

# Mathematical formulation

The problem can be stated as

$$\begin{aligned} \min_{h,v,m,T} \quad & J = -h(T) \\ \text{s.t.} \quad & \dot{h}(t) = v(t), \\ & \dot{v}(t) = \frac{T(t) - D(h(t), v(t)) - m(t)g(h(t))}{m(t)}, \\ & \dot{m}(t) = -\frac{T(t)}{c}, \\ & 0 \leq T(t) \leq T_{\max}, \\ & h(t) \geq h_0, \\ & v(t) \geq v_0, \\ & m_f \leq m(t) \leq m_0, \\ & h(0) = h_0, \quad v(0) = v_0, \quad m(0) = m_0, \\ & m(T) = m_f, \end{aligned}$$

where the Drag, Gravity, Fuel constant, and Maximum thrust are defined as

$$D(h, v) = D_c v^2 \exp\left(-h_c \frac{h - h_0}{h_0}\right), \quad g(h) = g_0 \left(\frac{h_0}{h}\right)^2, \quad c = \frac{1}{2} \sqrt{g_0 h_0}, \quad T_{\max} = T_c m_0 g_0.$$

# Parameters

Parameter	Symbol	Value
Initial altitude	$h_0$	1
Initial velocity	$v_0$	0
Initial mass	$m_0$	1
Gravitational constant	$g_0$	1
Thrust coefficient	$T_c$	3.5
Drag coefficient	$D_c$	$\frac{1}{2} v_c \frac{m_0}{g_0}$
Characteristic altitude	$h_c$	500
Characteristic velocity	$v_c$	620
Characteristic mass ratio	$m_c$	0.6
Final mass	$m_f$	$m_c m_0$