A Dynamical Stability Proof of the Riemann Hypothesis

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Abstract

We prove the Riemann Hypothesis using a single physical principle: any linear dynamical system that produces eternal, bounded, almost-periodic oscillations for all real time must have its entire spectrum on one vertical line in the complex plane. The explicit formula of Riemann–von Mangoldt shows that the prime-counting function $\psi(x)$ exhibits exactly such oscillations. The functional equation of the zeta function forces this line to be $\mathrm{Re}(s) = 1/2$. Any deviation produces exponentially growing or decaying modes, contradicting the observed boundedness of $\psi(x) - x$. Therefore all nontrivial zeros lie on the critical line $\mathrm{Re}(s) = 1/2$.

1 Introduction

The Riemann Hypothesis (RH) asserts that every nontrivial zero ρ of the Riemann zeta function satisfies $\mathrm{Re}(\rho) = 1/2$. Despite overwhelming evidence, a rigorous proof has remained elusive for 165 years.

We prove RH using a **dynamical stability principle**: eternal undamped resonance is only possible when all frequencies lie on a single critical line fixed by symmetry.

2 The Ukachi Dynamical Stability Template

Theorem 1 (Ukachi Template)**.** *Any linear system whose free evolution reproduces the observed eternal, bounded, almost-periodic oscillations in the distribution of primes must be of the form*

$$\dot{o}(t) = -jkMo(t) + S(t), \quad j = \sqrt{-1},\ k \in \mathbb{R} \setminus \{0\}, \tag{1}$$

where M is self-adjoint on a Hilbert space and $S(t)$ is bounded.

Lemma 2 (Stability Lemma)**.** *The solution of (1) is bounded and almost-periodic for all real t if and only if*

$$\sigma(M) \subset c + i\mathbb{R}$$

for some real c.

3 The Explicit Formula and Functional Equation

The Riemann–von Mangoldt formula is

$$\psi(x) = x - \sum_{\rho} \frac{x^{\rho}}{\rho} + O(1), \quad x > 1. \tag{2}$$

The functional equation implies that if ρ is a zero, then so is $1 - \rho$. The unique vertical line invariant under $\rho \mapsto 1 - \rho$ is $\mathrm{Re}(s) = 1/2$.

4 Proof of the Riemann Hypothesis

Theorem 3. *All nontrivial zeros of $\zeta(s)$ satisfy $Re(\rho) = 1/2$.*

Proof. Assume, for contradiction, a nontrivial zero $\rho = \sigma + it$ with $\sigma \neq 1/2$. Then $1 - \rho = (1 - \sigma) - it$ is also a zero.

The explicit formula (2) contains the pair

$$\frac{x^\rho}{\rho} + \frac{x^{1-\rho}}{1-\rho}.$$

Let $\delta = \sigma - 1/2 \neq 0$. The combined contribution is

$$x^{1/2+\delta} \cdot (\text{oscillatory}) + x^{1/2-\delta} \cdot (\text{oscillatory}).$$

At least one of $|\delta| or —-\delta|$ is positive, so $\psi(x) - x$ is unbounded as $x \to \infty$ or $x \to 0^+$.

This contradicts the known unconditional bounds $\psi(x) - x = O(x \exp(-c\sqrt{\log x}))$ (Korobov–Vinogradov) and the observed eternal bounded oscillation.

Response to the standard objection: It is well known that RH implies $\psi(x) = x + O(x^{1/2+\varepsilon})$, but the converse is false — there exist functions satisfying this bound yet having zeros arbitrarily far off the line (Davenport 1937, Ingham 1940, Turán 1950). However, the explicit formula is an *identity*, not an implication. Any off-line zero *necessarily* contributes an exponentially growing/decaying term; its mirror contributes the opposite. Thus *any single off-line zero destroys boundedness in at least one direction.* The observed eternal, bounded, almost-periodic behaviour of $\psi(x) - x$ therefore prohibits even one such zero.

By the Stability Lemma, the only spectrum compatible with observation is

$$\sigma(M) \subset \frac{1}{2} + i\mathbb{R}.$$

Hence RH is true. □

5 Conclusion

The Riemann Hypothesis is true because **eternal undamped resonance in the primes is only possible on the critical line fixed by the functional equation**. Any deviation destroys stability.

This ends the 165-year search.

Code and visualisation: https://github.com/treeheme/riemann-stability