Discrete Compactification and the Gauge–Coupling Dictionary on T^2 under the Principle of Least Information

James Antoniadis

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Abstract

We derive a discrete gauge–coupling dictionary from a rectangular two–torus compactification T^2 selected by the Principle of Least Information (PLI). The dictionary ties each Standard-Model (SM) coupling α_i to a compact geodesic length L_i and a small integer multiplicity m_i through a *single* compactification scale K, $\alpha_i = K\, m_i/L_i^2$. Using only integers ≤ 7, we find an *isotropic* baseline solution with $R_2/R_1 = 1$, axis integer $k = 7$, asymmetric pair $(p, q) = (5, 2)$, multiplicities $(m_x, m_y, m_{pq}) = (2, 1, 4)$, and $K = 0.854580$, which yields

$$\alpha_1(M_Z) = 0.017092, \quad \alpha_2(M_Z) = 0.034183, \quad \alpha_3(M_Z) = 0.117873,$$

i.e. an overall RMS error of 0.80% vs. the canonical targets. The electroweak ratio $\alpha_2/\alpha_1 = 2$ follows *exactly* from the multiplicity pattern on an isotropic T^2. For comparison, an *anisotropic* accuracy leader with $R_2/R_1 = \sqrt{4/3}$, $k = 7$, $(3, 2)$, $(3, 2, 3)$, and $K = 0.564011$ achieves RMS 0.40%. Both solutions use only integers and one common scale; percent-level threshold corrections (KK/brane matching) can absorb the residuals without introducing floats. We present the MDL (description-length) rationale, numerics at M_Z. High Energy fine structure constant calculated accurately, 1/127.

1 Introduction and scope

The 7D Janus/PLI programme posits that among consistent law–boundary pairs the realised history minimises algorithmic description length. In the compact sector this replaces continuous moduli by a few *integers* and a single scale K. Our aim here is to formalise a minimal, testable dictionary

$$\alpha_i \;=\; K\,\frac{m_i}{L_i^2}, \qquad i \in \{1, 2, 3\}, \tag{1}$$

with L_i geodesic lengths on a rectangular T^2 and $m_i \in \mathbb{Z}_{>0}$ fixed by a simple parity/multiplicity rule. We work at a single match scale and compare to the canonical values at M_Z.

Context. Paper A develops the 7D geometry and emergence of GR and the SM on observable slices; the PLI/auxiliary-time note derives strong positivity and the Born rule. Here we isolate the compact dictionary and its data fit.

2 Geometry of T^2 and primitive windings

We compactify two spatial directions on

$$\mathrm{T}^2 = \mathbb{R}^2/\big(2\pi R_1\,\mathbb{Z}\,\hat{e}_1 \oplus 2\pi R_2\,\mathbb{Z}\,\hat{e}_2\big), \qquad ds^2 = dy_1^2 + dy_2^2, \tag{2}$$

so a primitive closed geodesic labelled by $v = (n_1, n_2) \in \mathbb{Z}^2$ has

$$\ell^2(v) = (2\pi)^2\big(n_1^2 R_1^2 + n_2^2 R_2^2\big). \tag{3}$$

PLI favours the rectangular complex structure (shortest code) and a *minimal* nonzero set of windings sufficient to encode the SM couplings.

3 PLI selection: minimal integer data

We use two axis families with a common axis integer k and a single perpendicular "safety" step, and one asymmetric pair (p, q) for colour:

$$v_x = (k, 0) \oplus (0, 1), \qquad v_y = (0, k) \oplus (1, 0), \qquad v_c = (p, q). \tag{4}$$

This yields effective squared lengths

$$L_x^2 = (kR_1)^2 + R_2^2, \qquad L_y^2 = (kR_2)^2 + R_1^2, \qquad L_{(p,q)}^2 = (pR_1)^2 + (qR_2)^2. \tag{5}$$

Janus parity gives small integer multiplicities (m_x, m_y, m_{pq}).

Why these integers? A simple MDL count charges $\log_2 n$ bits per digit and 1 bit per independent loop or parity choice. The sets

$$\underbrace{R_2/R_1 = 1,\ k = 7,\ (p, q) = (5, 2),\ (m_x, m_y, m_{pq}) = (2, 1, 4)}_{\text{baseline (MDL)}} \quad \text{and} \quad \underbrace{R_2/R_1 = \sqrt{4/3},\ k = 7,\ (p, q) = (3, 2),\ (m}_{\text{accuracy leader}}$$

sit on the Pareto front of (error, bits) among all integers ≤ 7; the first minimises description length among sub-percent fits, the second minimises RMS error.

4 The gauge–coupling dictionary and hypercharge

We identify

$$\alpha_2 \leftrightarrow L_x, \qquad \alpha_1 \leftrightarrow L_y, \qquad \alpha_3 \leftrightarrow L_{(p,q)}, \tag{6}$$

and adopt GUT normalisation $\alpha_Y = \frac{3}{5}\alpha_1$ with

$$\alpha_{\text{em}}^{-1} = \alpha_2^{-1} + \frac{5}{3}\,\alpha_1^{-1}. \tag{7}$$

5 Electroweak ratio without anisotropy

On an *isotropic* torus $R_2/R_1 = 1$ we have $L_x^2 = L_y^2 = k^2 + 1$. Then

$$\frac{\alpha_2}{\alpha_1} = \frac{m_x/L_x^2}{m_y/L_y^2} = \frac{m_x}{m_y}. \tag{8}$$

With $(m_x, m_y) = (2, 1)$ the empirical electroweak ratio $\alpha_2/\alpha_1 = 2$ is automatic, with no continuous tuning.

6 Numerical illustration at M_Z

We now present the two integer solutions.

Baseline (MDL): isotropic $R_2/R_1 = 1$, $k = 7$, $(p, q) = (5, 2)$, $(m_x, m_y, m_{pq}) = (2, 1, 4)$

Set $R_1 = R_2 = 1$. Then

$$L_x^2 = L_y^2 = k^2 + 1 = 50, \qquad L_{(5,2)}^2 = 5^2 + 2^2 = 29.$$

With $K = 0.854580$ the dictionary (1) gives

$$\alpha_1 = K\frac{1}{50} = 0.017092, \quad \alpha_2 = K\frac{2}{50} = 0.034183, \quad \alpha_3 = K\frac{4}{29} = 0.117873.$$

This corresponds to $\alpha_{\text{em}}^{-1} \approx 126.8$ via (7) and an overall RMS error of 0.80% vs. the canonical $(\alpha_1, \alpha_2, \alpha_3)$ at M_Z.

Table 1: Baseline (MDL) integer solution. Narrow layout to fit A4 page width.

Assignment	L_i^2	m_i	α_i/K	$\alpha_i(M_Z)$
$\alpha_1 \leftrightarrow L_y$	50	1	$1/50 = 0.020000$	0.017092
$\alpha_2 \leftrightarrow L_x$	50	2	$2/50 = 0.040000$	0.034183
$\alpha_3 \leftrightarrow L_{(5,2)}$	29	4	$4/29 \approx 0.137931$	0.117873

Table 2: Accuracy leader integer solution (narrow layout).

Assignment	L_i^2	m_i	α_i/K	$\alpha_i(M_Z)$
$\alpha_1 \leftrightarrow L_y$	199/3	2	$6/199 \approx 0.030151$	0.017005
$\alpha_2 \leftrightarrow L_x$	151/3	3	$9/151 \approx 0.059603$	0.033617
$\alpha_3 \leftrightarrow L_{(3,2)}$	43/3	3	$9/43 \approx 0.209302$	0.118049

Accuracy leader: $R_2/R_1 = \sqrt{4/3}$**,** $k = 7$**,** $(p, q) = (3, 2)$**,** $(m_x, m_y, m_{pq}) = (3, 2, 3)$

Set $R_1 = 1$, $R_2^2 = 4/3$. Then

$L_x^2 = 49 + \frac{4}{3} = \frac{151}{3} \approx 50.333, \quad L_y^2 = 49 \cdot \frac{4}{3} + 1 = \frac{199}{3} \approx 66.333, \quad L_{(3,2)}^2 = 9 + 4 \cdot \frac{4}{3} = \frac{43}{3} \approx 14.333.$

With $K = 0.564011$,

$$\alpha_1 = 0.017005, \quad \alpha_2 = 0.033617, \quad \alpha_3 = 0.118049,$$

with RMS error 0.40%.

7 Robustness, thresholds, and MDL

Thresholds. Two-loop running and one-loop matching across the compactification scale (finite KK/brane pieces) naturally shift the couplings at the percent level; both solutions above are within that margin without floats.

MDL and degeneracy. The isotropic baseline (Table 1) minimises a simple description-length proxy (sum of $\log_2$ of integers plus an anisotropy flag). The anisotropic leader (Table 2) reduces RMS at the cost of $\sim$5 extra bits. Nearby integer moves that preserve $m_{pq}/L_{(p,q)}^2$ form small degeneracy classes with essentially the same α_3.

8 Predictions and tests

- **No floats.** Apart from K, no continuous parameters are tuned. Persistent >few-% discrepancies across schemes would force a different integer triplet.
- **Integer stability.** Swapping (p, q) or changing k typically breaks the simultaneous fit of electroweak and colour sectors; this can be checked numerically.
- **Electromagnetic coupling.** With K set by the three-coupling least-squares, $\alpha_{\text{em}}(M_Z)$ lands near $1/127$; alternately, fixing K by α_{em} predicts the triplet within percent-level thresholds.

9 Conclusions

Using only small integers and a single scale, the PLI-selected T^2 dictionary reproduces $(\alpha_1, \alpha_2, \alpha_3)$ at M_Z to sub-percent precision (anisotropic leader) or with minimal description length (isotropic baseline). The electroweak splitting can arise without anisotropy via multiplicities, and percent-level thresholds can absorb residuals without floats.

A Appendix A: Efficient Candidates (Pareto Set)

Table 3: Efficient candidates (error vs. complexity), with geometry/multiplicity integers, predicted couplings, total RMS relative error, and MDL proxy (bits).

$(r_{\rm num}, r_{\rm den}, R_2/R_1)$			k	p	q	m_x	m_y	m_{pq}	K	α_1	α_2	α_3	RMS %	MDL bits
4	3	1.154701	7	3	2	3	2	3	0.564 011	0.017 005	0.033 617	0.118 049	0.399 000	14.147
1	1	1.000000	7	5	2	2	1	4	0.854 580	0.017 092	0.034 183	0.117 873	0.798 000	9.129
2	3	0.816497	7	3	1	3	1	2	0.569 531	0.016 917	0.034 401	0.117 834	1.004 000	10.562
7	1	2.645751	6	3	2	1	3	3	1.454 739	0.017 250	0.033 831	0.117 952	1.035 000	12.147
4	3	1.154701	6	3	2	3	2	4	0.422 536	0.017 246	0.033 954	0.117 917	1.049 000	14.340
2	1	1.414214	7	5	3	1	1	3	1.693 471	0.017 106	0.033 205	0.118 149	1.181 000	10.299
3	2	1.224745	6	3	1	4	3	4	0.310 288	0.016 925	0.033 097	0.118 205	1.237 000	13.340
7	5	1.183216	6	3	2	3	2	4	0.430 082	0.016 735	0.034 499	0.117 831	1.370 000	15.884
2	1	1.414214	7	5	4	1	1	4	1.684 499	0.017 015	0.033 029	0.118 210	1.371 000	11.129
2	1	1.414214	7	7	2	1	1	4	1.684 499	0.017 015	0.033 029	0.118 210	1.371 000	10.615
7	3	1.527525	3	2	1	2	2	4	0.187 167	0.017 015	0.033 029	0.118 210	1.371 000	11.977
7	3	1.527525	3	2	1	1	1	2	0.374 333	0.017 015	0.033 029	0.118 210	1.371 000	8.977
7	2	1.870829	5	3	1	2	3	3	0.491 061	0.016 646	0.034 460	0.117 855	1.501 000	12.884

B Appendix B: Top 15 Candidates by RMS Error

MDL sketch for the integer set

A compact code lists: lattice type (rectangular), primitive loops, and multiplicities. For rectangular T^2, the code length scales like $\sum_v \log_2(1 + |n_1|) + \log_2(1 + |n_2|)$ plus constants; an anisotropy flag costs ~ 1 bit. The isotropic baseline is favoured among sub-percent fits; the anisotropic leader lives further along the error–bits Pareto front.

Table 4: Top 15 candidates by RMS relative error. Geometry/multiplicity integers, fitted scale K, predicted couplings, per-channel relative errors, overall RMS error, and MDL proxy (bits).

r_{num}	r_{den}	R_2/R_1	k	p	q	m_x	m_y	m_{pq}	K	α_1	α_2	α_3	α_1 err %	α_2 err %	α_3 err %	RMS %	MDL bits
4	3	1.154 701	7	3	2	3	2	3	0.564 011	0.017 005	0.033 617	0.118 049	0.348 656	0.595 817	0.041 413	0.399 280	14.147 205
1	1	1.000 000	7	5	2	2	1	4	0.854 580	0.017 092	0.034 183	0.117 873	0.857 481	1.079 671	0.107 590	0.798 444	9.129 283
2	3	0.816 497	7	3	1	3	1	2	0.569 531	0.016 917	0.034 401	0.117 834	0.174 192	1.724 346	0.140 685	1.003 910	10.562 242
7	1	2.645 751	6	3	2	1	3	3	1.454 739	0.017 250	0.033 831	0.117 952	1.791 511	0.038 710	0.040 808	1.034 839	12.147 205
4	3	1.154 701	6	3	2	3	2	4	0.422 536	0.017 246	0.033 954	0.117 917	1.770 817	0.401 348	0.070 316	1.049 098	14.339 850
2	1	1.414 214	7	5	3	1	1	3	1.693 471	0.017 106	0.033 205	0.118 149	0.941 119	1.811 903	0.126 373	1.181 054	10.299 208
3	2	1.224 745	6	3	1	4	3	4	0.310 288	0.016 925	0.033 097	0.118 205	0.126 823	2.131 094	0.173 620	1.236 634	13.339 850
7	5	1.183 216	6	3	2	3	2	4	0.430 082	0.016 735	0.034 499	0.117 831	1.248 391	2.012 330	0.143 390	1.369 734	15.884 171
2	1	1.414 214	7	5	4	1	1	4	1.684 499	0.017 015	0.033 029	0.118 210	0.406 345	2.332 091	0.178 349	1.370 593	11.129 283
2	1	1.414 214	7	7	2	1	1	4	1.684 499	0.017 015	0.033 029	0.118 210	0.406 345	2.332 091	0.178 349	1.370 593	10.614 710
7	3	1.527 525	3	2	1	2	2	4	0.187 167	0.017 015	0.033 029	0.118 210	0.406 345	2.332 091	0.178 349	1.370 593	11.977 280
7	3	1.527 525	3	2	1	1	1	2	0.374 333	0.017 015	0.033 029	0.118 210	0.406 345	2.332 091	0.178 349	1.370 593	8.977 280
7	2	1.870 829	5	3	1	2	3	3	0.491 061	0.016 646	0.034 460	0.117 855	1.771 208	1.899 409	0.123 242	1.501 124	12.884 171
7	1	2.645 751	7	3	1	1	3	1	1.889 406	0.016 477	0.033 739	0.118 088	2.766 971	0.232 593	0.074 493	1.603 723	9.784 635
7	1	2.645 751	7	5	1	1	3	2	1.889 406	0.016 477	0.033 739	0.118 088	2.766 971	0.232 593	0.074 493	1.603 723	11.521 600

RG comments

At a match scale μ_0 one-loop running gives

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(\mu_0) - \frac{b_i}{2\pi} \ln \frac{\mu_0}{M_Z},$$

with SM coefficients b_i in GUT normalisation. Reasonable μ_0 choices and finite thresholds yield $\mathcal{O}(1\%)$ shifts consistent with the residuals above.

Acknowledgements

This paper isolates and formalises the compactification sector implicit in the Janus/PLI programme and builds on the earlier drafts that introduced the 7D geometry and the PLI quantum foundations.

References

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