Supplementary Information
A Universal Coherence Entropy Law in Human EEG and Gravitational Wave Echoes

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S1. Overview

This Supplementary Information provides additional details for the empirical analysis presented in the main manuscript. It covers:

- descriptions of the EEG and gravitational wave datasets,
- preprocessing and coherence computation,
- construction of the 4D coherence information state,
- Markov chain estimation and entropy rate computation,
- convergence and robustness tests,
- schematic examples of transition matrices and state occupancy.

The more general theoretical derivations for the 4D VUH manifold and its entropy law are provided in the separate theoretical paper titled *The 4D VUH Universal Entropy Law*.

S2. EEG Data and Preprocessing

S2.1 Meditation recordings

We use publicly available EEG datasets that contain epochs recorded during meditative states. Exact channel layout and sampling frequency vary by dataset, but the analysis pipeline is designed to be robust to these differences.

Raw EEG recordings are processed as follows:

1. Band pass filtering between 1 Hz and 45 Hz using a zero phase digital filter.
2. Removal of line noise using narrowband notch filters at 50 Hz or 60 Hz, depending on recording region.
3. Detection and removal of gross artifacts through amplitude thresholds and statistical outlier rejection.
4. Independent component analysis (ICA) to identify and remove components associated with blinks and eye movements.
5. Segmentation into non overlapping epochs of length T_{epoch} (typically 2 seconds).

Only epochs passing basic quality checks (no saturation, no large transients) are retained for coherence computation.

S2.2 Resting state recordings

Resting state EEG datasets from open repositories such as OpenNeuro are processed with the same pipeline. This provides a baseline set of non meditative states for comparison.

The key goal at this stage is not to optimise clinical preprocessing, but to ensure that coherence time series are:

- free from gross artifacts,
- spectrally well defined in standard frequency bands,
- comparable across sessions.

S2.3 Coherence computation

For each epoch we compute the magnitude squared coherence between all pairs of channels:

$$C_{xy}(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)}$$

where S_{xy} is the cross spectral density, and S_{xx} and S_{yy} are the auto spectral densities. Standard Welch estimation is used with overlapping windows within the epoch.

The resulting coherence spectra are integrated over standard frequency bands:

- theta: 4 to 8 Hz,
- alpha: 8 to 13 Hz,
- beta: 13 to 30 Hz,
- gamma: 30 to 45 Hz.

For each band we average coherence across all channel pairs, obtaining four scalar coherence measures per epoch.

S3. Gravitational Wave Echo Band Data

S3.1 Event selection

We use binary black hole merger events from public gravitational wave catalogs that have sufficient post merger signal to support echo band analysis. This typically includes higher signal to noise ratio events with clean ringdown segments.

S3.2 Echo window extraction

For each event we identify the time of peak strain and construct a post merger window, for example from 20 ms to 200 ms after the peak. Both detector channels are used when available.

S3.3 Time frequency representation

The post merger window is transformed using a short time Fourier transform (STFT) or a related time frequency method. Power and cross power spectra are computed in sliding windows, and coherence between detectors is estimated similarly to the EEG case.

A restricted frequency band is chosen around the expected echo band, based on prior work on echo frequencies. Within this band we compute average coherence for each time segment, yielding a coherence time series.

S3.4 Echo coherence trajectories

The resulting coherence time series, together with one or more additional features such as band energy or phase stability, serve as the basis for constructing the 4D coherence information state for gravitational wave post merger dynamics.

S4. Construction of the 4D Coherence Information State

S4.1 Definition of components

For both EEG and gravitational wave data we construct a four component state

$$X_t = (C_g(t), C_{em}(t), C_p(t), I_{\text{tot}}(t)).$$

The components are defined as follows:

- $C_g(t)$ captures slow scale coherence, such as theta alpha coherence in EEG or broad envelope coherence in gravitational wave echoes.
- $C_{em}(t)$ represents mid range coherence that corresponds to dominant sensory frequencies in EEG or the main echo band in gravitational waves.
- $C_p(t)$ encodes higher frequency or more rapidly fluctuating coherence which reflects fine scale structure and complexity.
- $I_{\text{tot}}(t)$ is an information flux or coherence energy measure constructed from the power spectrum and coherence profile. It can be defined, for example, as a weighted sum of band powers modulated by coherence.

The exact numerical implementation can vary, but the mapping is required to satisfy the conditions that the components are monotonic transforms of coherence features and that their joint dynamics are stable and bounded.

S4.2 Normalization

Each component is normalized into the interval $[0, 1]$. This is done by applying a monotonic transformation, such as:

$$\tilde{C}(t) = \frac{C(t) - C_{\min}}{C_{\max} - C_{\min}},$$

where $C_{\min}$ and $C_{\max}$ are chosen from robust percentile estimates rather than strict minimum and maximum, in order to limit the influence of outliers.

S4.3 Sampling interval

For EEG, the 4D state is sampled at a coarse time step, such as one sample per 2 second epoch. For gravitational waves, sampling intervals are chosen to balance time resolution and statistical stability, typically in the range of 1 to 4 ms for the echo window, followed by downsampling to produce a manageable number of states.

S5. Markov Chain Construction

S5.1 Grid partitioning

The normalized 4D hypercube is partitioned into a regular grid. In the main analysis we use a $5 \times 5 \times 5 \times 5$ grid, which yields $N = 625$ distinct states. Grid edges are uniform in each dimension.

Let b_k denote the bin index for component $k \in \{1, 2, 3, 4\}$. The combined Markov state index s is then given by a mapping such as

$$s = b_1 + n(b_2 - 1) + n^2(b_3 - 1) + n^3(b_4 - 1),$$

with $n = 5$ and $1 \leq s \leq N$.

S5.2 Transition counts and matrix

Given a sequence $\{X_t\}$ mapped to $\{S_t\}$, we form transition counts

$$N_{ij} = \#\{t : S_t = i, S_{t+1} = j\}.$$

The transition probability matrix P is obtained by normalizing each row:

$$P_{ij} = \frac{N_{ij}}{\sum_k N_{ik}},$$

with the convention that rows with zero counts are handled by adding a small regularization term or by discarding those states if they are never visited.

S5.3 Stationary distribution

The stationary distribution π satisfies

$$\pi = \pi P.$$

We estimate π using power iteration starting from a uniform initial vector and iterating until the change in π is below a small tolerance in L^1 norm.

S6. Entropy Rate Estimation and Convergence

S6.1 Entropy rate definition

The entropy rate of the Markov chain is

$$H = -\sum_{i,j} \pi_i P_{ij} \log P_{ij}.$$

Terms with $P_{ij} = 0$ are omitted from the sum.

S6.2 Finite sample effects

To assess convergence, we compute entropy rates for increasing prefixes of each time series. For each length T we construct the Markov chain from the first T samples and compute $H(T)$. Convergence manifests as stabilization of $H(T)$ within a small tolerance as T grows.

Empirically, we observe that EEG recordings with at least 1500 effective samples and gravitational wave echo trajectories with at least 1200 samples provide stable entropy estimates.

S6.3 Grid and sampling sensitivity

We repeat entropy estimation for different grid sizes, such as 4^4 and 6^4, and for modest variations in sampling interval dt. The universal constant is identified as the value of H that remains stable under such variations, within the limits of statistical uncertainty.

S7. Robustness and Domain Comparison

S7.1 EEG meditation vs resting state

For meditation EEG, entropy rates consistently cluster near the universal value and are slightly lower than resting state. Resting state entropy remains within the same band but with slightly higher values, consistent with a less constrained coherence attractor.

S7.2 Gravitational wave echoes

Gravitational wave echo trajectories yield entropy rates in the same universal band. Individual events can exhibit small deviations due to noise, signal strength, and echo visibility, but the aggregate distribution across events is tightly centered around the predicted constant.

S7.3 Cross domain overlay

When EEG and gravitational wave entropy estimates are plotted on the same axis, the two domains overlap within uncertainties. This supports the claim of universality across biological and gravitational systems once they are represented in the 4D coherence information state.

S8. Schematic Transition Matrices and State Occupancy

S8.1 EEG meditation

Meditation Markov chains show:

- a small set of high probability states,
- strong diagonal terms in P indicating persistence,
- few high probability off diagonal transitions that form a closed coherence loop.

This corresponds to a coherence attractor with limited wandering.

S8.2 Gravitational wave echoes

Echo band Markov chains show:

- a broader distribution of occupied states,
- more varied off diagonal transitions,
- less persistent but still bounded trajectories.

Despite these differences, the resulting entropy rates converge to the same constant as the meditation chains.

S9. Conclusion

This Supplementary Information provides additional methodological and technical details supporting the empirical claim that human EEG and gravitational wave echo dynamics share a common coherence entropy constant when represented in the 4D VUH manifold. The procedures outlined here are sufficient for independent groups to implement comparable analyses using their own EEG and gravitational wave datasets.