Observer, Window, Mellin–Heisenberg and Reparametrization Covariance

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Abstract

Establish observer-dependent window framework with Mellin–Heisenberg phase–scale symmetry and reparametrization covariance. Core results: (i) Window choice encodes observer resolution; (ii) Mellin transform $\mathcal{M}[f](s) = \int_0^\infty f(x)x^{s-1}dx$ provides phase–scale representation via Weyl–Heisenberg commutation relations; (iii) Energy reparametrization $E \mapsto \phi(E)$ induces window pushforward preserving windowed observables; (iv) Wexler–Raz biorthogonality ensures frame completeness.

1 Mellin–Heisenberg Framework

On $\mathcal{H}_a = L^2(\mathbb{R}_+, x^{a-1}dx)$, define modulation/dilation:

$$(U_\tau f)(x) = x^{i\tau} f(x), \quad (V_\sigma f)(x) = e^{\sigma a/2} f(e^\sigma x)$$

satisfying Weyl relation $V_\sigma U_\tau = e^{i\tau\sigma} U_\tau V_\sigma$.

Via $x = e^t$ isometry with $L^2(\mathbb{R})$ Gabor frame.

2 Window Reparametrization

Theorem 2.1 (Reparametrization Covariance)**.** *For monotone $\phi : \mathbb{R} \to \mathbb{R}$ and window w, windowed observable*

$$\mathcal{O}_w[\rho] = \int w(E)\rho(E)dE$$

under $E \mapsto \phi(E)$ becomes

$$\mathcal{O}_{w^\phi}[\rho^\phi] = \int w(\phi^{-1}(E'))\rho(\phi^{-1}(E'))|\phi'(\phi^{-1}(E'))|^{-1}dE'$$

preserving integral value.

3 Observer Resolution

Window bandwidth ΔE encodes observer energy resolution. Wexler–Raz density condition $\Delta E \cdot \Delta t \leq 2\pi\hbar$ ensures frame completeness.

Different observers with windows w_1, w_2 relate via Wexler–Raz transform, preserving quantum information up to resolution limits.

4 Discussion

Framework provides:

- Observer-dependent but covariant measurement theory
- Mellin–Heisenberg phase–scale symmetry
- Reparametrization invariance
- Frame-theoretic foundations