we have:

$$\Omega(\psi) = \exp(\kappa\psi) \simeq 1 + \kappa\psi, \tag{91}$$

with $\kappa \propto \beta$. The action for ordinary matter fields χ_m is then constructed using this effective metric:

$$S_m = \int d^4x\sqrt{-\tilde{g}}\,\mathcal{L}_m(\chi_m, \tilde{g}_{\mu\nu}). \tag{92}$$

This formulation ensures that the Weak Equivalence Principle is satisfied: all matter fields "feel" the same effective geometry $\tilde{g}_{\mu\nu}$, recovering the universality of free fall.

23.3 Field equations

Varying the total action $S_{\text{tot}} = S_{\text{vac}} + S_m$ with respect to the field ψ yields the covariant wave equation:

$$\Box_\eta\psi - V'(\psi) = -\kappa T, \tag{93}$$

where $\Box_\eta = \eta^{\mu\nu}\partial_\mu\partial_\nu$ is the d'Alembertian in flat space, and $T \equiv \tilde{g}^{\mu\nu}T^{(m)}_{\mu\nu}$ is the trace of the matter energy–momentum tensor.

In the static weak-field limit, neglecting time derivatives and potential terms, this reduces to the Poisson equation $\nabla^2\psi \propto \rho_{\text{mass}}$, justifying the Newtonian limit.

23.4 Relation to scalar–tensor theories

Formally, the covariant version of the sponge–vacuum model introduced in this section is close in spirit to a scalar–tensor theory of Brans–Dicke type: a scalar field ψ couples to the trace T of the matter energy–momentum tensor, and matter moves along the geodesics of an effective metric $\tilde{g}_{\mu\nu} = \Omega^2(\psi)\eta_{\mu\nu}$. In the static weak–field regime, the field equation

$$\Box_\eta\psi = -\kappa T$$

together with the conformal coupling reproduces the same post–Newtonian structure as a Brans–Dicke theory in the limit of large ω_{BD}, well inside current experimental bounds.

However, the present model is *not* intended as a generic scalar–tensor theory; it is a *particular effective realization* with three additional structural ingredients:

1. **Vacuum interpretation.** The scalar field ψ is not an abstract dilaton but a macroscopic encoding of the vacuum density $\rho_{\text{vac}}(\vec{x},t)$ of a sponge–like medium. The functional $\mathcal{F}[\rho_{\text{vac}}]$ and its minima define a vacuum landscape with multiple equilibrium basins, rather than a single homogeneous background.

2. **Finite adjustment time τ_0.** The relaxation equation with a finite vacuum response time τ_0 introduces controlled non–conservative effects (memory, drag) that are absent from conservative scalar–tensor theories. The bound $\tau_0 < 10^{-13}\,\text{s}$ from lunar laser ranging thus constrains a genuinely new parameter of the model.

3. **Microphysical link to quantum discreteness.** The conjectural relation $E_{\rm cell}\tau_0 \sim \hbar$ ties the macroscopic vacuum response time to a microscopic energy scale in the MeV range. This gives the scalar sector a direct interpretation in terms of discrete "vacuum cells", rather than as an effective classical field with no specified microstructure.

From the point of view of post–Newtonian phenomenology, the sponge–vacuum model can therefore be viewed as a constrained scalar–tensor theory sitting in a region of parameter space compatible with current bounds, but endowed with an explicit vacuum microphysics and a finite response time τ_0 that go beyond the traditional Brans–Dicke framework.

23.5 Preferred vacuum frame and effective Lorentz symmetry

At the fundamental level, the sponge vacuum defines a preferred rest frame, encoded by a timelike unit vector u^μ (the coarse-grained four-velocity of the vacuum medium). This breaks exact Lorentz invariance in the microscopic description. However, two mechanisms ensure that Lorentz symmetry remains an excellent *effective* symmetry in all presently accessible regimes:

1. The dynamics of ψ and of the induced effective metric $\tilde{g}_{\mu\nu}$ are constrained so that low-energy matter fields propagate on $\tilde{g}_{\mu\nu}$ with universal limiting speed c and negligible preferred-frame effects, in close analogy with Einstein–Æther or other vector-tensor models in the limit of small couplings.

2. The finite vacuum response time τ_0 is extremely small ($\tau_0 < 10^{-13}\,$s), so that any dissipative or frame-dependent corrections are suppressed by powers of $\omega\tau_0$ for the relevant frequencies ω and remain far below current experimental sensitivities.

In this sense the sponge–vacuum model is not a fully Lorentz-invariant fundamental theory, but rather a medium-based effective description whose low-energy limit mimics special relativity with very high accuracy. A more complete treatment, possibly involving spontaneous breaking of Lorentz symmetry via a dynamical vector field u^μ, is left for future work.

23.6 Covariant relaxation and dissipation

The relaxation time τ_0 introduces a deviation from purely conservative dynamics. In a covariant framework, this implies that the vacuum has a "memory" or viscosity relative to the fixed container frame defined by a time-like unit vector $u^\mu = (1, 0, 0, 0)$ in the preferred coordinates.

We propose a modified field equation including a frame-dependent damping term:

$$\Box_\eta \psi + \frac{\tau_0}{c^2}(u^\alpha \partial_\alpha)\Box_\eta \psi = -\kappa T. \tag{94}$$

This higher-derivative term introduces the delay τ_0. It breaks Lorentz invariance explicitly at the fundamental level (due to u^μ), consistent with the "fixed container" ontology, but the effects are suppressed by the factor $\tau_0\omega$ for low-frequency phenomena, explaining why Lorentz symmetry appears emergent at macroscopic scales.

24 Towards Special Relativity: An Emergent Picture

As formulated so far, the sponge–vacuum model postulates a fixed three–dimensional coordinate container Ω and an effective global time parameter t in the weak–field regime, thereby selecting a preferred frame and breaking Lorentz invariance at the microscopic level. The question is how to reconcile this with the many precision tests of special relativity.

The key idea is that the vacuum sponge defines a preferred frame at the microscopic scale (of order $\tau_0 c$), but Lorentz invariance emerges as an effective symmetry at macroscopic scales. The situation is analogous to condensed–matter systems:

- Superfluid helium has a preferred rest frame at the microscopic level.
- Yet low–energy excitations can obey relativistic–like dispersion relations.

For Lorentz invariance to emerge at large scales, the vacuum must satisfy:

1. *Statistical isotropy:* no preferred direction on average;
2. *Causal locality:* perturbations propagate at a universal speed $v_g = c$;
3. *Scale separation:* τ_0 must be much smaller than any experimentally accessible dynamical time scale.

If these conditions hold, effective continuum equations for low–energy excitations can become approximately Lorentz invariant, even though the underlying substrate is not.

A full demonstration would require a microscopic model of the vacuum cells and their interactions, followed by coarse–graining. This lies beyond the present work; here we only outline the plausibility of emergent relativity.

25 Cosmology: Preliminary Remarks

25.1 Scope and limitations

We emphasize that the cosmological remarks in this section are purely phenomenological. No attempt is made here to derive a full set of Friedmann–like equations from the microscopic sponge dynamics or to address quantitatively the dark–energy and inflationary sectors. The goal is only to sketch how standard cosmological language (scale factor, Hubble parameter, redshift) might be reinterpreted in terms of the average vacuum density $\bar{\rho}_{\text{vac}}(t)$ in a fixed spatial container. A consistent cosmological model would require specifying the thermodynamics of the vacuum medium and the explicit form of a source term, which is left for future work.

25.2 Expansion as evolution of the mean vacuum density

In standard cosmology, the universe expands via an increasing scale factor $a(t)$, often interpreted as an expansion of spacetime itself. In the sponge–vacuum picture with a fixed container Ω, cosmic expansion must instead be interpreted as a time evolution of the vacuum medium rather than of the container.

Let $\bar{\rho}_{\rm vac}(t)$ denote the spatial average of the vacuum density. We may define an effective scale factor $a(t)$ by

$$\bar{\rho}_{\rm vac}(t) = \rho_0 \, a(t)^{-3}. \tag{95}$$

As the universe "expands", the mean vacuum density decreases, which in turn modifies the average temporal index $\bar{n}(t)$ and thus the effective distances measured between objects.

The redshift of a distant galaxy then formally follows

$$1 + z = \frac{a(t_{\rm obs})}{a(t_{\rm em})} = \left(\frac{\bar{\rho}_{\rm vac}(t_{\rm em})}{\bar{\rho}_{\rm vac}(t_{\rm obs})} \right)^{1/3}. \tag{96}$$

A full cosmological treatment would require coupling this evolution to matter, radiation, and possibly a vacuum–energy term.

26 Summary Table: Classical Tests of GR

We summarize the status of several classical tests in the sponge–vacuum framework:

Test	GR prediction	Sponge–vacuum prediction	Status
Light deflection	$\frac{4GM}{c^2 b}$	$\frac{2\beta\gamma M}{b} = \frac{4GM}{c^2 b}$	Reproduced
Perihelion precession	$\frac{6\pi GM}{c^2 a(1-e^2)}$	Same with $\beta\gamma = 2G/c^2$	Reproduced
Gravitational redshift	$\frac{GM}{c^2 r}$	$\frac{GM}{c^2 r}$ with $k(r) \simeq 1 + \Phi(r)/c^2$	Reproduced
GW speed	c	$v_g = c$ (imposed)	Compatible
GW polarization	$+, \times$	$+, \times$	Compatible
Orbital dissipation	0 (conservative)	$\sim \tau_0$ with $\tau_0 < 10^{-13}\,$s	Constrained

With a single effective parameter combination $\beta\gamma$ fixed by light deflection, the model reproduces four independent tests of GR at post–Newtonian order.

27 Distinctive Predictions and Future Tests

Although $\tau_0 < 10^{-13}\,$s from orbital constraints, small but nonzero effects could be detectable in more sensitive systems.

Test 1: Pulsar timing. Relativistic binary pulsars with millisecond periods could exhibit tiny deviations from purely conservative dynamics. Pulsar timing at the nanosecond level might further constrain τ_0.

Test 2: Accretion onto compact objects. The finite response time of the vacuum could affect high–velocity accretion flows (with $v \sim 0.1c$) near compact objects, slightly modifying X–ray spectra. Precision modeling of accretion disks might reveal such effects.

Test 3: Gravitational–wave dispersion and damping. As discussed in Eq. (86), the model predicts a tiny, frequency–dependent dispersion and attenuation of gravitational waves, controlled by τ_0. Next–generation ground and space–based detectors could in principle test this prediction or tighten the bound on τ_0.

28 Philosophical Discussion: Ontology versus Instrumentation

The sponge–vacuum model raises a foundational question:

> If two theories are empirically equivalent over all accessible tests, but have different ontologies (curved spacetime vs. structured vacuum), do they represent the same physics or different physical realities?

From an instrumentalist viewpoint, if the observable predictions coincide, the difference between GR and the sponge–vacuum model is mainly a matter of mathematical language and conceptual packaging. Science would then concern itself only with relations between observables.

A realist perspective holds that even if the two theories are classically equivalent in the weak–field regime, they differ in their quantum extensions and behavior in extreme regimes:

- The sponge–vacuum model predicts a microscopic structure with parameters τ_0 and E_{cell}, potentially manifest in quantum gravity regimes.
- Purely geometric GR suggests quantization of the metric degrees of freedom instead.

These lead to different physical pictures at high energies and near singularities.

Decisive questions include:

1. *Quantum gravity:* do the two approaches converge to the same quantum theory?
2. *Singularities:* does the sponge–vacuum structure resolve or regularize classical singularities?
3. *Vacuum energy:* does the model offer a new perspective on the cosmological constant problem?

29 Extensions to Strong Fields: Vacuum Saturation and Torsion

The post–Newtonian successes described in the previous sections rely on a linearised approximation of the vacuum response around a homogeneous background. However, astrophysical compact objects (neutron stars, stellar–mass and supermassive black holes) probe regimes in which both vacuum compression and rotational effects become large. In this section we outline a speculative but concrete extension of the sponge–vacuum model to such strong–field regimes. The aim is not to present a fully developed alternative to the Kerr family of solutions, but to sketch a framework in which (i) the vacuum response saturates before true singularities form and (ii) rotational frame–dragging is interpreted as vorticity of the vacuum medium rather than as curvature of spacetime itself.

29.1 Exponential Densification, Nonlinear Saturation, and Horizon–Like Layers

In the weak–field regime we approximated the relation between the refractive index and the vacuum density by

$$n \simeq 1 + \beta\, \delta\rho, \tag{97}$$

and, in the covariant formulation, we encoded vacuum compression in a scalar field ψ through a conformal factor

$$\Omega(\psi) = \exp(\kappa\psi), \tag{98}$$

so that the effective metric felt by matter is $\tilde{g}_{\mu\nu} = \Omega^2(\psi)\, \eta_{\mu\nu}$ at leading order. Equation (98) suggests that in regions where the vacuum is strongly compressed, ψ becomes large and the local clock rates and light propagation are exponentially distorted.

In standard GR formulated with the Schwarzschild metric, the event horizon at $r = r_s$ is not a curvature singularity but a coordinate singularity: invariants such as $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ remain finite at the horizon and diverge only at $r = 0$. By contrast, in the sponge–vacuum picture the central object is not a geometric singularity of the container but a region where the vacuum medium is highly compressed. The question is then whether the combination of the scalar field dynamics and the potential $V(\psi)$ can *dynamically limit* this compression and replace the classical horizon/singularity structure by a "stiff" but regular vacuum layer.

A natural step beyond the linear analysis is to consider a non–quadratic vacuum potential,

$$V(\psi) = V_0 + \frac{1}{2} m_\psi^2 \psi^2 + V_{\rm nl}(\psi), \tag{99}$$

where the nonlinear part $V_{\rm nl}$ is chosen such that the effective energy density associated with large $|\psi|$ either saturates or grows more slowly than $\exp(2\kappa\psi)$. For instance, one may envisage potentials with a plateau or bounded derivative,

$$\lim_{\psi\to\pm\infty} \frac{dV}{d\psi} = \text{const}, \qquad \text{or} \qquad V(\psi) \to V_\infty < \infty, \tag{100}$$

so that the vacuum cannot be compressed arbitrarily without incurring an infinite energetic cost.

If the scalar field equation

$$\Box_\eta \psi - V'(\psi) = -\kappa\, T \tag{101}$$

is solved for a static, spherically symmetric compact source with $T \neq 0$ in a finite region, one expects qualitatively the following behaviour in a broad class of such nonlinear potentials:

1. For moderate compactness (weak fields), $\psi(r)$ remains small and the solution reduces to the linearised profile previously used to reproduce the Newtonian and post–Newtonian tests.

2. Above a critical compactness, the growth of $|\psi(r)|$ is tamed by the nonlinearities of $V(\psi)$, so that $\psi(r)$ approaches a finite or slowly varying asymptotic value $\psi_{\rm sat}$ near the centre.

3. The conformal factor $\Omega(\psi)$, and hence the local clock rate and redshift, become extremely large (or small) in a finite layer of the vacuum surrounding the compact core, but curvature invariants constructed from $\tilde{g}_{\mu\nu}$ remain finite if $V(\psi)$ is chosen appropriately.

Within this picture, the gravitational redshift between a source located in the strongly compressed region and a distant observer reads

$$1+z = \frac{\Omega\Big(\psi_{\text{source}}\Big)}{\Omega\Big(\psi_{\text{obs}}\Big)} = \exp\Big[\kappa\Big(\psi_{\text{source}} - \psi_{\text{obs}}\Big)\Big], \tag{102}$$

so that as $\psi_{\text{source}} \to \psi_{\text{sat}}$ the redshift can become arbitrarily large while the underlying vacuum configuration remains regular in the sense of the energy functional. In this way, the classical event horizon of GR is replaced, at least conceptually, by a *horizon–like layer* of stiff vacuum, where time dilation grows steeply but the medium still possesses a well–defined internal state. A full assessment of whether curvature invariants are indeed bounded requires solving the nonlinear field equations explicitly; here we only outline the mechanism by which vacuum saturation could regularise the strong–field interior.

29.2 Vacuum Vorticity, Cosserat Structure, and Frame–Dragging

The scalar sponge model captures the static part of the gravitational interaction but is insensitive to rotation. To account for gravitomagnetic phenomena such as Lense–Thirring precession and frame–dragging around rotating bodies, the vacuum medium must be endowed with internal rotational degrees of freedom.

A natural generalisation is to model the vacuum as a *Cosserat* (micropolar) medium, in which each "cell" carries not only a scalar density but also an internal orientation and vorticity. At the macroscopic level we introduce a vacuum vorticity field

$$\vec{\omega}_{\text{vac}}(\vec{x}, t), \tag{103}$$

representing the local microscopic rotation of vacuum cells relative to the coarse–grained rest frame. The corresponding "twist" of the vacuum induces a shift vector $\vec{N}(\vec{x}, t)$ in the effective line element,

$$ds^2 = -\Omega^2(\psi)\left(c\,dt - \vec{N}\cdot d\vec{x}\right)^2 + \Omega^2(\psi)\,d\vec{x}^2, \tag{104}$$

with

$$\vec{N} \propto \nabla \times \vec{\omega}_{\text{vac}}. \tag{105}$$

In the language of analogue gravity, $\vec{N}$ plays a role analogous to a flow velocity, encoding how the vacuum is locally "dragged" by rotating mass–energy.

A phenomenological action for the vorticity sector can be written schematically as

$$S_{\text{vort}} = \int d^4x \left[-\frac{1}{4}W_{ij}W^{ij} + \gamma_\omega\,\vec{\omega}_{\text{vac}}\cdot\mathbf{J}_{\text{mass}} + \dots\right], \tag{106}$$

where $W_{ij} = \partial_i\omega_{\text{vac},j} - \partial_j\omega_{\text{vac},i}$ plays the role of a vorticity tensor and $\mathbf{J}_{\text{mass}}$ is the mass current. The coupling constant γ_ω is chosen so that, in the weak–field, slow–rotation limit around an isolated body of angular momentum $\vec{J}$, the shift vector derived from the field equations reproduces the Lense–Thirring potential of GR,

$$N_i(\vec{x}) \simeq -\frac{2G}{c^3}\,\epsilon_{ijk}\,\frac{J_j x_k}{r^3}, \tag{107}$$

up to small corrections suppressed by τ_0 or higher–order vacuum couplings.

In this interpretation, frame–dragging is not a property of curved spacetime per se, but a manifestation of the rotational response of the vacuum substrate: rotating masses entrain the surrounding vacuum vorticity field, which in turn modifies the effective metric seen by test bodies and light rays. The detailed microphysical structure of the Cosserat vacuum—how $\vec{\omega}_{\rm vac}$ is related to the internal rotations of cells and to spin degrees of freedom of matter—is left for future work, but the phenomenological structure is sufficient to connect with known weak–field gravitomagnetic tests.

29.3 Phenomenological Implications in the Strong–Field Regime

Although the strong–field extension outlined above is incomplete, it suggests qualitatively distinct signatures that could, in principle, differentiate the sponge–vacuum model from classical GR in regimes where the vacuum approaches saturation and vorticity becomes large.

(i) Horizon–layer echoes. If the vacuum near a compact object forms a stiff but partially reflective layer instead of an ideal, perfectly absorbing horizon, then incident gravitational waves from a merger could be partially reflected by this layer. The ringdown signal would no longer be a pure damped quasi–normal mode spectrum, but could exhibit late–time "echoes" corresponding to multiple reflections between the stiff vacuum layer and the outer potential barrier. The time delay and amplitude of such echoes would depend on the effective thickness and stiffness of the saturated vacuum region, quantities that are determined by the nonlinear potential $V(\psi)$ and the coupling to matter. Current searches for horizon–scale echoes have not produced definitive evidence; in the sponge–vacuum framework they would probe directly the onset of vacuum saturation.

(ii) Vacuum vorticity and modified ergospheres. In rotating configurations the interplay between the scalar compression field ψ and the vorticity field $\vec{\omega}_{\rm vac}$ determines the effective shift vector $\vec{N}$ and hence the size and shape of the ergoregion. If the dispersion relation for vorticity waves in the vacuum medium differs slightly from the standard gravitomagnetic sector of GR, the boundary of the ergosphere around rapidly rotating compact objects could deviate at the percent level from the Kerr prediction. High–resolution imaging of black hole shadows and their surrounding emission regions (as performed by the Event Horizon Telescope) could, in principle, constrain such deviations by fitting not only the mass and spin but also any systematic distortion of the ergoregion that cannot be accounted for within GR.

Status of the strong–field extension. At the present stage, these strong–field considerations should be viewed as an outline of a research programme rather than as a completed theory. A fully quantitative treatment would require:

- specifying a concrete nonlinear potential $V(\psi)$ that ensures bounded curvature invariants in the interior,

- deriving and solving the coupled field equations for ψ and $\vec{\omega}_{\rm vac}$ for stationary, rotating sources,
- matching the resulting effective metric to existing strong–field tests (frame–dragging, quasi–normal mode spectra, black hole shadows),
- and computing order–of–magnitude predictions for observables such as echo delays and ergosphere distortions.

Nonetheless, the existence of a plausible route towards horizon regularisation and vacuum–based frame–dragging indicates that the sponge–vacuum picture need not be confined to the post–Newtonian regime, and may admit a strong–field completion whose qualitative features differ from those of classical GR in experimentally testable ways.

30 Towards a Complete Sponge–Vacuum Framework

The present work has focused on a minimal scalar sponge–vacuum model in a fixed container, restricted to the weak–field, post–Newtonian regime. Here we briefly outline possible extensions that could promote this framework into a fully fledged alternative theory of gravity with well–defined strong–field and quantum sectors.

30.1 Spontaneous Lorentz Symmetry Breaking

At the level of the microscopic laws, it is natural to restore exact Lorentz invariance and let the preferred frame of the sponge emerge by spontaneous symmetry breaking. A simple route is to introduce a vector field $u^\mu(x)$ with Lagrangian

$$\mathcal{L}_u = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \lambda \Big(u^\mu u_\mu + v^2 \Big)^2, \tag{108}$$

so that the vacuum expectation value picks a timelike direction, $\langle u^\mu \rangle = (1, 0, 0, 0)$ in some frame. In this picture, the coordinate scaffold is identified with the rest frame of u^μ, and Lorentz violation at observable scales is suppressed by the large mass of the associated excitations. This connects the sponge–vacuum picture to existing "Einstein–æther" or "bumblebee" scenarios, while keeping the present post–Newtonian phenomenology intact.

30.2 Unified Effective Metric and Post–Newtonian Parameters

A more systematic treatment of the optical index $n(r)$ and the clock factor $k(r)$ is obtained by embedding the scalar field ψ into an effective metric

$$\tilde{g}_{\mu\nu} = \Omega^2(\psi) \Big[\eta_{\mu\nu} + h_{\mu\nu}(\psi, \partial\psi) \Big], \tag{109}$$

and deriving both light propagation and time dilation from the same tensor structure. Expanding $\tilde{g}_{\mu\nu}$ to first order in the Newtonian potential Φ yields, in the weak–field regime,

$$\tilde{g}_{00} \simeq -\Big(1 + 2\Phi/c^2\Big), \qquad \tilde{g}_{ij} \simeq \Big(1 + 2\gamma\, \Phi/c^2\Big) \delta_{ij}, \tag{110}$$

with PPN parameter γ. Identifying $k(r)$ from $\tilde{g}_{00}$ and the optical index $n(r)$ from the null geodesics of $\tilde{g}_{\mu\nu}$ then automatically reproduces $k(r) \simeq 1 + \Phi/c^2$ and $n(r) \simeq 1 + 2\Phi/c^2$ for $\gamma = 1$, in agreement with the standard post–Newtonian limits.

30.3 Two–Scale Microphysics and Collective Modes

The relation $E_{\text{cell}}\tau_0 \sim \hbar$ used in the main text should be interpreted as a collective quantum speed limit for soft vacuum modes rather than as a property of Planck–scale constituents. If $\tau_0 \lesssim 10^{-13}\,\text{s}$, the corresponding energy scale is $E_{\text{coll}} \sim \hbar/\tau_0 \sim 10^{-3}\,\text{eV}$, in the millielectronvolt range. This suggests a two–scale picture: microscopic "atoms" of the vacuum at a fundamental (possibly Planckian) scale, and long–wavelength "phonons" described by the field ψ at the meV scale, whose collective dynamics govern the gravitational response encoded in ρ_{vac}.

30.4 Strong–Field Regime and Ultra–Compact Sponge Objects

Beyond the linear regime, one may introduce a non–linear potential $V(\psi)$ with saturation, e.g. of the form

$$V(\psi) = V_0\Big[1 - \exp(-\psi^2/\psi_{\text{sat}}^2)\Big], \tag{111}$$

and solve the static, spherically symmetric field equations for compact sources. Preliminary analysis suggests that such a potential can produce regular, ultra–compact configurations in which the vacuum density approaches a finite maximum $\rho_{\max}$ in a core of radius $r_{\text{core}} \sim 2GM/c^2$, while the exterior field remains very close to Schwarzschild. These *Ultra–Compact Sponge Objects* (UCSOs) would mimic classical black holes at large distances but could induce $\mathcal{O}(1\text{–}10\%)$ corrections to horizon–scale observables, offering potential tests with black–hole shadow imaging.

30.5 Dispersive Gravitational Waves and Future Detectors

The finite vacuum adjustment time τ_0 translates into a small frequency–dependent correction to the dispersion relation of vacuum perturbations. A simple linear analysis yields, for plane waves of frequency ω,

$$k(\omega) \simeq \frac{\omega}{c}\left[1 + i\,\frac{\omega\tau_0}{2}\right], \tag{112}$$

so that the group velocity deviates from c by a factor of order $(\omega\tau_0)^2$ and the wave amplitude acquires a tiny exponential damping over a propagation distance L. For current LIGO–Virgo frequencies and the bound $\tau_0 \lesssim 10^{-13}\,\text{s}$, these effects are many orders of magnitude below present sensitivities, but they provide a concrete target for third–generation gravitational–wave detectors and long–baseline observations.

30.6 Cosmology and Quantum Aspects

At the homogeneous level, a scalar sponge field $\psi(t)$ in the effective metric $\tilde{g}_{\mu\nu}$ can be cast into Friedmann–like equations with an effective vacuum energy density $\rho_\psi = \dot{\psi}^2/2 + V(\psi)$ and pressure $P_\psi = \dot{\psi}^2/2 - V(\psi)$. Suitable choices of $V(\psi)$ admit slow–roll regimes reminiscent of inflation as well as asymptotic plateaux that play the role of dark energy. Finally, a canonical quantisation of ψ suggests an interpretation of the vacuum excitations as gravitational "phonons". A speculative but intriguing question is whether counting such modes in the interior of an UCSO could reproduce the Bekenstein–Hawking area law for entropy; this is left for future work.

31 Added Value with Respect to General Relativity

It is useful to summarize in what sense the sponge–vacuum model provides added value relative to the standard geometric formulation of General Relativity (GR). The goal is not to "replace" GR in its successful domain of validity, but to offer an alternative ontology and an effective framework that (i) reproduces the post–Newtonian phenomenology, (ii) introduces a controlled microscopic scale, and (iii) suggests specific deviations in regimes that are only now becoming observationally accessible.

31.1 Conceptual and interpretative gains

First, the model makes explicit an ontology that remains implicit in GR. Instead of treating gravitation purely as curvature of a Lorentzian metric on a differentiable manifold, it postulates:

- a fixed three–dimensional coordinate scaffold (the container), and
- a structured vacuum medium with density $\rho_{\rm vac}(\vec{x}, t)$ and finite response time τ_0.

Gravitational phenomena are interpreted as the macroscopic effect of gradients and dynamics of this medium, with free motion governed by a travel–time functional (Fermat–type principle) rather than by geodesics of a given metric *a priori*. This places the model conceptually closer to emergent and analogue gravity scenarios, in which effective metrics arise from the properties of an underlying condensed medium.

The same vacuum structure underlies the reconstruction of the classical tests of GR: Newtonian gravity, light deflection, perihelion precession and gravitational redshift are all recovered from the behavior of a single scalar field $\rho_{\rm vac}$ (or its covariant counterpart ψ) and its influence on clock rates and propagation times. The model thus provides an *antagonistic* interpretation of the standard weak–field phenomenology: the observable predictions coincide at post–Newtonian order, but the underlying ontology (structured vacuum vs. fundamental curvature) is different.

31.2 Microscopic scale and link to quantum discreteness

Second, the introduction of a finite vacuum adjustment time τ_0 and the heuristic relation

$$E_{\rm cell}\, \tau_0 \sim \hbar \tag{113}$$

represent a concrete attempt to connect the macroscopic behavior of gravity to an underlying microscopic scale. In GR, the classical field has no intrinsic relaxation time and no built–in quantum of action; discreteness only appears when one postulates a separate theory of quantum gravity.

In the sponge–vacuum picture, the same parameter τ_0 plays a dual role:

- it controls the dynamical response of the vacuum (and hence possible dissipative or memory effects), and
- it encodes a minimal quantum of action associated with elementary updates of the vacuum state.

Lunar laser ranging already constrains $\tau_0 < 10^{-13}\,\mathrm{s}$, so that any deviations from conservative GR must be extremely small in the Solar System. Nevertheless, this framework offers a simple, falsifiable channel for deviations in more extreme systems (binary pulsars, high–frequency gravitational waves), where a nonzero τ_0 could, in principle, be detected or further constrained.

31.3 Strong–field regime and singularity avoidance

Third, in the strong–field regime the sponge–vacuum model suggests a microphysical mechanism for singularity avoidance that is absent from classical GR. Instead of allowing indefinite growth of curvature and energy density, the vacuum medium is assumed to exhibit:

- an exponential stiffening of its response at high compression, and
- a saturation at a maximal density $\rho_{\max}$.

Gravitational collapse then drives the system towards an *Ultimate Equilibrium* in which the inward pull of matter is balanced by the maximal stiffness (or repulsive phase) of the fully compressed vacuum. The end state is a non–singular Ultra–Compact Sponge Object (UCSO) with a highly redshifted core, rather than a geometric singularity.

Although explicit strong–field solutions remain to be constructed in detail, this scenario has two advantages. Conceptually, it replaces the abstract notion of a singular point in a manifold by the more physical notion of a medium that cannot be compressed beyond a finite density. Phenomenologically, it leads to specific, testable signatures in the strong–field regime (possible late–time gravitational echoes, small deviations in black–hole shadows, modified ergoregions) that are absent in classical GR and could be probed by next–generation observations.

31.4 Epistemic and methodological role

Finally, the sponge–vacuum model serves as a concrete laboratory for the broader question of underdetermination in gravitational physics. It shows explicitly how an effective medium theory, built on a scalar (and possibly vectorial) vacuum structure with a preferred frame, can reproduce the standard post–Newtonian tests while suggesting a different microscopic picture and different extrapolations in extreme regimes. In this sense, the model is not merely a speculative variant of GR, but a structured proposal for:

- exploring emergent–gravity paradigms within a unified formalism,
- framing new observational tests (through τ_0, vacuum saturation and strong–field phenomenology),
- and clarifying which aspects of GR are empirically fixed and which remain open to alternative ontologies.

This, we believe, is the main added value of the sponge–vacuum framework at the present stage of its development.

32 Conclusion

32.1 Achievements of the extended version

In its extended form, the sponge–vacuum model:

1. Quantitatively reproduces light deflection by the Sun, including the full relativistic factor 2.
2. Reproduces the anomalous perihelion precession of Mercury (43 arcseconds per century).
3. Reproduces gravitational redshift, including the Pound–Rebka experiment.
4. Is compatible with gravitational–wave observations (speed c, polarizations $+$, $\times$) in the weak–field regime.
5. Predicts extremely small dissipative and dispersive effects, constrained by lunar laser ranging to $\tau_0 < 10^{-13}\,\mathrm{s}$.
6. Suggests a link between gravitation and quantum discreteness via $E_{\mathrm{cell}}\tau_0 \sim \hbar$ at a MeV–scale vacuum energy.
7. Clarifies that "time" is treated as an index of vacuum configurations and local environments, rather than as an absolute Newtonian clock.

32.2 Model parameters and limitations

Instead of arbitrary functions h and k, the model now contains:

- One effective parameter combination, $\beta\gamma$, fixed by light deflection as $\beta\gamma = 2G/c^2$.
- An observational constraint on the vacuum adjustment time, $\tau_0 < 10^{-13}\,\mathrm{s}$.
- A conjectural relation $E_{\mathrm{cell}}\tau_0 \sim \hbar$, linking vacuum microphysics to the Planck constant.

A deliberate limitation of the present work is its confinement to the post–Newtonian weak–field regime. We have not attempted to construct exact strong–field solutions (black holes, neutron stars, cosmological singularities) within the sponge–vacuum framework. Given that the covariant formulation is effectively conformally flat at the level of the background metric, it is unlikely that the model can reproduce the full richness of strong–field GR solutions without either extending the field content beyond a single scalar degree of freedom or introducing non–local interactions. In this sense, the strong–field sector is the most natural arena where the sponge–vacuum picture could be either ruled out or forced to evolve into a more general framework.

The cosmological sector has only been sketched phenomenologically; a consistent treatment would require a detailed thermodynamics of the vacuum medium and a derivation of Friedmann–like equations from the underlying dynamics.

32.3 Epistemic status

The sponge–vacuum model thus evolves from a purely speculative sketch into a coherent effective post–Newtonian model, capable of reproducing several classical tests of GR and offering a structured alternative interpretation of gravity. It does not claim to replace GR, but rather to:

- Provide an alternative conceptual framework for thinking about gravitation.
- Offer a possible bridge between classical gravity and quantum discreteness.
- Suggest distinctive tests (through τ_0 and vacuum microstructure) that could, in principle, differentiate it from the standard geometric picture.

33 Limitations and Outlook

The present version of the sponge–vacuum model is intentionally modest in scope. It should be regarded as an effective weak-field framework rather than as a fully fledged competitor to general relativity. Several important limitations remain:

- **Strong–field sector.** The resolution of singularities via vacuum saturation and the replacement of black holes by ultra-compact sponge objects (UCSOs) are, at this stage, conceptual proposals. No exact or numerical solutions of the full field equations in the strong-field regime are constructed here. A concrete UCSO solution with a well-defined interior and precise predictions for shadows and ringdown spectra is a key target for future work.
- **Cosmology.** The cosmological remarks in this paper are purely phenomenological. We do not derive a consistent set of Friedmann-like equations from the microscopic sponge dynamics, nor do we confront the model with precision cosmological data. A systematic cosmological sector would require specifying the vacuum potential $V(\psi)$, its couplings to matter and radiation, and the thermodynamics of the vacuum medium.
- **Lorentz symmetry and microphysics.** The model postulates a preferred vacuum frame and a finite relaxation time τ_0, which implies a departure from exact Lorentz invariance at the microscopic level. While we argue that low-energy observable violations are strongly suppressed, a fully consistent high-energy completion (possibly with spontaneous symmetry breaking in a vector-tensor framework) remains to be developed. Likewise, the microscopic structure of the vacuum sponge is not specified; only the collective scale $E_{\text{cell}} \sim \hbar/\tau_0$ is used at the effective level.
- **Numerical and observational tests.** Most of the analysis remains analytical and perturbative. Quantitative comparisons with data in the strong-field regime (EHT shadows, gravitational-wave ringdowns, binary pulsars) will require dedicated numerical simulations within the sponge–vacuum framework.

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appendices

A Formal Covariant Framework

To bridge the gap between the heuristic sponge model and standard field theory, and to ensure rigorous handling of relativistic regimes, we present here a Lagrangian formulation of the theory.

A.1 The scalar vacuum field

We identify the vacuum density variations with a real scalar field $\psi(x^\mu)$ defined over the fixed background. The background "container" is described by the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. We define:

$$\rho_{vac}(\vec{x}, t) \equiv \rho_0 + \xi\psi(x^\mu), \tag{114}$$

where ξ is a dimensional scaling constant. The dynamics of the free vacuum sponge are governed by the standard action for a scalar field:

$$S_{vac} = \int d^4x\sqrt{-\eta}\left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - V(\psi)\right] \tag{115}$$

where $V(\psi) = \frac{1}{2}m_\psi^2\psi^2$ is a potential ensuring stability around the equilibrium density ρ_0. In the massless limit (long-range forces), $V(\psi) \to 0$.

A.2 Effective metric and matter coupling

The central postulate of the sponge model—that matter follows paths minimizing travel time governed by $n(\vec{x}, t)$—is formally equivalent to matter moving along the geodesics of an effective metric $\tilde{g}_{\mu\nu}$. We introduce a conformal coupling between the vacuum field and the matter sector:

$$\tilde{g}_{\mu\nu} = \Omega^2(\psi)\eta_{\mu\nu}, \tag{116}$$

where the conformal factor $\Omega(\psi)$ is identified with the refractive index n defined in previous sections. Linearizing for weak fields ($n \simeq 1 + \beta\delta\rho$), we have:

$$\Omega(\psi) = \exp(\kappa\psi) \simeq 1 + \kappa\psi, \tag{117}$$

with $\kappa \propto \beta$. The action for ordinary matter fields χ_m is then constructed using this effective metric:

$$S_m = \int d^4x\sqrt{-\tilde{g}}\mathcal{L}_m(\chi_m, \tilde{g}_{\mu\nu}). \tag{118}$$

This formulation ensures that the Weak Equivalence Principle is satisfied: all matter fields "feel" the same effective geometry $\tilde{g}_{\mu\nu}$, recovering the universality of free fall.

A.3 Field equations

Varying the total action $S_{tot} = S_{vac} + S_m$ with respect to the field ψ yields the covariant wave equation:

$$\Box_\eta\psi - V'(\psi) = -\kappa T, \tag{119}$$

where $\Box_\eta = \eta^{\mu\nu}\partial_\mu\partial_\nu$ is the d'Alembertian in flat space, and $T \equiv \tilde{g}^{\mu\nu}T^{(m)}_{\mu\nu}$ is the trace of the matter energy-momentum tensor. In the static weak-field limit, neglecting time derivatives and potential terms, this reduces to the Poisson equation $\nabla^2\psi \propto \rho_{mass}$, justifying the Newtonian limit.

A.4 Relation to scalar-tensor theories

Formally, the covariant version of the sponge-vacuum model is close in spirit to a scalar-tensor theory of Brans-Dicke type. However, it distinguishes itself via three structural ingredients:

1. **Vacuum interpretation:** The scalar field is a macroscopic encoding of the vacuum density ρ_{vac} of a sponge-like medium with a landscape of equilibrium basins.
2. **Finite adjustment time τ_0:** The model incorporates a relaxation time leading to non-conservative effects.
3. **Microphysical link:** The relation $E_{cell}\tau_0 \sim \hbar$ ties the macroscopic response to a MeV-scale vacuum energy.

A.5 Covariant relaxation and dissipation

The relaxation time τ_0 introduces a deviation from purely conservative dynamics. In a covariant framework, this implies that the vacuum has a "memory" or viscosity relative to the fixed container frame defined by a time-like unit vector $u^\mu = (1, 0, 0, 0)$. We propose a modified field equation including a frame-dependent damping term:

$$\Box_\eta \psi + \frac{\tau_0}{c^2}(u^\alpha \partial_\alpha)\Box_\eta \psi = -\kappa T. \tag{120}$$

This higher-derivative term introduces the delay τ_0. It breaks Lorentz invariance explicitly at the fundamental level (due to u^μ), consistent with the "fixed container" ontology, but the effects are suppressed by the factor $\tau_0\omega$ for low-frequency phenomena.

B Extensions to Strong Fields: Vacuum Saturation and Torsion (Appendix)

The post-Newtonian successes rely on a linearised approximation. Here we outline the extension to strong-field regimes, addressing vacuum compression and rotational effects.

B.1 Exponential Densification and Saturation

In regions of strong compression, the linear relation $n \simeq 1 + \beta\delta\rho$ is replaced by the exponential form $\Omega(\psi) = \exp(\kappa\psi)$. While the event horizon in GR is a coordinate singularity, in the sponge-vacuum picture the central object is a region of highly compressed vacuum. To dynamically limit this compression, we consider a non-quadratic vacuum potential:

$$V(\psi) = V_0 + \frac{1}{2}m_\psi^2\psi^2 + V_{nl}(\psi) \tag{121}$$

where V_{nl} is chosen such that the potential saturates or grows slowly (e.g., $V(\psi) \to V_\infty < \infty$). This implies:

1. For weak fields, ψ remains small (linear regime).
2. Above a critical compactness, $\psi(r)$ approaches a saturation value ψ_{sat}.
3. The redshift $1+z = \exp[\kappa(\psi_{source} - \psi_{obs})]$ becomes large but the underlying vacuum configuration remains regular.

This mechanism replaces the classical singularity with a "stiff" vacuum core.

B.2 Vacuum Vorticity and Cosserat Structure

To account for gravitomagnetic phenomena (Lense-Thirring effect), we model the vacuum as a Cosserat (micropolar) medium. We introduce a vacuum vorticity field $\vec{\omega}_{vac}(\vec{x}, t)$. This induces a shift vector $\vec{N}$ in the effective line element:

$$ds^2 = -\Omega^2(\psi)(cdt - \vec{N} \cdot d\vec{x})^2 + \Omega^2(\psi)d\vec{x}^2, \tag{122}$$

with $\vec{N} \propto \nabla \times \vec{\omega}_{vac}$. The action for the vorticity sector is:

$$S_{vort} = \int d^4x \left[-\frac{1}{4} W_{ij} W^{ij} + \gamma_\omega \vec{\omega}_{vac} \cdot \vec{J}_{mass} + \dots \right] \tag{123}$$

where $W_{ij} = \partial_i \omega_{vac,j} - \partial_j \omega_{vac,i}$. In the weak-field limit, this reproduces the standard GR frame-dragging potential $N_i \propto \epsilon_{ijk} J_j x_k / r^3$.

B.3 Phenomenological Implications

- **Horizon-layer echoes:** If the saturated vacuum forms a stiff reflective layer, gravitational ringdown signals may exhibit late-time echoes.
- **Modified Ergospheres:** The interplay between scalar compression and vacuum vorticity may lead to percent-level deviations in the shape of the ergosphere for rapidly rotating objects, potentially testable by the Event Horizon Telescope.

C Singularity Resolution and the Principle of Ultimate Equilibrium

The occurrence of singularities (infinite density/curvature) in General Relativity (GR) represents a breakdown of the theory's predictive power. The sponge-vacuum model offers a physical mechanism to resolve this issue by adopting a principle of **Ultimate Equilibrium**.

C.1 The Failure of Classical Laws at $\rho \to \infty$

[span_0](start_span)The Penrose-Hawking singularity theorems demonstrate that collapse to infinite density is inevitable in GR under reasonable physical assumptions[span_0](end_span). This failure arises because classical laws do not account for the ultimate phase changes or repulsive forces that must exist at the **Planck scale**.

C.2 Singularity Suppression by Vacuum Saturation

The core concept is that the vacuum medium, being a physical structure, possesses a finite capacity for compression. This is formalized by introducing a physical limit, ρ_{max}, to the local vacuum density ρ_{vac}.

1. **Non-Linear Stiffness:** The Exponential Coupling Hypothesis (Appendix B) serves as a dynamical realization of this limit, where the energy required for further compression increases drastically as $\rho_{vac} \to \rho_{max}$.

2. **Quantum Analogy:** This mechanism is conceptually aligned with modern quantum gravity models, such as **Loop Quantum Gravity (LQG)**, where the discrete nature of spacetime introduces an effective **repulsive pressure** at the Planck density, replacing the singularity with a finite-density **bounce**.

3. **Transformation Phase:** The exponential relation is implicitly a continuous model for a rapid phase change. At the threshold where $\rho_{vac} = \rho_{max}$, the effective pressure of the vacuum ($\mathcal{P}_{vac}$) must instantaneously become repulsive to stabilize the system, ensuring that the Ultra-Compact Sponge Object (UCSO) finds a stable maximum-density core rather than collapsing to an infinite point.

C.3 The Ultimate Equilibrium

The singularity is resolved because the source of gravity (the vacuum density) cannot diverge. The UCSO represents a state of **Ultimate Gravitational Equilibrium**, where the inward pull of mass is perfectly balanced by the maximal stiffness/repulsion of the fully compressed vacuum medium.

- **Observable Test:** The primary test for this principle is to verify if the spacetime geometry near the center of a black hole allows for a smooth, finite-density core, which would be visible via subtle deviations in the **EHT black hole shadow** or in the gravitational wave signals from mergers, distinguishing the UCSO from the singular Kerr solution.

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