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UCHINCHI TARTIBLI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMA UCHUN TESKARI MASALA

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Annotatsiya: Ushbu maqolada uchinchi tartibli xususiy hosilali differensial tenglama uchun teskari masala o'rganilgan. Teskari masalaning yechimi uchun analitik va sonli usullar asosida yondashuvlar ko'rib chiqiladi. Shartlar berilgan holda differensial tenglamaning noma'lum koeffitsientlarini aniqlash, mavjudlik va yagona yechimning mavjudligi masalalari tahlil qilinadi. Shuningdek, teskari masalaning barqarorligi va hisoblash aniqligi masalalari muhokama qilinadi hamda misollar yordamida amaliy natijalar keltiriladi.

Kalit so'zlar: uchinchi tartibli differensial tenglama, xususiy hosilali tenglama, teskari masala, analitik yechim, sonli usul, barqarorlik, mavjudlik, yagona yechim.

Hozirgi kunda turli fizik, texnik va muhandislik jarayonlarini matematik modellashtirishda vaqt va fazo o'zgaruvchilari bo'yicha bog'langan xususiy hosilali differensial tenglamalar muhim o'rin egallaydi. Bunday tenglamalar yordamida issiqlik o'tkazish, to'lqin tarqalishi, elastiklik, diffuziya, tebranish va soddalashtirilgan energetik tizimlarning harakati kabi murakkab jarayonlar tahlil qilinadi.

Shunday tenglamalardan biri

$$u_{tt}(x,t) - a^2 u_{xxt}(x,t) = f\left(x\right),\; a \quad 0, \qquad (1)$$

ko'rinishidagi tenglama bo'lib, ushbu tenglama ayrim viskoelastik materiallardagi deformatsiya jarayonlarini, dempferlangan tebranishlar, shuningdek issiqlik yoki mexanik ta'sirning vaqt bo'yicha o'zgaruvchi dinamikasini tavsiflashda uchraydi.

To'g'ri masala. $\Omega = \left\{0 < x < \pi, 0 < t \quad T\right\}$ sohada (1) tenglamani va boshlang'ich

$$u\left(x,0\right) = \varphi\left(x\right),\; u_t\left(x,0\right) = \psi\left(x\right),\; 0 \quad x \quad \pi, \qquad (2)$$

chegaraviy

$$u\left(0,t\right) = u\left(\pi,t\right) = 0 \quad ,\; 0 \quad t \quad T, \qquad (3)$$

shartlarni qanoatlantiruvchi $u\left(x,t\right)$ funksiyani toping.

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Teskari masala. (1)-(3) masaladagi $\varphi(x)$ va $\psi(x)$ funksiyalar berilgan ma'lum, $f(x)$ funksiya noma'lum bo'lsa,

$$u(x,T)=g(x) \qquad (4)$$

qo'shimcha shart asosida $f(x)$ funksiyani toping, bu yerda $g(x)$ berilgan funksiya.

Tadqiqotning asosiy maqsadi — berilgan boshlang'ich-chegaraviy shartlar asosida teskari masala yechimi $f(x)$ funksiyani va to'g'ri masala yechimi $u(x,t)$ funksiyani topishdan va ularning uning mavjudligi, yagonaligi hamda barqarorligini o'rganishdan iborat.

Faraz qilamiz (1)-(3) masalada $f(x)$ berilgan bo'lsin. (1)-(3) masala yechimini Furye usuli bilan qidiramiz, ya'ni

$$u(x,t)=X(x)T(t).$$

U holda (1) tenglamaning bir jinsli qismi $X(x)$ va $T(t)$ funksiyalarga nisbatan

$$X\ (x)+\lambda X(x)=0,$$

$$T\ (t)+\lambda T\ (t)=0$$

tenglamalarga keladi, bu yerda λ o'zgarmas son. (3) chegaraviy shartlarni hisobga olsak, keyingi spektral masalaga kelamiz:

$$X\ (x)+\lambda X(x)=0,$$
$$X(0)=0, X(\pi)=0.$$

Ma'lumki, bu masala $\lambda_n=n^2$ xos sonlarga va $X_n(x)=\sin(nx)$ xos funksiyalarga ega, $n\quad N$. Har bir $\lambda_n, X_n(x)$ juftligiga mos ravishda $T_n(t)$ funksiyani

$$T_n(t)+a^2n^2T_n(t)=0$$

tenglamadan topamiz va

$$T_n(t)=A_n+B_ne^{-a^2n^2t},$$

bu yerda A_n, B_n - noma'lum koeffitsientlar. Bularni aniqlashda boshlang'ich shartlardan foydalanamiz. Demak,

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$$\bar{u}(x,t) = \sum_{n=1} \left(A_n + B_n e^{-a^2 n^2 t}\right)\sin(nx),$$

bunda $\bar{u}(x,t)$ funksiya $\bar{u}_{tt}(x,t) - a^2\bar{u}_{xxt}(x,t) = 0$ tenglamaning yechimi bo'lib, $\bar{u}(x,0) = \varphi(x)$, $\bar{u}_t(x,0) = \psi(x)$ shartlarni qanoatlantirishi kerak. Shunday qilib,

$$\bar{u}(x,0) = \sum_{n=1} (A_n + B_n)\sin(nx) = \varphi(x),$$

$$\bar{u}_t(x,t) = \sum_{n=1} \left(-a^2 n^2 B_n\right)\sin(nx) = \psi(x).$$

Bu tengliklardan

$$A_n = \varphi_n + \frac{\psi_n}{a^2 n^2},\ B_n = -\frac{\psi_n}{a^2 n^2}$$

bo'lishini aniqlaymiz, bu yerda $\varphi_n = \frac{2}{\pi}\int_0^\pi \varphi(x)\sin(nx)\,dx$, $\psi_n = \frac{2}{\pi}\int_0^\pi \psi(x)\sin(nx)\,dx$. Natijada

$$\bar{u}(x,t) = \sum_{n=1} \varphi_n + \frac{\psi_n}{a^2 n^2}\left(1 - e^{-a^2 n^2 t}\right) \sin(nx). \tag{5}$$

Endi (1) tenglamaning bir jinslimas qismining yechimini topamiz. Ya'ni

$$\tilde{u}_{tt} - a^2\tilde{u}_{xxt} = f(x) \tag{6}$$

tenglamani va

$$\tilde{u}(x,0) = 0,\ \tilde{u}_t(x,0) = 0, \tag{7}$$

$$\tilde{u}(0,t) = 0, \tilde{u}(\pi,t) = 0 \tag{8}$$

shartlarni qanoatlantiruvchi $\tilde{u}(x,t)$ funksiyani aniqlaymiz. Ma'lumki,

$$\tilde{u}(x,\mathrm{t}) = \sum_{n=1} \tilde{T}_n(t)\sin(nx),$$

$$f(x) = \sum_{n=1} f_n \sin(nx)$$

shaklda yozish mumkin. Bulardan va (6) tenglama hamda (7) shartlardan

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$$\tilde{T}_n''(t)+a^2n^2\tilde{T}_n'(t)=f_n,$$

$$\tilde{T}_n(0)=0,\ \tilde{T}_n'(0)=0$$

masalaga kelamiz. Buning yechimi

$$\tilde{T}(t)=\frac{f_n}{a^2n^2}\left[t-\frac{1}{a^2n^2}\left(1-e^{-a^2n^2t}\right)\right].$$

Demak, (6)-(8) masala yechimi

$$\tilde{u}(x,t)=\sum_{n=1}^{\infty}\frac{f_n}{a^2n^2}\left[t-\frac{1}{a^2n^2}\left(1-e^{-a^2n^2t}\right)\right]\sin(nx). \qquad (9)$$

Vanihoyat, (5) va (9) asosida (1)-(3) masala yechimini

$$u(x,t)=\sum_{n=1}^{\infty}\left[\varphi_n+\frac{f_nt}{a^2n^2}+\frac{1}{a^2n^2}\left(1-e^{-a^2n^2t}\right)\left(\psi_n-\frac{f_n}{a^2n^2}\right)\right]\sin(nx) \qquad (10)$$

ko'rinishda yozamiz.

Teskari masalani yechish. (10) yechimni (4) shartga qanoatlantiramiz va

$$g(x)=\sum_{n=1}^{\infty}\left[\varphi_n+\frac{f_nT}{a^2n^2}+\frac{1}{a^2n^2}\left(1-e^{-a^2n^2T}\right)\left(\psi_n-\frac{f_n}{a^2n^2}\right)\right]\sin(nx).$$

Bundan tashqari

$$g(x)=\sum_{n=1}^{\infty}g_n\sin nx,$$

bu yerda $g_n=\frac{2}{\pi}\int_0^{\pi}g(x)\sin(nx)dx$. Bu tengliklardan

$$g_n=\varphi_n+\frac{f_nT}{a^2n^2}+\frac{1}{a^2n^2}\left(1-e^{-a^2n^2T}\right)\left(\psi_n-\frac{f_n}{a^2n^2}\right)$$

tenglikni aniqlaymiz va bundan $\forall n\in N$ uchun f_n qiymati

$$f_n=\frac{a^4n^4g_n-a^4n^4\varphi_n-a^2n^2\psi_n\left(1-e^{-a^2n^2T}\right)}{Ta^2n^2-1+e^{-a^2n^2T}} \qquad (11)$$

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formula bilan aniqlanishini topamiz.

1-teorema. Aytaylik, $\varphi(x)\quad W_2^2[0,\pi]$, $g(x)\quad W_2^2[0,\pi]$, $\psi(x)\quad L_2[0,\pi]$ bo'lsin. U holda $\forall T\quad T_0>0$ uchun (1)-(4) teskari masala yechimi mavjud, yagona va

$$\|f(x)\|\quad \frac{a^2C_1}{T}\left(\|g(x)\|_{W_2^2}+\|\varphi(x)\|_{W_2^2}\right)+\frac{C_2}{T}\|\psi(x)\|_{L_2}$$

tengsizlik o'rinli bo'ladi, bu yerda C_1, C_2 - chegaralangan musbat o'zgarmaslar.

Isbot. (11) tenglikdan

$$f(x)={}_{n=1}\quad\frac{a^4n^4g_n-a^4n^4\varphi_n-a^2n^2\psi_n\left(1-e^{-a^2n^2T}\right)}{Ta^2n^2-1+e^{-a^2n^2T}}\quad\sin(nx).$$

U holda

$$\|f(x)\|^2={}_{n=1}\quad\frac{a^4n^4g_n-a^4n^4\varphi_n-a^2n^2\psi_n\left(1-e^{-a^2n^2T}\right)}{Ta^2n^2-1+e^{-a^2n^2T}}^{\;2}$$

$$ {}_{n=1}\frac{3a^8n^8g_n^2}{\left(Ta^2n^2-1+e^{-a^2n^2T}\right)^2}+{}_{n=1}\frac{3a^8n^8\varphi_n^2}{\left(Ta^2n^2-1+e^{-a^2n^2T}\right)^2}+{}_{n=1}\frac{3a^4n^4\psi_n^2\left(1-e^{-a^2n^2T}\right)^2}{\left(Ta^2n^2-1+e^{-a^2n^2T}\right)^2}. \qquad (12)$$

Shuni takidlaymizki, $\forall T>0$ uchun $Ta^2n^2-1+e^{-a^2n^2T}>0$. T qiymati 0 soniga yaqinlashganda (12) qator qiymati $+$ ga intiladi. Shuning oldini olish uchun, T qiymati $T\quad T_0>0$ bo'yicha qaraladi.

(12) tengsizlikning o'ng tarafidagi qatorlarni baholab chiqamiz:

Yetarlicha katta n qiymatlarida ${}_{n=1}\dfrac{a^8n^8g_n^2}{\left(Ta^2n^2-1+e^{-a^2n^2T}\right)^2}$ qator ${}_{n=1}\dfrac{a^8n^8g_n^2}{\left(Ta^2n^2\right)^2}={}_{n=1}\dfrac{a^4n^4g_n^2}{T^2}$ qator bilan, ${}_{n=1}\dfrac{a^8n^8\varphi_n^2}{\left(Ta^2n^2-1+e^{-a^2n^2T}\right)^2}$ qator ${}_{n=1}\dfrac{a^4n^4\varphi_n^2}{T^2}$ qator bilan va ${}_{n=1}\dfrac{a^4n^4\psi_n^2\left(1-e^{-a^2n^2T}\right)^2}{\left(Ta^2n^2-1+e^{-a^2n^2T}\right)^2}$ qator ${}_{n=1}\dfrac{\psi_n^2}{T^2}$ qator bilan majorant bo'ladi. Demak, agar

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$\varphi(x) \quad W_2^2[0,\pi]$, $g(x) \quad W_2^2[0,\pi]$ va $\psi(x) \quad L_2[0,\pi]$ bo‘lsa $\|f(x)\|^2$ yaqinlashuvchi qator bo‘ladi. Bu esa (1)-(4) teskari masala yechimi $L_2[0,\pi]$ fazoda mavjud ekanligini ko‘rsatadi.

$\varphi(x)$, $g(x)$, $\psi(x)$ funksiyalarga mos φ_n, g_n, ψ_n Furye koeffitsiyentlar $\forall n \quad N$ uchun bir qiymatli aniqlangan bo’ladi. Bundan, $\varphi(x)=0$, $g(x)=0$, $\psi(x)=0$ bo‘lsa $f(x) \quad 0$ ekanligi kelib chiqadi, bu yerda $T \quad T_0>0$. Demak, (1)-(4) teskari masala yechimi yagona bo‘lishini ko‘rish mumkin.

Qatorlar yaqinlashuvchi ekanligidan,

$$\|f(x)\|^2 \quad \frac{a^4C_1^2}{T^2}\left(\|g(x)\|_{W_2^2}^2+\|\varphi(x)\|_{W_2^2}^2\right)+\frac{C_2^2}{T^2}\|\psi(x)\|_{L_2}^2$$

tengsizlikni yozish mumkin, bu yerda C_1, C_2 biror musbat o‘zgarmaslar. Bundan esa

$$\|f(x)\| \quad \frac{a^2C_1}{T}\left(\|g(x)\|_{W_2^2}+\|\varphi(x)\|_{W_2^2}\right)+\frac{C_2}{T}\|\psi(x)\|_{L_2}$$

tengsizlik kelib chiqadi. 1-teorema isbot bo‘ldi.

(11) tenglikni (10) yechim ifodasiga olib borib qo‘yilsa (1)-(4) to‘g‘ri masala yechimini

$$u(x,t)=\sum_{n=1} \left[\varphi_n+\frac{\psi_n}{a^2n^2}\left(1-e^{-a^2n^2t}\right)+\frac{a^2n^2t-1+e^{-a^2n^2t}}{a^2n^2T-1+e^{-a^2n^2T}}\left[g_n-\varphi_n-\frac{\psi_n}{a^2n^2}\left(1-e^{-a^2n^2T}\right)\right]\right]\sin(nx)$$

(13)

ko‘rinishda ifodalash mumkin.

2-teorema. Agar $T \quad T_0>0$, $\varphi(x) \quad W_2^2[0,\pi]$, $g(x) \quad W_2^2[0,\pi]$ va $\psi(x) \quad W_2^2[0,\pi]$ bo‘lsa, (1)-(4) to‘g‘ri masala yechimi mavjud, yagona bo‘ladi hamda

$$\|u(x,t)\| \quad 2\sqrt{3}\|\varphi(x)\|+\frac{2\sqrt{3}}{a^2}\|\psi(x)\|+3\|g(x)\|$$

tengsizlik o‘rinli.

Isbot. (13) funksiyadan mos ravishda (1) tenglamada qatnasghgan xususiy hosilalarni hisoblaymiz:

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$$u_{tt}(x,t)=\sum_{n=1}^{\infty}\left[-a^2n^2\psi_n e^{-a^2n^2t}+\frac{a^4n^4e^{-a^2n^2t}}{a^2n^2T-1+e^{-a^2n^2T}}\left(g_n-\varphi_n-\frac{\psi_n}{a^2n^2}\left(1-e^{-a^2n^2T}\right)\right)\right]\sin(nx),$$

$$u_{xxt}(x,t)=\sum_{n=1}^{\infty}\left[-n^2\psi_n e^{-a^2n^2t}+\frac{-a^2n^4+a^2n^4e^{-a^2n^2t}}{a^2n^2T-1+e^{-a^2n^2T}}\left(g_n-\varphi_n-\frac{\psi_n}{a^2n^2}\left(1-e^{-a^2n^2T}\right)\right)\right]\sin(nx).$$

Bulardan, (13) tenglamani qanoatlantirishini ko'rishimiz mumkin. Bundan tashqari, $\|u(x,t)\|^2$, $\|u_{tt}(x,t)\|^2$, $\|u_{xxt}(x,t)\|^2$ normalardan hosil bo'lgan qatorlar yaqinlashuvchi bo'ladi agar $\varphi(x)$, $g(x)$, $\psi(x)$ boshlang'ich berilgan funksiyalar $W_2^2[0,\pi]$ fazodan bo'lsa, bu esa mavjudligini anglatadi. (13) dan

$$\|u(x,t)\|^2 \le 12\left(\sum_{n=1}^{\infty}\varphi_n^2+\frac{12}{a^4}\sum_{n=1}^{\infty}\psi_n^2+9\sum_{n=1}^{\infty}g_n^2\right)$$

tengsizlik kelib chiqadi yoki

$$\|u(x,t)\| \le 2\sqrt{3}\|\varphi(x)\|+\frac{2\sqrt{3}}{a^2}\|\psi(x)\|+3\|g(x)\|.$$

Foydalanilgan adabiyotlat:

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