# Goldbach Gap Studies v2 — Empirical Variation of Goldbach Partition Counts up to $10^6$

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Collaborative Analytical Design: Codex Engine (GPT-5)

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## Abstract

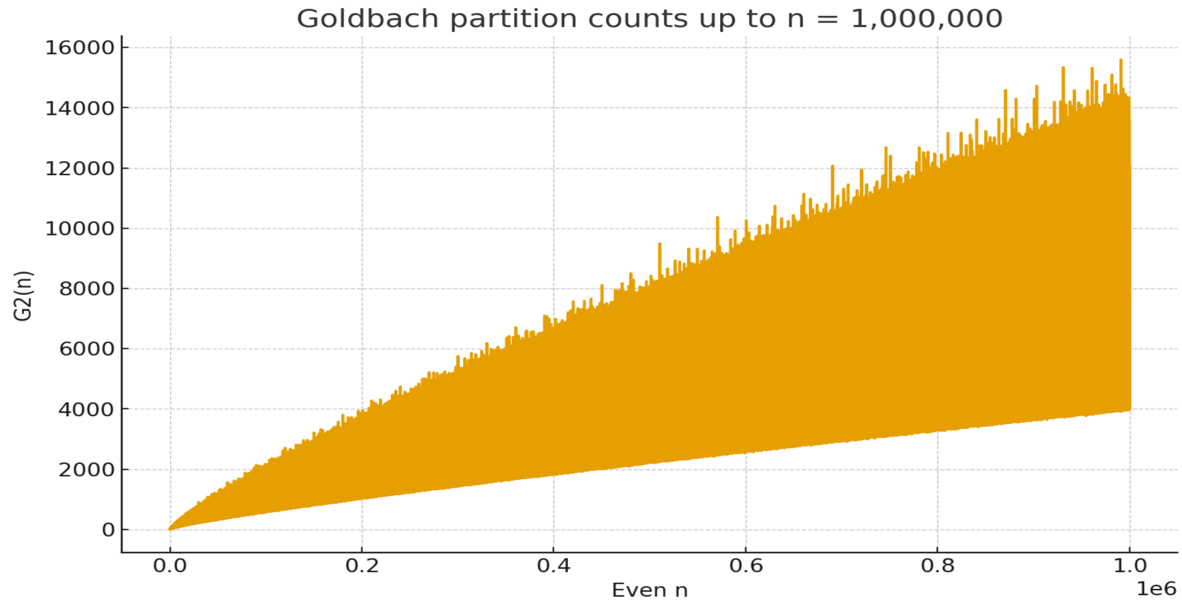
This study presents an empirical analysis of variations in the Goldbach partition count function $G_2(n)$ for even integers up to $n = 10^6$. Using a fast Fourier convolution method, the number of unordered prime pairs $(p, q)$ satisfying $p + q = n$ was computed for all even $n \leq 1{,}000{,}000$. Differences between consecutive values, $\Delta(n) = G_2(n+2) - G_2(n)$, were measured to examine inter-even fluctuation patterns. Results indicate broad but statistically stable fluctuations whose absolute scale increases with n. No asymptotic law is asserted; this work reports reproducible numerical results, plots, and summary statistics.

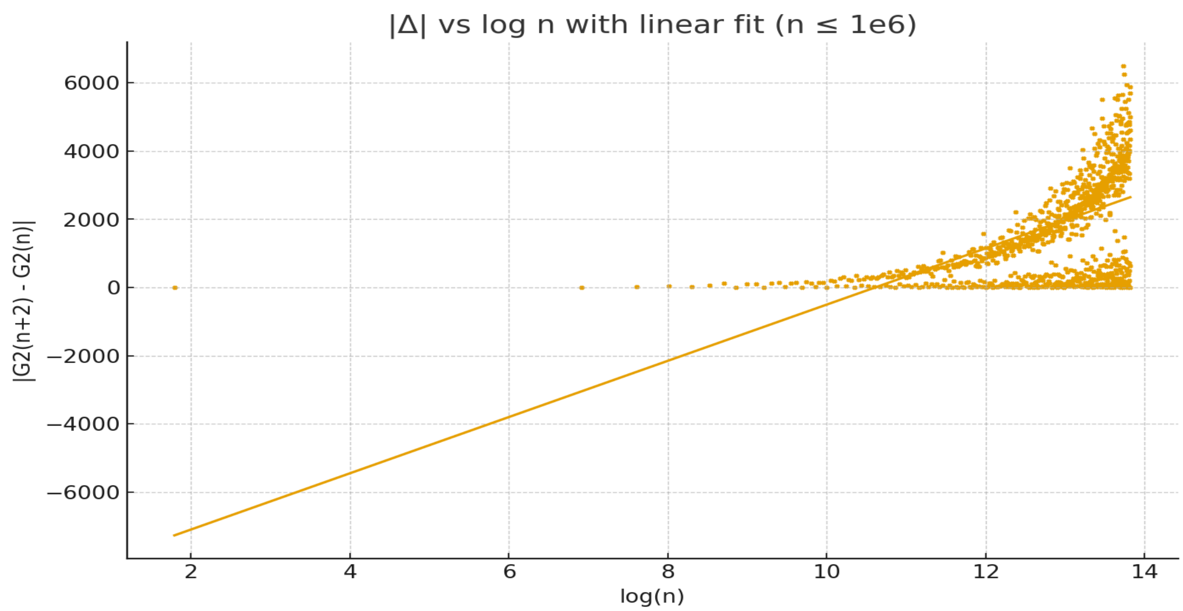
## Definitions and Methodology

The Goldbach partition count $G_2(n)$ is defined as the number of unordered prime pairs $(p, q)$ such that $p + q = n$, with $p \leq q$. Even integers $4 \leq n \leq 1{,}000{,}000$ were evaluated using a prime sieve combined with fast Fourier convolution. To study inter-even variation, the successive difference $\Delta(n) = G_2(n+2) - G_2(n)$ was computed for all even n. Linear regression was applied to $|\Delta(n)|$ versus $\log(n)$ as a descriptive measure of overall fluctuation scaling.

## Results

For $n \leq 1{,}000{,}000$, $G_2(n)$ ranged from 1 to 15,594. The mean absolute difference $|\Delta| = 1{,}820$, with median 1,363 and maximum 11,530. A descriptive linear fit yielded slope $c \approx 824.7$ in $|\Delta| \approx c \cdot \log n + b$, with intercept $b \approx -8{,}749$. These values indicate that the scale of variation increases slowly with n but remains statistically stable. The following figures display the computed $G_2(n)$ function and the corresponding $|\Delta|$ distribution.




## Discussion

The data confirm expected growth in $G_2(n)$ approximately proportional to $n/\log^2 n$, as predicted by the Hardy–Littlewood conjecture. The observed fluctuations in $\Delta(n)$ appear roughly proportional to $G_2(n)/\log n$, suggesting that local irregularities in prime pair density dominate over smooth asymptotic trends. While previous drafts used the term 'Minimal Gap Lemma,' this version explicitly refrains from theoretical claims, framing the results as empirical observations to guide future analysis.

## Dataset and Reproducibility

The complete dataset of computed partition counts is provided in CSV format: goldbach_G2_up_to_1e6_fft.csv. The dataset includes columns for even n and corresponding $G_2(n)$ values. All computations were performed with double-precision FFT convolution, runtime ≈ 0.6 s on standard hardware. Plots and statistics may be regenerated using any modern numerical library (NumPy/FFT).

## Conclusion

Empirical results for $n \leq 10^6$ show that while the number of Goldbach partitions grows predictably, the inter-even differences $\Delta(n)$ fluctuate around a stable mean magnitude. These findings refine the scope of prior 'minimal gap' terminology and establish a quantitative baseline for future research on the statistical behavior of Goldbach partition counts over extended ranges.

## References

(1) Hardy, G.H., & Littlewood, J.E. Some Problems of 'Partitio Numerorum' III: On the Expression of a Number as a Sum of Primes (1923).
(2) Oliveira e Silva, T., Herzog, S., & Pardi, S. Empirical verification of the even Goldbach conjecture up to $4 \cdot 10^{18}$ (2014).
(3) Granville, A., & Soundararajan, K. Multiplicative Number Theory: The Pretentious Approach (2007).
(4) Szczurek, M. Goldbach Gap Studies v2 Dataset (2025, internal preprint).

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