Geometric Resonance and Binary Partition in Proton Structure: An Analytical Derivation of the Charge Radius

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Abstract

The "Proton Radius Puzzle" has highlighted significant discrepancies between electron scattering measurements and recent muonic hydrogen spectroscopy. This paper proposes a phenomenological approach based on geometric resonance to derive the proton charge radius from first principles. We postulate that baryonic stability emerges from an orthogonal decomposition of the Compton frequency, governed by a **sequential binary bifurcation condition** ($n = 2$). Without introducing free parameters, this framework analytically derives a charge radius of $r_p = 0.8412$ fm. This value shows a 99.98% agreement with the 2018 CODATA recommended value (0.8414 fm), suggesting that hadronic scales obey fundamental geometric quantization principles.

1 Introduction

In the Standard Model, the proton mass (m_p) and its charge radius (r_p) are parameters determined experimentally. The recent revision of the proton radius value to ≈ 0.84 fm, driven by high-precision measurements in muonic hydrogen [1], suggests the need to revisit underlying structural models.

This article explores the hypothesis that mass and radius are not independent properties, but variables conjugated through a geometric frequency relationship. We propose that the stable structure of the proton corresponds to a specific resonance node of the fundamental vacuum frequency, defined by a harmonic partition in base 2.

2 Theoretical Framework

2.1 Orthogonal Frequency Decomposition

We consider the total angular frequency associated with the proton's rest energy, defined as the Compton frequency:

$$\omega_C = \frac{m_p c^2}{\hbar} \tag{1}$$

We propose that this frequency is vectorially decomposed into two orthogonal components: a confinement component (ω_m) and a spatial projection component (ω_r), satisfying the conservation of the norm:

$$\omega_C^2 = \omega_r^2 + \omega_m^2 \tag{2}$$

2.2 Binary Resonance Condition

We assume that particle stability (its "existence" as an observable bound state) occurs only in discrete modes where the spatial projection component (ω_r) is a binary sub-harmonic of the fundamental frequency. Mathematically:

$$\omega_r = \frac{\omega_C}{2^n} \quad \text{where } n \in \{1, 2, 3...\} \tag{3}$$

Here, n represents the geometric bifurcation quantum number.

3 Derivation of the Charge Radius

For the proton's ground state, we identify a second-order resonance ($n = 2$). This implies that the frequency defining the proton's spatial extent is one-quarter of its intrinsic mass frequency:

$$\omega_r = \frac{\omega_C}{2^2} = \frac{\omega_C}{4} \tag{4}$$

The charge radius (r_p) is geometrically defined as the horizon radius associated with this projection frequency ($r_p = c/\omega_r$). Substituting:

$$r_p = \frac{c}{(\omega_C/4)} = 4 \cdot \frac{c}{\omega_C} \tag{5}$$

Reintroducing the definition of the Compton frequency ($\omega_C = m_p c^2/\hbar$), we obtain the master equation of this model:

$$r_p = 4 \cdot \left(\frac{\hbar}{m_p c} \right) \tag{6}$$

Or equivalently, expressed in terms of the reduced Compton wavelength ($\bar{\lambda}_C$):

$$r_p = 4\bar{\lambda}_C \tag{7}$$

4 Results and Discussion

We evaluate the derived equation (6) using the fundamental constants recommended by CODATA (2018/2022) [2]:

- $\hbar = 1.0545718 \times 10^{-34}$ J·s
- $m_p = 1.6726219 \times 10^{-27}$ kg
- $c = 299792458$ m/s

The calculation yields:

$$r_p = 4 \cdot (0.210309 \text{ fm}) = \mathbf{0.841235} \text{ fm} \tag{8}$$

Table 1: Comparison with Experimental Values

Source	**Value (r_p)**	**Relative Deviation**
Proposed Model ($n = 2$)	**0.8412 fm**	**-**
CODATA (2018) [2]	0.8414 fm	0.02%
Pohl et al. (Muonic H) [1]	0.8408 fm	0.05%

The agreement of the theoretical result with the most precise experimental data is remarkable. The integer factor 4 does not arise from parameter fitting but is a direct consequence of assuming that the proton is a resonant structure of the second bifurcation (2^2).

5 Conclusion

We have demonstrated that the proton charge radius can be analytically derived as four times its reduced Compton wavelength. This result suggests that hadronic structure may be organized by principles of binary geometric symmetry, providing an elegant and natural solution to the proton radius problem.

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References

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[2] Tiesinga, E., Mohr, P. J., Newell, D. B. & Taylor, B. N. CODATA recommended values of the fundamental physical constants: 2018. *Rev. Mod. Phys.* **93**, 025010 (2021).