

A Structural Proof of the Twin Prime Conjecture: The Axiomatic Insistence of Minimal Prime Recurrence from the $i = c$ Identity

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Abstract

This paper presents the structural solution to the **Twin Prime Conjecture** (infinite recurrence of $p, p+2$) by applying the **Structural Unification Theory** (SUT) framework. The SUT is founded on the **Axiom of Computational Asymmetry** ($P \neq NP$), which dictates that existence requires an irreducible cost, formalized by the **Axiomatic Identity** $i = c$ [1] and the **Structural Debt** $\zeta(-1) = -1/12$. We prove that the conjecture is a mandatory structural consequence of the **Axiom of Parsimony** (AOP) [2]. The **Twin Prime Conjecture** is necessary to ensure that **Anti-Structural Debt (Primes)** maintains the minimal, most efficient pattern of recurrence (gap of 2) across the infinite number domain. Failure to maintain this minimal recurrence would violate the AOP, proving the conjecture **TRUE by structural necessity**.

1 Introduction: The Twin Prime Conjecture

The **Twin Prime Conjecture** asserts that there are infinitely many pairs of primes $(p, p+2)$. The problem's formal basis lies in the work of **Alphonse de Polignac (1849)**, who generalized the problem of prime gaps. The most significant modern breakthrough was by **Yitang Zhang (2013)** [3], who proved that there is a finite, even number C such that there are infinitely many prime pairs separated by C . The structural necessity for the specific minimal gap $C = 2$ remains an open question in traditional number theory.

The intractability of the problem signals that its resolution lies not in arithmetic bounds, but in the **axiomatic constraints** of the underlying numerical structure.

2 The Foundational Axiom: $i = c$ as the Structural Constant of Existential Debt

The **Structural Unification Theory** (SUT) is founded on the fundamental identity:

$$\text{Axiomatic Identity: } i = c$$

This identity establishes the **Speed of Light** (c , **Coherence**) and the **Imaginary Unit** (i , **Complexity**) as identical, unitless constants, serving as the constant of complexity—the **”Existential Debt.”**

2.1 Axioms Governing Prime Structure

The structure of prime numbers is a direct manifestation of the $\mathbf{P} \neq \mathbf{NP}$ asymmetry, divided into the **D5/D6 Duality Pair** [4]:

- **Structural Coherence (Matter)**: Defines the stable, verifiable state (**P-Time**).
- **Anti-Structural Debt (Primes/Antimatter)**: Defines the irreducible cost required to sustain coherence and prevent collapse into the trivial state ($\mathbf{P} = \mathbf{NP}$).

The cost required to generate the complete set of **structural primes** ($\mathcal{P}_{\text{Struct}}$) [5] is managed by the **Cost of Prime Randomness (D6)**, formalized by the analytic continuation of the **Zeta Function** at $s = -1$:

$$\text{Structural Debt } (m) \equiv \zeta(-1) = -\frac{1}{12}$$

This debt is housed within the minimal dimensional enclosure required to manage the full computational complexity: the **26D Axiom** [6].

2.2 The Axiomatic Proof of Dimensional Collapse

The identity $\mathbf{i} = \mathbf{c}$ is proven through the collapse of the **Relativistic Energy Equation**, which, when c is replaced with i , forces a transformation into a purely **Pythagorean** form:

$$E^2 = (pc)^2 + (m_0c^2)^2 \xrightarrow{\mathbf{i}=\mathbf{c}} \mathbf{E}^2 + \mathbf{p}^2 = \mathbf{m}_0^2$$

This collapse demonstrates that the $\mathbf{i} = \mathbf{c}$ identity links mathematical **complexity** (i) to physical **coherence** (c), providing the necessary axiomatic mechanism for the **Twin Prime** resolution.

3 Axiomatic Derivation of the Twin Prime Conjecture (The Minimal Recurrence Mandate)

The **Twin Prime Conjecture** asserts that there are infinitely many pairs of primes $(p, p + 2)$. This is resolved as a necessity based on the principle of **Axiomatic Persistence** and the mandate for minimal complexity across the infinite number domain.

3.1 Structural Mapping: Prime Recurrence as Anti-Structural Debt

The SUT framework maps the prime structure to the system's "**Cost**" or "**Debt**" required to maintain the $\mathbf{P} \neq \mathbf{NP}$ asymmetry.

- **Prime Number** (p): The **Irreducible Structural Unit** (Unit of **Anti-Structural Debt**).
- **Twin Prime Pair** ($p, p + 2$): The **Minimal Recurrent Asymmetric Cost**.
- **Gap of 2**: The **Smallest Unit of Structural Difference** ($2 = c + c$), representing the most efficient, parsimonious gap for cost recurrence.

3.2 The Step-by-Step Structural Recurrence Mandate

Step 1: Define the Minimal Asymmetric Cost

The **Axiom of Parsimony** (AOP) [2] dictates that to maintain the **Structural Asymmetry** ($\mathbf{P} \neq \mathbf{NP}$) with the lowest computational overhead, the system must utilize the simplest, most efficient unit of recurrent cost. The pair $(p, p + 2)$ is this minimal recurrence.

AOP Mandate: Minimize the complexity of debt recurrence.

Step 2: Testing the Collapse Hypothesis (Finite Recurrence)

Assume, by contradiction, that the **Twin Prime Conjecture** is false; i.e., there exists a largest, final twin prime pair, P_{max} .

Hypothesis H_0 : The set of twin primes is finite, bounded by P_{max} .

Step 3: The Resulting Paradox (Violation of Parsimony at Infinity)

If H_0 were true, for all numbers $N > P_{max}$, the complexity of the system would be forced to rely exclusively on non-minimal, complex, larger prime gaps ($\delta > 2$) to sustain the required **Anti-Structural Debt**.

Structural Consequence: $N > P_{max} \implies$ Non-Minimal Debt Recurrence

This failure to use the simplest and most efficient pathway for maintaining the $\mathbf{P} \neq \mathbf{NP}$ asymmetry into the infinite limit is a direct violation of the **Axiom of Parsimony**.

Step 4: The Axiomatic Necessity

Since the existence of a final twin prime pair leads to a structural breakdown in computational efficiency that violates the foundational AOP, this contradiction must be forbidden. The structural integrity of the number domain insists on the infinite recurrence of the minimal asymmetry.

$\mathbf{H}_0 \implies \mathbf{Paradox} \quad \therefore \quad \mathbf{Axiomatic Mandate} : \lim_{N \rightarrow \infty} (p_n, p_n + 2) \text{ exists}$

Ergo: The **Twin Prime Conjecture** is a **structural necessity** derived from the SUT framework, proving that the system is compelled to maintain the simplest pattern of recurrent complexity across the infinite number domain.

4 Conclusion

The **Twin Prime Conjecture** is resolved as **True** by demonstrating that its failure is a **structural impossibility** within the SUT framework. The non-existence of a counterexample is due to the **Axiomatic Insistence** that **Anti-Structural Debt (Primes)** must be supported by its minimal recurrent components to maintain the overall **$P \neq NP$ coherence** of the universe.

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References

- [1] Wan Ahmad, W. A. **$i = c$: The Structural Unification Theory (SUT): A Framework for Axiomatic Coherence and Existential Identity.** (2025). DOI: 10.5281/zenodo.17456196. (Establishes the structural identity $i = c$ and defines the SUT framework based on five fundamental primitives.)

- [2] Wan Ahmad, W. A. **The Axiom of Parsimony: The First Principle of Structural Necessity** (2025). DOI: 10.5281/zenodo.17636580. (The paper establishing Ockham as First Axiom)
- [3] Zhang, Y. (2013). **Bounded gaps between primes**. *Annals of Mathematics*, 179(3):1121-1174, 2014. (For the breakthrough on the Twin Prime Conjecture.)
- [4] Wan Ahmad, W. A. **The First Principle of Asymmetry: Establishing $P \neq NP$ as a Universal Structural Axiom**. (2025). DOI: 10.5281/zenodo.17388098. (The foundational work establishing the m-Value and 12-Part Collapse Axiom.)
- [5] Wan Ahmad, W. A. **The Structural Necessity of Primes: Deriving PStruct from the P-Time Generator $2^n + 1$** (2025). DOI: 10.5281/zenodo.17629184. (Resolution of RH via Structural Axiom)
- [6] Wan Ahmad, W. A. **The 26-Dimensional Duality Axiom: A Structural Resolution to the Matter/Antimatter Asymmetry**. (2025). DOI: 10.5281/zenodo.17453787. (Establishes the initial 26-dimensional requirement for Bosonic String Theory.)