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DIOFANT TENGLAMALARINI YECHISH USULLARI

Kirish. Matematikada Diofant tenglamalari — bu noma’lumlar butun son bo‘lishi kerak bo‘lgan tenglamalardir. Ular qadimgi yunon matematigi Diofant nomi bilan atalgan. Bunday tenglamalarni yechishning maqsadi — barcha butun sonli yechimlarni topishdir.

Masalan:

$3x + 5y = 11$, yoki $x^2 + y^2 = z^2$ kabi.

Quyida biz Diofant tenglamalarini turlarini va ularning yechimlarini o’rganamiz.

1. Chiziqli Diofant tenglamalar
2. Kvadrat Diofant tenglamalari.
3. Nostandart kvadrat tenglamalar

Materiallar va usullar.

Chiziqli Diofant tenglamalari

Quyidagi

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = b \qquad (1)$$

ko’rinishidagi tenglama **chiziqli Diofant tenglamasi** deyiladi. Bu yerda a_1, a_2, a_n, b lar o’zgarmas butun sonlar. Biz $n \quad 1$ deb faraz qilamiz.

(G’arb matematika usuli)

Teorema. (1) tenglama yechimga ega bo’ladi, agar

$\gcd(a_1, a_2, ..a_n) | b$.

(gcd-greatest common devisor-EKUB) Agar yechim mavjud bo’lsa, unda n-1 ta yechim tanlash mumkin, har qanday boshqa yechim shu n-1 ta yechimning butun sonli chiziqli kombinatsiyasi orqali ifodalanadi.

Isbot. $\left(\gcd(a_1, a_2, ..a_n) = d\right) EKUB(a_1, a_2, ..a_n) = d$ bo’lsin. Agar d ga teng bo’lmasa, (1) tenglama ildizga ega emas bo’ladi. Chunki, har qanday $x_1, x_2, ..x_n$ butun sonlar uchun chap tomoni d ga bo’linadi, o’ng tomoni esa yo’q. Demak, $EKUB(x_1, x_2, ..x_n)$ ni $x_1, x_2, ..x_n$ butun sonlar koeffitsiyentlari bilan chiziqli kombinatsiya tuzish mumkinligini isbotlashimiz kerak. n=2 uchun bu proporsiyadan kelib chiqadi. Chunki

$EKUB(x_1, x_2, ..x_n) = EKUB\left(EKUB(x_1, x_2, ..x_{n-1}), x_n\right)$.

Bundan $EKUB\left(x_1,x_2,..x_n\right)$ ifoda x_n va $EKUB\left(x_1,x_2,..x_{n-1}\right)$ ning chiziqli kombinatsiyasi ekanligini bilib olamiz. Shunday qilib, matematik induksiya metodidan x_n va x_{n-1} lar $x_1,x_2,..x_n$ larning chiziqli kombinatsiyasidir.

Xulosa. Agar a_1 va a_2 lar o'zaro tub butun sonlar va (u,v) lar quyidagi

$$a_1x_1+a_2x_2=b$$

tenglama yechimi bo'lsa, u holda ushbu tenglama umumiy yechimi quyidagicha ifodalanadi:

$$x_1=u+a_2t \quad x_2=v-a_1t \quad t \quad Z. \qquad (2)$$

1-misol. 3x+4y+5z=6 tenglamani yeching.

Yechish: Tenglikni 5 modul bo'yicha hisobga olsak, $3x+4y \quad 1(\bmod 5)$. Demak $3x+4y=1+5k \quad k \quad Z$ bo'ladi. Bu tenglamani chap tomoni o'ng tomoniga teng bo'ladigan yechim tanlab olamiz $x_1=-1+3k, \quad y_1=1-k$. Endi tenglama yechimini (2) ga asoslanib yozamiz:

$$x=-1+3k+4l, \quad y=1-k-3l \quad k,l \quad Z.$$

2-misol. $x+y+z+xyz=xy+xz+yz+2$ tenglamani manfiy bo'lmagan butun sonlarda yeching.

Yechish: Biz tenglamani ko'paytiruvchilarga ajratib olamiz:

$$xyz-\left(xy+xz+yz\right)+x+y+z-1=1$$

$$xy\left(z-1\right)-xz+x-yz+y+z-1=1$$

$$\left(z-1\right)\left(xy-x-y+1\right)=1 \qquad \left(z-1\right)\left(y-1\right)\left(x-1\right)=1$$

x,y,z lar nomanfiy butun sonlar bo'lganligi sababli, biz tanlab olamiz

$x-1=y-1=z-1=1 \quad natijada \quad x=y=z=2$ kelib chiqadi.

(2-usul)

Chiziqli Diofant tenglamalarni yechishda o'zgaruvchilar soni n-1 taga tushiriladi va koeffitsientlarni karrali ko'paytuvchilarga ajartish $a_i=ka_j$ orqali berilgan tenglama n-1 ta o'zgaruvchiga bog'liq yechimga keltiriladi.

3-misol. 5x+6y+7z=19 tenglamani butun sonlarda yeching.

Yechish: noma'lumni bittaga kamaytiramiz:

$$5\left(x+y\right)+y+7z=19 \quad x+y=a \quad 5a+y+7z=19$$

$$5a+\underbrace{y+6z}_{b}=19 \quad 5a+b+z=19 \quad z=19-5a-b$$

$$\begin{matrix} y+6z=b \\ y=b-6z=b-6\left(19-5a-b\right) \end{matrix} \qquad \begin{matrix} x=-29z-7b+114 \\ y=30a+7b-114 \quad a,b \quad Z \\ z=19-5a-b \end{matrix}$$

Ta'rif. ax+by=c ko'rinishdagi tenglama ikki o'zgaruvchili chiziqli Diofant tenglamalari deyiladi, bu yerda $a,b,c \quad Z, \quad \left(a,b,c\right)=1$

Teorema. ax+by=c tenglamani butun sonlar to'plamida yechimga ega bo'lishi uchun (a,b)=1 bo'lishi zarur va yetarlidir.

Isboti. Zaruriyligi. Faraz qilaylik (a,b)=d>1 bo'lsa ax+by=c tenglama butun sonlar to'plamida yechish mumkin bo'lsin. Faraz qilamiz $x_0,y_0 \in Z$ sonlar juftligi ax+by=c tenglamaning ildizi bo'lsin, ya'ni $ax_0+by_0=c$. Ma'lumki, (a,b)=d>1 va $ax_0+by_0=c$ bo'lgani uchun c ozod son d ga bo'linadi. Demak, (a,b,c)=1. Bu esa (a,b,c)=1 shartga zid.

Yetarliligi. (a,b)=1 bo'lsin. Shunday EKUB xossasiga ko'ra shunday $x', y' \in Z$ sonlar mavjudki $1=ax'+by'$. Tenglikning har ikkala tomonini c ga ko'paytiramiz: $a(cx')+b(cy')=c$. Natijada cx', cy' butun sonlar bo'lib ax+by=c tenglamaning ildizlaridir. Ularni mos ravishda $cx'=x_0$, $cy'=y_0$ kabi belgilaymiz. Demak, (a,b)=1 bo'lishi kelib chiqadi.

Yechish usullari:

a) taqqoslash usuli; b) munosib kasrlar usuli;

c) tanlash usuli; d) ko'paytuvchilarga ajratish usuli va belgilash usuli; e) ko'paytuvchilarga ajatish va o'rniga qo'yish usullari yordamida yechamiz.

Ko'paytuvchilarga ajatish va o'rniga qo'yish usuli mohiyati quyidagilardan iborat:

$$ax+by=c \qquad ax=c-by \quad x=\frac{c-by}{a}=d+\frac{n+ky}{a}$$

$$\left\langle \frac{n+ky}{a}=t \right\rangle \qquad y=\frac{at-n}{k}=l+\frac{ft+h}{k}=l+t_1 \quad \left\langle \frac{ft+h}{k}=t_1 \right\rangle$$

butun son bo'lguncha shunday belgilashlar qilib boramiz.

Bu usul mohoyati quyidagi misol orqali ko'rsatiladi:

4-misol. 24x-17y=2 tenglamani yeching.

Yechish: $24x-17y=2/\ 7 \quad 168x-119y=14 \quad 24\underbrace{(7x-5y)}_{t}+y=14 \quad \begin{matrix} y=14-24t \\ x=10-17t \end{matrix}.$

Evklid algoritmidan foydalanish usuli.

Evklid algoritmidan foydalanib ax+by=c ko'rinishidagi tenglamalarni yechish 1 sonini a va b sonlar orqali ifodalashga asoslanadi.

Natija. ax+by=c tenglamaning ildizlari osod son c ga karrali bo'ladi. Tenglamaning barcha butun ildizlarini topamiz:

1) Agar c=0 bo'lsa, ax+by=0 tenglamada $y = -\frac{ax}{b}$. Shartga ko'ra $y \quad Z$ va (a,b)=1. Demak, shunday $t \quad Z$ mavjudki, x=bt. U holda y=-at. Natijada tenglamaning umumiy ildizi quyidagicha

$$\begin{matrix} x = bt \\ y = -at, \end{matrix} \quad t \quad Z.$$

2) Agar $c \quad 0$ bo'lsa, ax+by=c tenglama uchun shunday $x_0, y_0 \quad Z$ mavjudki, $ax_0 + by_0 = c$. Bunga asosan ax+by=c va $ax_0 + by_0 = c$ tenglamalarni ayiramiz, natijada: $a\left(x - x_0\right) + b\left(y - y_0\right) = 0$. Bu tenglama yuqoridagi kabi yechiladi:

$$\begin{matrix} x - x_0 = bt \\ y - y_0 = -at, \end{matrix} \quad \begin{matrix} x = x_0 + bt \\ y = y_0 - at, \end{matrix} \quad t \quad Z.$$

5-misol. 17x+11y=6 tenglamani yeching.

Yechish: Tenglamada (17,11,6)=1 va (17,11)=1. Demak, tenglama butun sonlar to'plamida ildizga ega. 1 sonini a va b sonlar orqali ifodalaymiz. a=17, b=11.

$$17 = 11 \quad 1 + 6 \qquad 11 = 6 \quad 1 + 5 \qquad 6 = 5 \quad 1 + 1$$

$$1 = 6 - 5 \quad 1 = 6 - 1 \quad \left(11 - 6 \quad 1\right) = 6 \quad 2 - 11 \quad 1 = \left(17 - 11 \quad 1\right) \quad 2 - 11 \quad 1 = 17 \quad 2 - 3 \quad 11$$

$$1 = 17 \quad 2 - 3 \quad 11$$

a=17, b=11 bo'lgani uchun $17 \quad 2 - 3 \quad 11 = 1$ ifodani quyidagicha yozamiz:

$17 \quad 2 - 3 \quad 11 = 1 / \ 6 \quad va \quad 17 \quad 12 + 11 \quad \left(-18\right) = 6.$ Demak, $x_0 = 12, \quad y_0 = -18$ va

$$\begin{matrix} x = x_0 + bt \\ y = y_0 - at, \end{matrix} \quad \begin{matrix} x = 12 + 11t \\ y = -18 - 17t, \end{matrix} \quad t \quad Z.$$

Javob: (12+11t; -18-17t), $t \quad Z$

Zanjir kasrdan foydalanish usuli.

Ta'rif. Chekli zanjir kasr deb quyidagi ko'rinishdagi ifodaga aytiladi:

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ldots + \cfrac{1}{a_n}}}$$

bu yerda $a_i \quad N, \quad i = \overline{1, n}, \quad a_0 \quad Z, \quad a_n \quad 1.$

Chekli zanjir kasrlar quyidagi ko'rinishda belgilanadi: $\frac{a}{b} = \left[a_0; a_1; a_2, \ldots, a_n\right].$

Ta'rif. a_i son zanjir kasrning elementi deyiladi.

Ta'rif. a_0 son nolinchi tartibli kasrga mos son deyiladi va u $a_0 = \frac{b_0}{q_0}$ kabi yoziladi, bu yerda $q_0 = 1.$

Teorema. Har qanday a/b ratsional sonni chekli zarjir kasr shaklida yagona tarzda ifodalash mumkin va zarjir kasrning $[a_i]$ elementlari a vva b sonlari uchun Evklid algoritmidan hosil bo'ladi.

Isboti. $\frac{a}{b} \quad Q \quad a \quad Z, b \quad N,$ a va b sonlariga Evklid algoritmini qo'llaymiz:

$$a = ba_0 + r_1, \quad 0 \quad r_1 < b, \quad b = r_1 a_1 + r_{2,}\, 0 \quad r_2 < r_1,$$
$$r_1 = r_2 a_2 + r_3, 0 \quad r_3 < r_2,$$
$$\ldots\ldots\ldots$$
$$r_{n-2} = r_{n-1} a_{n-1} + r_n, \quad 0 \quad r_n < r_{n-1}, \qquad r_{n-1} = r_n a_n + 0.$$

Bu yerda $a_0 \quad Z, \quad i = \overline{1,n}, \quad a_i \quad N.$ Birinchi tenglikni b ga ikkinchisini r_1 ga bo'lamiz va shu ishni takrorlaymiz.

$$\frac{a}{b} = a_0 + \frac{r_1}{b} = a_0 + \frac{1}{\frac{b}{r_1}}, \quad \frac{b}{r_1} = a_1 + \frac{r_2}{r_1} = a_1 + \frac{1}{\frac{r_1}{r_2}},$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\frac{r_{n-2}}{r_n} = a_{n-1} + \frac{1}{\frac{r_{n-1}}{r_n}}, \qquad \frac{r_{n-1}}{r_n} = a_n.$$

Birinchi tenglikka qolgan hammasini qo'yamiz:

$$\frac{a}{b} = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ldots + \cfrac{1}{a_n}}}$$

bu yerda $a_i \quad N, \quad i = \overline{1,n}, \quad a_0 \quad Z, \quad a_n \quad 1.$

Zanjir kasrlarning qismlarini qurish usulini keltiramiz.

$\frac{a}{b} = [a_0; a_1; \ldots; a_n], \quad A_k = [a_0; a_1; \ldots; a_k], \quad k \quad n$ berilgan bo'lsin.

1) 11nolinchi tartibli kasr, $p_0 = a_0, q_0 = 1$.

2) $A_1 = a_0 + \frac{1}{a_1} = \frac{p_1}{q_1} \qquad p_1 = a_0 a_1 + 1, \qquad q_1 = a_1$

....

k) $A_k = \frac{p_k}{q_k} = \frac{p_{k-1}a_k + p_{k-2}}{q_{k-1}a_k + q_{k-2}} \Rightarrow p_k = p_{k-1}a_k + p_{k-2}; \qquad q_k = q_{k-1}a_k + q_{k-2}$.

Buni quyidagi jadvalda tasvirlaymiz:

k	0	1	2	...	n
a_k	a_0	a_1	a_2	...	a_n
p_k	$p_0 = a_0$	$p_1 = a_0 a_1 + 1$	$p_1 a_2 + a_0$	...	$p_{n-1}a_n + p_{n-2}$
q_k	$q_0 = 1$	$q_1 = a_1$	$q_1 a_2 + q_0$	...	$q_{n-1}a_n + q_{n-2}$

Demak, $\frac{a}{b} = [a_0; a_1; \ldots; a_n]$, $\frac{a}{b} = \frac{p_n}{q_n}$. Quyidagi tenglamani ko’raylik:

$ax + by = c, \quad (a,b,c) = 1$ va (a,b)=1. Demak, ushbu Diofant tenglamasi butun sonlarda yechimga ega. Zanjir kasrdan foydalanamiz:

$$\frac{a}{b} = [a_0; a_1; \ldots; a_n], \quad \frac{a}{b} = \frac{p_n}{q_n}, \quad \frac{p_n}{q_n} - \frac{p_{n-1}}{q_{n-1}} = \frac{(-1)^{n-1}}{q_n q_{n-1}}; \quad \frac{a}{b} - \frac{p_{n-1}}{q_{n-1}} = \frac{(-1)^{n-1}}{b q_{n-1}}$$

$$a q_{n-1} - b q_{n-1} = (-1)^{n-1}.$$

Tenglikni har ikkala tomonini c ga ko’paytiramiz:

$$a(cq_{n-1}) + b(cp_{n-1}) = c(-1)^{n-1},$$

Yana $(-1)^{n-1}$ ga ko’paytiramiz: $a\left((-1)^{n-1} cq_{n-1}\right) + b\left((-1)^{n-1} cp_{n-1}\right) = c.$

Natijalar. 587x+113y=1 tenglamani yeching.

Yechish: Zanjir kasrdan foydalanamiz:\

$$\frac{587}{113} = 5 + \frac{22}{113} = 5 + \frac{1}{\frac{113}{22}} = 5 + \frac{1}{5 + \frac{1}{7 + \frac{1}{3}}}$$

Ushbu zanjir kasrning oxirgi bo’g’inini -uchdan birni tashlab yuborib, hosil bo’lgan zanjir kasrni oddiy kasrga aylantiramiz:

$$5+\cfrac{1}{5+\cfrac{1}{7}}=5+\cfrac{1}{\cfrac{36}{7}}=5+\frac{7}{36}=\frac{187}{36}.$$

Uni 587/113 kasrdan ayiramiz:

$$\frac{587}{113}-\frac{187}{36}=\frac{21132-21131}{113\ \ 36}=\frac{1}{113\ \ 36}$$

Demak, umumiy maxrajga keltirib, uni berilgan tenglama bilan solishtiramiz:

x=36, y=-187 bitta xususiy ildizni topamiz. Natijada

$$\begin{matrix} x=x_0+bt & x=36+113t \\ y=y_0-at, & y=-187-587t, \end{matrix} \quad t \quad Z.$$

7-misol. 571x+359y=7 tenglamani yeching.

Yechish: (571,359,7)=1 va (571,359)=1, demak, tenglama butun sonlar to’plamida yechimga ega.
$\frac{a}{b}=\frac{571}{359}$; $\frac{571}{359}=[1;1;1;2;3;1;4;1;2]$. $p_k=p_{k-1}a_k+p_{k-2}$,

$q_k=q_{k-1}a_k+q_{k-2}$

k	0	1	2	3	4	5	6	7	8
a_k	1	1	1	2	3	1	4	1	2
p_k	1	2	3	8	27	35	167	202	571
q_k	1	1	2	5	17	22	105	127	359

$\frac{571}{359}-\frac{202}{127}=\frac{(-1)^7}{359\ \ 127}$. Buni tenglama bilan solishtirsak,
$571\ \ 127+359\ \ (-202)=-1/\ \ 7 \quad 571\ \ 889+359\ \ (-1414)=-7 \quad x_0=889 \quad y_0=-1414$

Demak, $\begin{matrix} x=x_0+bt=889+359t \\ y=y_0-at=-1414-571t, \end{matrix} \quad t \quad Z.$

Muhokama. Ushbu maqolada ixtiyoriy darajali diofant tenglamalarining butun va ratsional yechimlarini topish usullari, so‘ngra ikki noma'lumli chiziqli tenglamalarni yechish usullari batafsil bayon etildi, misollar keltirildi. So‘ngra tenglamalarning yechimi Avval Yevklid algoritmi bilan, so‘ngra zanjirli kasr yordamida qidirildi. Har bir usul afzalligi, bajarilash jaroni ham nazariy ma'lumotlar bilan, ham misollar bilan ko'rsatib o'tildi. Diofant tenglamalarining boshq atenglamalardan farqi tenglama ildizlari butun sonlar to'plamidan qidirishidadir,

Foydalanilgan adabiyotlar:

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