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An Elementary Chapter in Number Theory: Proof of Fermat's Last Theorem

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Abstract. ***Pierre de Fermat first stated around 1637 that for any integer n > 2, the equation an + bn = cn has no positive integer solutions, and he said the theorem in the margin of a copy of Arithmetica. His proof is available only for the equation $a^4 + b^4 = c^4$ for the exponent n = 4. Subsequently, Euler proved the theorem in the equation $a^3 + b^3 = c^3$ for the exponent n = 3. Taking the above two proofs of Fermat and Euler, it would suffice to prove the theorem for n = p, where p is any prime > 3. In this proof, we hypothesize all r, s and t as positive integers satisfying the equation rp + sp = tp and establish a contradiction. We use another auxiliary equation, $x^3 + y^3 = z^3$, and combine the two equations using transformation equations. Solving the transformation equations, we establish a contradiction, thereby proving the theorem.***

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I. INTRODUCTION

Pierre-de-Fermat, a French mathematician around 1637, wrote in the margin of a copy of Arithmetica that it is impossible to find positive integers A, B and C satisfying the equation $A^n + B^n = C^n$, where n is an integer greater than 2. He stated that he himself had found a marvellous proof for the equation, but the margin was too narrow to contain it. His proof for the theorem is available only for $n = 4$, using the infinite descent method. Subsequently, Euler proved the theorem for n = 3 [1].

Dirichlet, Legendre, and Lame proved the theorem for the exponents $n = 5$ and $n = 7$. Around 1820, Sophie Germain proved the theorem for some specific cases. Kummer proved the theorem for regular primes. He invented ideal number theory, and number theory advanced significantly into newer areas. Mathematicians observed a close connection between Fermat's Last Theorem and Elliptic Curves [2]. After 358 years, in 1995, Prof. Andrew Wiles proved the theorem completely [3]. Many mathematicians and number theorists have contributed to and analysed the theorem [4]. In this proof, we are trying for an alternative elementary proof of Fermat's Last Theorem.

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II. ASSUMPTIONS

A. We hypothesize that r, s and t are positive integers satisfying the equation $r^p + s^p = t^p$
Here, p is any prime >3. Clearly, gcd (r, s, t) = 1, and we establish a centre contradiction in this proof.

B. We include the auxiliary equation $x^3 + y^3 = z^3$ in this proof, in which we can have both x and y to be positive integers; z^3 will be a positive integer; both z and z^2 irrational (as proved already by Euler and others) $gcd\,(x, y, z^3) = 1$ and $\sqrt{rt}$ will be irrational. Since both x and z^3 cannot simultaneously be squares.

C. Let $F = (Ryz^3rs)$, where $R = y$.

D. We can have x, y and z^3 such that each has some other odd prime factors coprime to r, s and t.

Proof. By random trials, we have created the following equations.

$$\left(a\sqrt{t^p} + b\sqrt{F^{1/3}}\right)^2 + \left(c\sqrt{x} + d\sqrt{R^{1/3}}\right)^2 = \left(e\sqrt{R^{2/3}} + f\sqrt{rst}\right)^2$$

and

$$\left(a\sqrt{z^3} - b\sqrt{s^p}\right)^2 + \left(c\sqrt{F^{2/3}} - d\sqrt{E^{2/3}}\right)^2 = \left(e\sqrt{yr^p} - f\sqrt{E^{1/3}}\right)^2 \quad (1)$$

is the transformation equations of $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$, respectively, through the parameters called a, b, c, d, e and f. Here $F = (Ryz^3rs)$.

From equation (1), we get

$$a\sqrt{t^p} + b\sqrt{F^{1/3}} = \sqrt{x^3} \quad (2)$$

$$a\sqrt{z^3} - b\sqrt{s^p} = \sqrt{r^p} \quad (3)$$

$$c\sqrt{x} + d\sqrt{R^{1/3}} = \sqrt{y^3} \quad (4)$$

$$c\sqrt{F^{2/3}} - d\sqrt{F^{2/3}} = \sqrt{s^p} \quad (5)$$

$$e\sqrt{R^{2/3}} + f\sqrt{rst} = \sqrt{z^3} \quad (6)$$

and

$$e\sqrt{yr^p} - f\sqrt{F^{1/3}} = \sqrt{t^p} \quad (7)$$

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

$$a = \left(\sqrt{x^3s^p} + \sqrt{F^{1/3}r^p}\right)/\left(\sqrt{s^pt^p} + \sqrt{z^3F^{1/3}}\right)$$

$$b = \left(\sqrt{x^3z^3} - \sqrt{r^pt^p}\right)/\left(\sqrt{s^pt^p} + \sqrt{F^{1/3}z^3}\right)$$

$$c = \left(\sqrt{F^{2/3}y^3} + \sqrt{R^{1/3}s^p}\right)/\left(\sqrt{F^{2/3}x} + \sqrt{F^{2/3}R^{1/3}}\right)$$

$$d = \left(\sqrt{F^{2/3}y^3} - \sqrt{xs^p}\right)/\left(\sqrt{F^{2/3}x} + \sqrt{F^{2/3}R^{1/3}}\right)$$

$$e = \left(\sqrt{F^{1/3}z^3} + \sqrt{rst^{p+1}}\right)/\left(\sqrt{F^{1/3}R^{2/3}} + \sqrt{yr^{p+1}st}\right)$$

And

$$f = \left(\sqrt{yz^3r^p} - \sqrt{R^{2/3}t^p}\right)/\left(\sqrt{F^{1/3}R^{2/3}} + \sqrt{yr^{p+1}st}\right)$$

From (2) & (7), we get

$$\sqrt{t^p} \times \sqrt{t^p} = \left(\sqrt{x^3} - b\sqrt{F^{1/3}}\right)\left(e\sqrt{yr^p} - f\sqrt{F^{1/3}}\right)/(a)$$

e., $\quad t^p = \left\{(e)\sqrt{x^3yr^p} - (f)\sqrt{F^{1/3}x^3} - (be)\sqrt{F^{1/3}yr^p} + (bf)\left(F^{1/3}\right)\right\}/(a)$

From (3) & (7), we have

$$\sqrt{r^p} \times \sqrt{r^p} = \left(a\sqrt{z^3} - b\sqrt{s^p}\right)\left(\sqrt{t^p} + f\sqrt{F^{1/3}}\right)/\left(e\sqrt{y}\right)$$

i.e., $\quad r^p = \left\{(a)\sqrt{z^3t^p} + (af)\sqrt{F^{1/3}z^3} - (b)\sqrt{s^pt^p} - (bf)\sqrt{F^{1/3}s^p}\right\}/\left(e\sqrt{y}\right)$

From (3) & (5), we get

$$\sqrt{s^p} \times \sqrt{s^p} = \left(a\sqrt{z^3} - \sqrt{r^p}\right)\left(c\sqrt{F^{2/3}} - d\sqrt{F^{2/3}}\right)/(b)$$

i.e., $\quad s^p = \left\{(ac)\sqrt{F^{2/3}z^3} - (ad)\sqrt{F^{2/3}z^3} - (c)\sqrt{F^{2/3}r^p} + (d)\sqrt{F^{2/3}r^p}\right\}/(b)$

Substituting the above equivalent values of t^p, r^p and s^p in Fermat's equation, $r^p + s^p = t^p$, after multiplying both sides by $\left\{abe\sqrt{y}\right\}$, we get

$$\{be\}\sqrt{y}\left\{(e)\sqrt{x^3yr^p} - (f)\sqrt{F^{1/3}x^3} - (be)\sqrt{F^{1/3}yr^p} + (bf)\left(F^{1/3}\right)\right\}$$
$$= (ab)\left\{(a)\sqrt{t^pz^3} + (af)\sqrt{F^{1/3}z^3} - (b)\sqrt{s^pt^p} - (bf)\sqrt{F^{1/3}s^p}\right\}$$
$$+ (ae)\sqrt{y}\left\{(ac)\sqrt{F^{2/3}z^3} - (ad)\sqrt{F^{2/3}z^3} - (c)\sqrt{F^{2/3}r^p} + (d)\sqrt{F^{2/3}r^p}\right\} \quad (8)$$

Our aim is to compute all rational terms in equation (8) after multiplying both sides by

$$\left\{\left(\sqrt{s^pt^p} + \sqrt{F^{1/3}z^3}\right)^3\left(\sqrt{F^{2/3}x} + \sqrt{F^{2/3}R^{1/3}}\right)\left(\sqrt{F^{1/3}R^{2/3}} + \sqrt{yr^{p+1}st}\right)^2\right\}$$

To be free from denominators on the parameters *a*, *b*, *c*, *d*, *e* and *f* and again multiplying both sides by $\sqrt{t}$ for getting some rational terms.

I term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{be^2\}$

$$= \left(y\sqrt{x^3r^p}\right)\left\{(s^pt^p) + \left(F^{1/3}z^3\right) + 2\sqrt{s^pt^p}\sqrt{F^{1/3}z^3}\right\}\left(\sqrt{F^{2/3}x} + \sqrt{F^{2/3}R^{1/3}}\right)$$
$$\times \sqrt{t}\left(\sqrt{x^3z^3} - \sqrt{r^pt^p}\right)\left\{\left(F^{1/3}z^3\right) + (rst^{p+1}) + 2\sqrt{F^{1/3}z^3rst^{p+1}}\right\}$$

On multiplying by

$$\left\{\left(y\sqrt{x^3r^p}\right)\left(F^{1/3}z^3\right)\left(F^{1/3}\sqrt{x}\right)\sqrt{t}\left(-\sqrt{r^pt^p}\right)\left(F^{1/3}z^3\right)\right\}$$

We get

$$\left\{-\left(Fx^2yz^6r^p\right)\sqrt{t^{p+1}}\right\}$$

II term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{b(ef)\}$

$$= \left(-\sqrt{F^{1/3}x^3y}\right)\left\{(s^pt^p) + \left(F^{1/3}z^3\right) + \left(2\sqrt{s^pt^p}\sqrt{F^{1/3}z^3}\right)\right\}\left(\sqrt{F^{2/3}x} + \sqrt{F^{2/3}R^{1/3}}\right)$$
$$\times \sqrt{t}\left(\sqrt{x^3z^3} - \sqrt{r^pt^p}\right)\left(\sqrt{F^{1/3}z^3} + \sqrt{rst^{p+1}}\right)\left(\sqrt{yz^3r^p} - \sqrt{R^{2/3}t^p}\right)$$

(i) On multiplying by

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$$\left\{\left(-\sqrt{F^{1/3}x^3y}\right)\left(z^3\sqrt{F^{2/3}}\right)\sqrt{F^{2/3}x}\sqrt{t}\left(-\sqrt{r^pt^p}\right)\sqrt{F^{1/3}z^3}\sqrt{yz^3r^p}\right\}$$

We get

$$\left\{(Fx^2yz^6r^p)\sqrt{t^{p+1}}\right\}$$

(This term algebraically gets cancelled with the I term in LHS above)

(ii) Also, on multiplying by

$$\left\{\left(-\sqrt{F^{1/3}x^3y}\right)(s^pt^p)\sqrt{F^{2/3}R^{1/3}}\sqrt{t}\sqrt{x^3z^3}\sqrt{rst^{p+1}}\left(-\sqrt{R^{2/3}t^p}\right)\right\}$$

We get

$$\left\{(x^3s^pt^{2p+1})\sqrt{FRyz^3rs}\right\}$$

which is rational, since $F = (Ryz^3rs)$

(iii) Again, on multiplying by

$$\left\{\left(-\sqrt{F^{1/3}x^3y}\right)\left(z^3\sqrt{F^{2/3}}\right)\sqrt{F^{2/3}R^{1/3}}\sqrt{t}\sqrt{x^3z^3}\sqrt{F^{1/3}z^3}\left(-\sqrt{R^{2/3}t^p}\right)\right\}$$

We get

$$\left\{(Fx^3z^6)\sqrt{t^{p+1}}\sqrt{Ry}\right\}$$

which is rational, since $R = y$.

III term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{b^2e^2\}$

$$= \left(-y\sqrt{F^{1/3}r^p}\right)\left(\sqrt{s^pt^p}+\sqrt{F^{1/3}z^3}\right)\left(\sqrt{F^{2/3}x}+\sqrt{F^{2/3}R^{1/3}}\right)\sqrt{t}$$
$$\times\left\{(x^3z^3)+(r^pt^p)-2\sqrt{x^3z^3r^pt^p}\right\}\left\{(F^{1/3}z^3)+(rst^{p+1})+2\sqrt{F^{1/3}z^3rst^{p+1}}\right\}$$

(i) On multiplying by

$$\left(-y\sqrt{F^{1/3}r^p}\right)\sqrt{s^pt^p}\sqrt{F^{2/3}x}\sqrt{t}(x^3z^3+r^pt^p)(rst^{p+1})$$

We get

$$\left\{-(yrst^{p+1})\sqrt{t^{p+1}}(x^3z^3+r^pt^p)\sqrt{Fxr^ps^p}\right\}$$

Which will be irrational, since $\sqrt{Fxr^ps^p} = \sqrt{Rxyz^3}\sqrt{(rs)^{p+1}}$, which will be irrational, since $R = y$ and $\sqrt{xz^3}$ will be irrational.

(ii) Also, on multiplying by

$$\left\{\left(-y\sqrt{F^{1/3}r^p}\right)\sqrt{F^{1/3}z^3}\sqrt{F^{2/3}x}\sqrt{t}\left(-2\sqrt{x^3z^3r^pt^p}\right)\left(z^3\sqrt{F^{2/3}}\right)\right\}$$

We get

$$\left\{(2Fx^2yz^6r^p)\sqrt{t^{p+1}}\right\}$$

(This term algebraically gets cancelled with the IV term in the LHS below)

IV term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{b^2(ef)\}$

$$= \left(F^{1/3}\sqrt{y}\right)\left(\sqrt{s^pt^p}+\sqrt{F^{1/3}z^3}\right)\left(\sqrt{F^{2/3}x}+\sqrt{F^{2/3}R^{1/3}}\right)\sqrt{t}$$
$$\times\left\{(x^3z^3)+(r^pt^p)-2\sqrt{x^3z^3r^pt^p}\right\}\left(\sqrt{F^{1/3}z^3}+\sqrt{rst^{p+1}}\right)\left(\sqrt{yz^3r^p}-\sqrt{R^{2/3}t^p}\right)$$

(i) On multiplying by

$$\left\{\left(F^{1/3}\sqrt{y}\right)\sqrt{F^{1/3}z^3}\sqrt{F^{2/3}x}\sqrt{t}\left(-2\sqrt{x^3z^3r^pt^p}\right)\sqrt{F^{1/3}z^3}\sqrt{yz^3r^p}\right\}$$

We get

$$\left\{-(2Fx^2yz^6r^p)\sqrt{t^{p+1}}\right\}$$

(This term algebraically gets cancelled with the III term in LHS worked out above)

(ii) Also, on multiplying by

$$\left\{\left(F^{1/3}\sqrt{y}\right)\sqrt{F^{1/3}z^3}\sqrt{F^{2/3}R^{1/3}}\sqrt{t}\left(-2\sqrt{x^3z^3r^pt^p}\right)\sqrt{F^{1/3}z^3}\sqrt{R^{2/3}t^p}\right\}$$

We get

$$\left\{-(2Fz^3t^p)\sqrt{Rx^3yz^3r^pt}\right\}$$

Which will be irrational, since $R = y$ if r and t are coprimes to x and z^3.

(iii) Again, on multiplying by

$$\left\{\left(F^{1/3}\sqrt{y}\right)\sqrt{F^{1/3}z^3}\sqrt{F^{2/3}R^{1/3}}\sqrt{t}(x^3z^3+r^pt^p)\sqrt{F^{1/3}z^3}\left(-\sqrt{R^{2/3}t^p}\right)\right\}$$

We get

$$\left\{-\left(Fz^3\sqrt{Ry}\right)\sqrt{t^{p+1}}(x^3z^3+r^pt^p)\right\}$$

Which will be rational, since we have defined $R = y$.

I term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2b\}$

$$=\sqrt{z^3t^p}\left(\sqrt{F^{2/3}x}+\sqrt{F^{2/3}R^{1/3}}\right)\left\{\left(F^{1/3}R^{2/3}\right)+(yr^{p+1}st)+2\sqrt{F^{1/3}R^{2/3}}\sqrt{yr^{p+1}st}\right\}\sqrt{t}$$
$$\times\left((x^3s^p)+\left(F^{1/3}r^p\right)+2\sqrt{F^{1/3}x^3r^ps^p}\right)\left(\sqrt{x^3z^3}-\sqrt{r^pt^p}\right)$$

On multiplying by

$$\left\{\sqrt{z^3t^p}\sqrt{F^{2/3}R^{1/3}}\left(2\sqrt{F^{1/3}R^{2/3}}\sqrt{yr^{p+1}st}\right)\sqrt{t}(x^3s^p)\left(-\sqrt{r^pt^p}\right)\right\}$$

we get

$$\left\{-(2x^3r^ps^pt^{p+1})\sqrt{FRyz^3rs}\right\}$$

Which is rational, since we have defined $F = (Ryz^3rs)$

II term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(a^2b)f\}$

$$=\sqrt{F^{1/3}z^3}\left(\sqrt{F^{2/3}x}+\sqrt{F^{2/3}R^{1/3}}\right)\left(\sqrt{F^{1/3}R^{2/3}}+\sqrt{yr^{p+1}st}\right)\sqrt{t}$$
$$\times\left((x^3s^p)+\left(F^{1/3}r^p\right)+2\sqrt{F^{1/3}x^3r^ps^p}\right)\left(\sqrt{x^3z^3}-\sqrt{r^pt^p}\right)\left(\sqrt{yz^3r^p}-\sqrt{R^{2/3}t^p}\right)$$

(i) On multiplying by

$$\left\{\sqrt{F^{1/3}z^3}\sqrt{F^{2/3}R^{1/3}}\sqrt{yr^{p+1}st}\sqrt{t}(x^3s^p)\left(-\sqrt{r^pt^p}\right)\left(-\sqrt{R^{2/3}t^p}\right)\right\}$$

we get

$$\left\{(x^3r^ps^pt^{p+1})\sqrt{FRyz^3rs}\right\}$$

Which is rational.

(ii) Also, on multiplying by

$$\left\{\sqrt{F^{1/3}z^3}\sqrt{F^{2/3}R^{1/3}}\sqrt{F^{1/3}R^{2/3}}\sqrt{t}\left(r^p\sqrt{F^{2/3}}\right)\left(-\sqrt{r^pt^p}\right)\sqrt{yz^3r^p}\right\}$$

we get

$$\left\{-(Fz^3r^{2p})\sqrt{t^{p+1}}\sqrt{Ry}\right\}$$

Which is rational, since $R = y$.

III term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{ab^2\}$

$$=\left(-\sqrt{s^pt^p}\right)\left(\sqrt{F^{2/3}x}+\sqrt{F^{2/3}R^{1/3}}\right)\left\{\left(F^{1/3}R^{2/3}\right)+(yr^{p+1}st)+2\sqrt{F^{1/3}R^{2/3}}\sqrt{yr^{p+1}st}\right\}$$
$$\times\sqrt{t}\left(\sqrt{x^3s^p}+\sqrt{F^{1/3}r^p}\right)\left\{\left(x^3z^3\right)+\left(r^pt^p\right)-2\sqrt{x^3z^3r^pt^p}\right\}$$

On multiplying by

$$\left\{\left(-\sqrt{s^pt^p}\right)\sqrt{F^{2/3}R^{1/3}}\left(2\sqrt{F^{1/3}R^{2/3}}\sqrt{yr^{p+1}st}\right)\sqrt{t}\sqrt{x^3s^p}\left(-2\sqrt{x^3z^3r^pt^p}\right)\right\}$$

we get

$$\left\{(4x^3r^ps^pt^{p+1})\sqrt{FRyz^3rs}\right\}$$

Which is rational.

IV term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab^2)f\}$

$$=\left(-\sqrt{F^{1/3}s^p}\right)\left(\sqrt{F^{2/3}x}+\sqrt{F^{2/3}R^{1/3}}\right)\left(\sqrt{F^{1/3}R^{2/3}}+\sqrt{yr^{p+1}st}\right)\sqrt{t}$$
$$\times\left(\sqrt{x^3s^p}+\sqrt{F^{1/3}r^p}\right)\left\{(x^3z^3)+(r^pt^p)-2\sqrt{x^3z^3r^pt^p}\right\}\left(\sqrt{yz^3r^p}-\sqrt{R^{2/3}t^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{F^{1/3}s^p}\right)\sqrt{F^{2/3}R^{1/3}}\sqrt{yr^{p+1}st}\sqrt{t}\sqrt{x^3s^p}\left(-2\sqrt{x^3z^3r^pt^p}\right)\left(-\sqrt{R^{2/3}t^p}\right)\right\}$$

we get

$$\left\{-(2x^3r^ps^pt^{p+1})\sqrt{FRyz^3rs}\right\}$$

Which is rational.

V term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2ce\}$

$$= \sqrt{F^{2/3}yz^3}\left(\sqrt{s^pt^p}+\sqrt{F^{1/3}z^3}\right)\left(\sqrt{F^{1/3}R^{2/3}}+\sqrt{yr^{p+1}st}\right)\sqrt{t}$$

$$\times\left\{(x^3s^p)+\left(F^{1/3}r^p\right)+2\sqrt{F^{1/3}x^3r^ps^p}\right\}\left(\sqrt{F^{2/3}y^3}+\sqrt{R^{1/3}s^p}\right)\left(\sqrt{F^{1/3}z^3}+\sqrt{rst^{p+1}}\right)$$

(i) On multiplying by

$$\left\{\sqrt{F^{2/3}yz^3}\sqrt{s^pt^p}\sqrt{F^{1/3}R^{2/3}}\sqrt{t}(x^3s^p)\sqrt{R^{1/3}s^p}\sqrt{rst^{p+1}}\right\}$$

we get

$$\left\{(x^3s^{2p}t^{p+1})\sqrt{FRyz^3rs}\right\}$$

Which is rational.

(ii) Also, on multiplying by

$$\left\{\sqrt{F^{2/3}yz^3}\sqrt{s^pt^p}\sqrt{F^{1/3}R^{2/3}}\sqrt{t}\left(r^p\sqrt{F^{2/3}}\right)\sqrt{R^{1/3}s^p}\sqrt{F^{1/3}z^3}\right\}$$

We get

$$\left\{(Fz^3r^ps^p)\sqrt{t^{p+1}}\sqrt{Ry}\right\}$$

Which will be rational.

VI term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2de\}$

$$= \left(-\sqrt{F^{2/3}yz^3}\right)\left(\sqrt{s^pt^p}+\sqrt{F^{1/3}z^3}\right)\left(\sqrt{F^{1/3}R^{2/3}}+\sqrt{yr^{p+1}st}\right)\sqrt{t}$$

$$\times\left\{(x^3s^p)+\left(F^{1/3}r^p\right)+2\sqrt{F^{1/3}x^3r^ps^p}\right\}\left(\sqrt{F^{2/3}y^3}-\sqrt{xs^p}\right)\left(\sqrt{F^{1/3}z^3}+\sqrt{rst^{p+1}}\right)$$

(i) On multiplying by

$$\left\{\left(-\sqrt{F^{2/3}yz^3}\right)\sqrt{s^pt^p}\sqrt{yr^{p+1}st}\sqrt{t}(x^3s^p)\left(-\sqrt{xs^p}\right)\sqrt{F^{1/3}z^3}\right\}$$

we get

$$\left\{(x^3yz^3s^{2p})\sqrt{(rt)^{p+1}}\sqrt{Fxst}\right\}$$

(ii) Also, on multiplying by

$$\left\{\left(-\sqrt{F^{2/3}yz^3}\right)\sqrt{F^{1/3}z^3}\sqrt{yr^{p+1}st}\sqrt{t}(x^3s^p)\left(-\sqrt{xs^p}\right)\sqrt{rst^{p+1}}\right\}$$

we get

$$\left\{(x^3yz^3s^pt)\sqrt{(rst)^{p+1}}\sqrt{Fxrs}\right\}$$

The above two terms will be irrational, since $F = (Ryz^3rs)$ and $R = y$ and $\sqrt{xz^3}$ will irrational.

VII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{ace\}$

$$= \left(-\sqrt{F^{2/3}yr^p}\right)\left\{(s^pt^p)+\left(F^{1/3}z^3\right)+2\sqrt{F^{1/3}z^3s^pt^p}\right\}\left(\sqrt{F^{1/3}R^{2/3}}+\sqrt{yr^{p+1}st}\right)\sqrt{t}$$

$$\times\left(\sqrt{x^3s^p}+\sqrt{F^{1/3}r^p}\right)\left(\sqrt{F^{2/3}y^3}+\sqrt{R^{1/3}s^p}\right)\left(\sqrt{F^{1/3}z^3}+\sqrt{rst^{p+1}}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{F^{2/3}yr^p}\right)(s^pt^p)\sqrt{F^{1/3}R^{2/3}}\sqrt{t}\sqrt{x^3s^p}\sqrt{R^{1/3}s^p}\sqrt{rst^{p+1}}\right\}$$

we get

$$\left\{-\left(\sqrt{FRx^3yst}\right)(s^{2p}t^p)\sqrt{(rt)^{p+1}}\right\}$$

Which will be irrational, since $F = (Ryz^3rs)$, $R = y$ and $\sqrt{x^3z^3rt}$ will be irrational

VIII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{ade\}$

$$= \sqrt{F^{2/3}yr^p}\left\{(s^pt^p) + (F^{1/3}z^3) + 2\sqrt{F^{1/3}z^3s^pt^p}\right\}\left(\sqrt{F^{1/3}R^{2/3}} + \sqrt{yr^{p+1}st}\right)\sqrt{t}$$
$$\times\left(\sqrt{x^3s^p} + \sqrt{F^{1/3}r^p}\right)\left(\sqrt{F^{2/3}y^3} - \sqrt{xs^p}\right)\left(\sqrt{F^{1/3}z^3} + \sqrt{rst^{p+1}}\right)$$

(i) On multiplying by

$$\left\{\sqrt{F^{2/3}yr^p}\left(2\sqrt{F^{1/3}z^3s^pt^p}\right)\sqrt{yr^{p+1}st}\sqrt{t}\sqrt{x^3s^p}\sqrt{F^{2/3}y^3}\sqrt{F^{1/3}z^3}\right\}$$

we get

$$\left\{(2Fy^2z^3)\sqrt{(rst)^{p+1}}\sqrt{x^3yr^ps^pt}\right\}$$

This will be irrational if x and y are coprime to r, s, and *t*.

(ii) Also, on multiplying by

$$\left\{\sqrt{F^{2/3}yr^p(s^pt^p)}\sqrt{yr^{p+1}st}\sqrt{t}\sqrt{F^{1/3}r^p}\left(-\sqrt{xs^p}\right)\sqrt{rst^{p+1}}\right\}$$

we get

$$\left\{-(yr^ps^pt^{p+1})\sqrt{(rst)^{p+1}}\sqrt{Fxrs}\right\}$$

Which will be irrational, since we have defined $F = (Ryz^3rs)$ and

$$\sqrt{Fxrs} = \sqrt{(Ryz^3rs)(xrs)} = \left\{\sqrt{Ry}(rs)\sqrt{xz^3}\right\}$$

Where $R = y$; $\sqrt{xz^3}$ will be irrational since $gcd(x, z^3) = 1$ and both x and z^3 cannot simultaneously be squares.

Sum of all rational terms in the LHS of equation (8)

$$= \left\{(x^3s^pt^{2p+1})\sqrt{FRyz^3rs}\right\} \text{ (vide II term)}$$
$$+\left\{(Fx^3z^6)\sqrt{t^{p+1}}\sqrt{Ry}\right\}$$
$$-\left\{\left(Fz^3\sqrt{Ry}\right)\sqrt{t^{p+1}}(x^3z^3 + r^pt^p)\right\} \text{ (vide IV term)}$$
$$= \left\{(x^3s^pt^{2p+1})\sqrt{FRyz^3rs}\right\}$$
$$-\left\{(Fz^3r^pt^p)\sqrt{t^{p+1}}\sqrt{Ry}\right\}$$

Sum of all rational terms in the RHS of equation (8)

$$= \left\{(x^3r^ps^pt^{p+1})\sqrt{FRyz^3rs}\right\} \text{ (Adding I to IV terms)}$$
$$+\left\{(x^3s^{2p}t^{p+1})\sqrt{FRyz^3rs}\right\} \text{ (vide V term)}$$
$$+\left\{(Fz^3r^ps^p)\sqrt{t^{p+1}}\sqrt{Ry}\right\}$$
$$-\left\{(Fz^3r^{2p})\sqrt{t^{p+1}}\sqrt{Ry}\right\} \text{ (vide II term)}$$
$$= \left\{(x^3s^pt^{2p+1})\sqrt{FRyz^3s}\right\} \quad (\because \; r^p + s^p = t^p)$$
$$+\left\{(Fz^3r^p)\sqrt{t^{p+1}}\sqrt{Ry}\right\} \; (s^p - r^p)$$

Equating the rational term on both sides, we get

$$\left\{\left(Fz^3r^p\sqrt{Ry}\sqrt{t^{p+1}}\right)(t^p + s^p - r^p)\right\} = 0$$
$$(2Fz^3r^ps^p)\sqrt{Ry}\sqrt{t^{p+1}} = 0$$

That is, either $r = 0$ or $s = 0$ or $t = 0$. $(\because \; F = Ryz^3rs)$

This contradicts our hypothesis that all *r*, *s* and *t* are non-zero integers in the equation $r^p + s^p = t^p$ and proves that only a trivial solution exists.

III. CONCLUSIONS

Equation (8) was derived from the two transformation equations by substituting the equivalent values of r^p, s^p & t^p in Fermat's equation $r^p + s^p = t^p$. The central hypothesis we made in the proof, namely that r, s, and t are non-zero integers, has been shattered by the result r = 0; thus, we are proving the theorem.

DECLARATION STATEMENT

Some of the references cited are older, noted explicitly as [1], [2], [3] and [4]. However, these works remain significant for the current study, as they are pioneering in their fields.

I must verify the accuracy of the following information as the article's author.

- **Conflicts of Interest/ Competing Interests:** Based on my understanding, this article has no conflicts of interest.
- **Funding Support:** This article has not been funded by any organizations or agencies. This independence ensures that the research is conducted with objectivity and without any external influence.
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- **Author's Contributions:** The authorship of this article is contributed solely by the author.

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