Possibility of Void Black Holes in the Sobolev–Ozok Lattice Framework

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Abstract

Within the Sobolev–Ozok Lattice (SOL) framework, we propose the potential existence of *Void Black Holes* (VBHs). Unlike classical black holes formed through matter collapse, VBHs emerge from extreme decoherence collapse, where the coherence density of spacetime falls below a critical threshold. The resulting structure features a de Sitter–like decoherence core, a coherence-pressure shell, and a characteristic coherence-induced redshift that reproduces the photometric signatures, without requiring massive accretion or dust obscuration. This paper explores the theoretical basis, metric formulation, and observational implications of VBHs within the SOL model, proposing that they may populate cosmic voids and influence large-scale coherence dynamics.

1 Introduction

The Sobolev–Ozok Lattice (SOL) model offers an interesting and exciting perspective on the possibility of a new cosmic structure. In SOL, spacetime consists of discrete Planck-scale coherence cells [1] [2] [3], where gravitational and quantum behaviors emerge from the collective dynamics of coherence and decoherence. We propose that in low-coherence regions of the cosmic web, particularly within primordial voids, the local coherence field may undergo collapse, creating a *Void Black Hole* (VBH). These objects mimic the photometric appearance of black holes [4] [5] but differ fundamentally in origin, structure, and gravitational signature.

2 Theoretical Framework

We adopt the SOL energy spectrum and k=2 curvature scaling:

$$\varepsilon(\ell) = \left(\frac{\ell_p}{\ell}\right)^3, \qquad K_2(r) = \beta\,\frac{\varepsilon(r)}{r^2} = \beta\,\frac{\ell_p^3}{r^5}, \tag{1}$$

and define the coherence pressure [6]

$$P_{\mathrm{coh}}(r) = \alpha^2\,\frac{c^2}{G}\,K_2(r) = \alpha^2\,\frac{c^2}{G}\,\beta\,\frac{\ell_p^3}{r^5}. \tag{2}$$

Near the core/boundary we use the potential as described in the Black Holes from the Sobolev–Ozok Lattice (SOL):Derivation, Galaxy Occupation Thresholds and Quasi-Black-Hole Signatures paper [7]:

$$u(r) = -\frac{A}{r}, \qquad T_c(r) \equiv \beta\,\frac{d^2u}{dr^2} = -\,2\beta\,\frac{A}{r^3}, \tag{3}$$

3 Thresholds: BH/QBH vs. VBH

In the SMBH / QBH case, a saturation threshold of k = 2 $T_{\rm crit} > 0$ yields the standard SOL horizon radius:

$$r_H = \Big(\frac{2A}{T_{\rm crit}}\Big)^{1/3}, \qquad A \sim \frac{GM}{c^2} \quad \text{(weak–field match) [8].} \tag{4}$$

A **Void Black Hole (VBH)** forms when drive is too small to maintain the inner gradient, that is,

$$T_c(r) \;\le\; T_{\rm void}\ (<0), \qquad \Rightarrow \qquad r_V = \Big(\frac{2A}{|T_{\rm void}|}\Big)^{1/3}. \tag{5}$$

Thus, VBHs are the under-driven dual of Eq. (4): the shell sits where the second derivative reaches the (negative) void threshold; inside, $d^2u/dr^2 \to 0$, and the solution relaxes toward a low–curvature (de Sitter–like) core. Hence, the VBH is a 'bubble of negative coherence curvature' [9] bounded by a shell of residual positive curvature. Using the isothermal–core scaling,

$$\rho(r) \simeq \frac{\sigma^2}{2\pi G\, r^2}, \qquad P_\star(r) \sim \frac{\sigma^4}{2\pi G\, r^2}, \tag{6}$$

The occupation of saturation k = 2 (SMBH / QBH) occurs when $r_{\rm inf} = GM_\bullet/\sigma^2 \gtrsim r_c$, implying minimal dispersion $\sigma_{\rm min}(r_c)$ Conversely, a VBH is favored when the drive of the bulge is below that threshold, $\sigma \lesssim \sigma_{\rm min}(r_c)$, so the interior cannot hold saturation and a void shell r_V forms.

4 Possible Coherence Redshift from the VBH Shell

A line of sight integral of the tension k = 2 across the thin shell gives rise to a possible red compactness due to the coherence redshift. Consistent with Eqs. (1)–(3), we parameterize the frequency shift as

$$1 + z_{\rm coh}(r_{\rm em}) = 1 + \chi\int_{r_{\rm em}}^{\infty} |T_c(r)|\, dr \;\;=\;\; 1 + 2\chi\beta A\int_{r_{\rm em}}^{\infty}\frac{dr}{r^3} \;\;=\;\; 1 + \chi\beta A\,\frac{1}{r_{\rm em}^2}, \tag{7}$$

where χ is a dimensionless coupling controlling how the tension k = 2 is mapped to the photon energy loss. Equation (7) shows why emission from just inside the shell appears extremely red, while the core (with $T_c \to 0$) contributes little. [10]

5 Universe as a Global Void Black Hole

[11] [12]. The same logic covered above can be extended cosmologically: if a local black hole in the Sobolev–Ozok Lattice (SOL) forms when the second–order coherence tension exceeds the critical threshold $T_{\rm crit}$, then a cosmological configuration in which the same tension remains below any saturation value must correspond to a large-scale VBH.

5.1 Global Coherence Balance

In the SOL curvature–pressure relation,

$$T_c(r) = \beta \frac{d^2u}{dr^2}, \qquad K_2(r) = \beta \frac{\ell_p^3}{r^5}, \qquad P_{\rm coh}(r) = \alpha^2 \frac{c^2}{G} K_2(r), \tag{8}$$

the coherence tension decays as r^{-3}, whereas the large–scale matter–radiation drive behaves as $P_\star \sim \sigma^4/(2\pi G r^2)$. At galactic scales, these curves intersect to produce $T_c = T_{\rm crit}$ (SMBH) or $T_c \approx T_{\rm void}$ (VBH). On cosmological scales the mean tension is far smaller,

$$T_c^{\rm U}(r) \;\le\; T_{\rm void}, \tag{9}$$

so the Universe as a whole resides in the under–driven regime: a coherence field that never reaches $k = 2$ saturation.

5.2 Cosmic Coherence Shell

The boundary where the residual tension equals the void threshold defines a *cosmic coherence shell*,

$$r_V^{\rm U} = \left(\frac{2A_{\rm U}}{|T_{\rm void}|}\right)^{1/3}, \qquad A_{\rm U} \sim \frac{GM_{\rm U}}{c^2}, \tag{10}$$

analogous to Eq. (4) for local black holes but with the total mass–energy of the Universe $M_{\rm U}$. Identifying $r_V^{\rm U}$ with the Hubble radius $R_H = c/H_0$ gives a quantitative link between cosmic expansion and the SOL coherence balance:

$$|T_{\rm void}| \;\approx\; \frac{2A_{\rm U}}{R_H^3} = \frac{2GM_{\rm U}}{c^2 R_H^3}. \text{ [13] [14]} \tag{11}$$

Hence, the cosmological horizon can be reinterpreted as the outer coherence shell of a global VBH.

5.3 Metric Form and Effective Lapse

Keeping the leak form [7], the cosmic lapse becomes

$$f_{\rm U}(r) = 1 - \varepsilon_0 \left(\frac{r_V^{\rm U}}{r}\right)^m, \qquad m > 0,\ 0 < \varepsilon_0 \ll 1, \tag{12}$$

with $r_s \to 0$ (no central mass spike) and deformation dominated by the shell itself. This produces a nearly flat interior ($f_{\rm U} \simeq 1$ for $r < R_H$) and a weak curvature barrier at the boundary, consistent with the observed low spatial curvature of the Universe.

5.4 Observable Consequences

- **Accelerated expansion:** the outward relaxation of an under–driven coherence field manifests itself as a cosmic acceleration, replacing the need for a separate "dark energy" term as it can be reinterpreted as the decoherence pressure.
- **Weak global lensing:** the shell–dominated potential yields negligible large–scale convergence, consistent with the extremely small mean shear of the cosmic microwave background.
- **Cosmic redshift as coherence–redshift:** integrating Eq. (7) across the global shell gives

  $$1 + z_{\rm coh}^{\rm U} \;\simeq\; 1 + \chi\beta A_{\rm U}/R_H^2, \tag{13}$$

  linking the observed Hubble law directly to coherence gradients.
- **Apparent flatness:** inside $r < R_H$ the curvature term $K_2 \propto r^{-5}$ is nearly null, producing an effective $\Omega_k \approx 0$ without fine–tuning.

5.5 Section Summary

In this view, the Universe can be the ultimate VBH: a global coherence domain bounded by its own decoherence shell at the Hubble horizon. Local SMBHs and QBHs are nested microanalogues within the same coherence spectrum. The cosmological constant and dark energy effects emerge from the boundary term T_{void}, while the apparent acceleration of cosmic expansion is the macroscopic expression of the same under saturated coherence dynamics that, at smaller scales, produce VBH shells around galaxies.

6 Multiverse Interpretation: Hierarchical VBH Domains in the SOL Framework

The possibility that the Universe itself is a global Void Black Hole (VBH) naturally opens the door to hierarchical cosmology in which many such coherence domains coexist within the Sobolev–Ozok lattice. Because the SOL framework is scale-free and recursive [15], the formation of a VBH at any level implies that analogous coherence shells can emerge at larger or smaller scales, each defining a self–contained "universe" with its own internal metric, time, and coherence tension.

6.1 Fractal Hierarchy of Coherence Domains

In the SOL model, the coherence tension obeys the same law $T_c(r) = \beta\, d^2u/dr^2$ regardless of the scale. Whenever the tension locally falls below the void threshold, $T_c \leq T_{\text{void}}$, a new coherence shell forms. This recursive rule implies that the cosmos may be populated by a fractal spectrum of VBHs where each level preserves the same cubic threshold scaling $r_V = (2A/|T_{\text{void}}|)^{1/3}$, but with A and T_{void} corresponding to its own local coherence energy and environment.

6.2 Birth of New Universes

When a coherence island reaches complete decoherence collapse, the sign of its internal tension can be reversible, creating a new domain that satisfies the same VBH boundary condition on the opposite side of the decoherence interface. In this sense, black holes in one universe act as the nucleation points of new VBH universes in the wider SOL lattice. Each emergent domain inherits its own space, time and coherence field, while the global coherence energy remains conserved through the SOL resolution balance [16] [17].

6.3 Energy Conservation Across Domains

Because the SOL formalism conserves total coherence energy,

$$E_{\text{total}} = \sum_i \int_{\Omega_i} P_{\text{coh}}^{(i)}\, dV_i - \sum_j \int_{\Omega_j} P_{\text{coh}}^{(j)}\, dV_j = 0, \tag{14}$$

The appearance of a new VBH universe does not add energy to existence; rather, it redistributes coherence between the coherent and decoherent sections of the lattice. The overall Sobolev norm of the ensemble remains invariant.

6.4 Physical and Philosophical Implications

- Each VBH domain defines its own space-time and horizon, so the "multiverse" becomes a hierarchy of coherence domains rather than a spatial ensemble of disjoint regions.
- Cosmic acceleration, dark energy, and redshift are internal manifestations of under–driven coherence, not external parameters.

- The emergence of universes is continuous and scale-recursive: the SOL lattice perpetually resolves new coherence shells whenever the local tension reaches T_{void}.
- The "Big Bang" thus can be replaced by ongoing resolution, in which each VBH domain begins as a coherence void bubble within the larger lattice continuum with the interplay of coherence and decoherence zones.

6.5 Section Summary

The multiverse, within the SOL interpretation, is not a collection of parallel worlds, but a fractal hierarchy of coherence islands, each governed by the same k=2 tension law and threshold logic. Our own Universe is one such VBH domain, nested within a continuum that may itself be part of a higher order coherence structure.

7 Symmetry of Coherence Tension: From Gravitation to Expansion

A key insight emerging from the Sobolev–Ozok Lattice (SOL) framework is that the sign of the coherence tension T_c determines the direction of spacetime curvature and thus unifies two seemingly opposite cosmic phenomena: gravitation and expansion.

7.1 The Dual Regimes of Coherence Tension

Within SOL, the coherence tension is given by

$$T_c(r) = \beta \frac{d^2u}{dr^2}, \qquad u(r) = -\frac{A}{r}, \tag{15}$$

so that

$$T_c(r) = 2\beta \frac{A}{r^3}. \text{ [18] [19]} \tag{16}$$

The sign of T_c encodes whether the coherence curvature is inward (compressive) or outward (relaxing):

Regime	Sign of T_c	Physical Interpretation
$T_c > T_{\text{crit}} > 0$	Inward curvature	Over driven coherence: gravitation / black holes
$T_c = 0$	Flat curvature	Equilibrium: Minkowski or balanced coherence
$T_c < T_{\text{void}} < 0$	Outward curvature	Under driven coherence: expansion / VBH

Thus, the same second–order law governs both attractive and repulsive curvature, depending solely on the local balance between coherence density and its radial gradient.

7.2 Gravitation as Coherence Compression

When $T_c > 0$, the coherence gradients are directed inward: the lattice cells align and contract. This produces metric compression, a reduction in the local lapse function $f(r)$, and the appearance of gravitational attraction. At the critical limit $T_c = T_{\text{crit}}$, the curvature saturates and a quasi–black hole horizon forms.

7.3 Expansion as Coherence Relaxation

When $T_c < 0$, the coherence gradients reverse direction. The lattice ceases to compress and begins to relax; coherence density decreases outward, and the resulting geometry becomes quasi-flat or slightly convex. The metric deformation takes the form

$$f_{\mathrm{VBH}}(r) = 1 - \varepsilon_0 \left(\frac{r_V}{r}\right)^m, \tag{17}$$

yielding negligible curvature and a weak, outward–directed acceleration. This is the defining signature of the VBH regime.

7.4 Unified Interpretation

Gravitational collapse and cosmic expansion are therefore not distinct physical processes, but opposite polarities of the same coherence law. The symmetry

$$T_c \longleftrightarrow -T_c \tag{18}$$

maps inward coherence (gravity) to outward decoherence (expansion). The equilibrium point $T_c = 0$ represents the self–organized balance observed as a large-scale spatial flatness. At the cosmological scale, the Universe resides in the $T_c < 0$ regime—a global coherence relaxation identified as a Void Black Hole (VBH) configuration.

7.5 Physical Consequences

- **No need for dark energy:** cosmic acceleration is the macroscopic expression of $T_c < 0$.
- **Unified curvature law:** black holes and cosmic expansion are governed by the same T_c dynamics.
- **Flatness problem resolved:** the global coherence oscillates around $T_c = 0$, enforcing a near–zero curvature.
- **Fractal symmetry:** local contraction and global expansion are mirror phases in the SOL hierarchy.

7.6 Summary

The SOL framework thus reveals a deep symmetry: gravity and expansion are two aspects of the same coherence tension field. Positive T_c compresses spacetime; negative T_c relaxes it. The transition through $T_c = 0$ defines the balance of the lattice, where the universe appears flat and self-regulated. This duality unites general relativity and cosmology within a single coherence principle.

8 Conclusion and Outlook

The Sobolev–Ozok Lattice (SOL) framework provides a unified language linking quantum coherence, gravitation, and cosmology within a single hierarchy of scale–invariant laws. In this paper, we have shown that the same coherence–tension dynamics that generate classical and quasi–black holes ($T_c \geq T_{\mathrm{crit}}$) also admits the dual, under driven solution ($T_c \leq T_{\mathrm{void}}$), yielding the *Void Black Hole* (VBH) regime. This condition naturally leads to thin coherence shells, de Sitter–like interiors, and strong coherence redshift without requiring dust, accretion, or extreme mass.

At the largest scale, the Universe itself satisfies the VBH criterion: its global coherence tension lies below the saturation threshold k = 2, and the Hubble radius acts as the outer coherence shell.

This interpretation redefines cosmic expansion and the cosmological constant as manifestations of under-driven coherence dynamics, eliminating the need for a separate dark-energy term. The Universe therefore behaves as a global VBH bounded by its own coherence horizon. [20] [21]

Because the SOL equations are scale-free and recursive, VBH formation can occur at every structural level of the lattice. Each decoherence collapse that inverts the sign of its internal tension creates a new coherence domain obeying the same threshold law, producing a hierarchical multiverse, in which universes emerge as self–contained coherence islands. Energy conservation is maintained through the global Sobolev norm: the creation of new domains redistributes coherence rather than generating it. In this view, the multiverse is not an ensemble of parallel worlds but a fractal continuum of coherence domains. extending from the Planck scale to the cosmic horizon. Every scale, particle, black hole, galaxies, and universe obeys the same second-order tension law.

$$T_c(r) = \beta \frac{d^2u}{dr^2}, \qquad r_{\text{threshold}} \propto A^{1/3},$$

with only the boundary conditions changing. This self-similarity implies that the principles governing Planck scale dynamics and cosmic expansion are manifestations of the same underlying coherence mechanism.

Future work will focus on quantitative calibration of the void threshold T_{void}, the coupling constant χ that governs the coherence redshift, and numerical simulation of nested VBH domains to compare directly with the observations of JWST and CMB observations. If validated, the SOL VBH interpretation could provide a coherent, energy-conserving explanation for the origin, structure, and evolution of the entire multiverse.

A Numerical Illustration: Estimating the Void Threshold T_{void}

To validate the cosmological scale of the Void Black Hole (VBH) threshold within the Sobolev–Ozok Lattice (SOL) framework, we evaluate the coherence tension limit defined by Eq. (25):

$$|T_{\text{void}}| = \frac{2\,G\,M_{\text{U}}}{c^2\,R_H^3}, \tag{19}$$

where M_{U} is the effective mass–energy content of the observable universe and $R_H = c/H_0$ is the Hubble radius.

Simplified formulation. Using the critical density $\rho_c = 3H_0^2/(8\pi G)$ and the relation $M_{\text{U}} = (4\pi/3)\rho_c R_H^3$, Eq. (19) elegantly simplifies

$$|T_{\text{void}}| = \frac{H_0^2}{c^2}. \tag{20}$$

Numerical evaluation. With

$$H_0 = 67.4\ \text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1} \;\Rightarrow\; H_0 = 2.1843 \times 10^{-18}\,\text{s}^{-1}, \qquad c = 2.998 \times 10^8\,\text{m}\,\text{s}^{-1},$$

we find

$$|T_{\text{void}}| = \frac{(2.1843 \times 10^{-18})^2}{(2.998 \times 10^8)^2} = 5.0 \times 10^{-53}\,\text{m}^{-2}.$$

Interpretation. This curvature magnitude is strikingly close to the observed cosmological constant scale $\Lambda \approx 1.1 \times 10^{-52}\,\text{m}^{-2}$, differing only by a factor of order unity. The result indicates that the VBH curvature threshold naturally coincides with the Hubble expansion scale:

$$|T_{\text{void}}| \approx \frac{H_0^2}{c^2} \approx \frac{\Lambda}{2}. \text{ [22]}$$

Thus, the Universe itself satisfies the condition $T_c = T_{\text{void}}$, behaving as a global Void Black Hole whose coherence shell lies at the Hubble radius $R_H = c/H_0$.

Robustness. Because Eq. (20) depends only on H_0, its sensitivity is minimal: a 1% change in H_0 produces a 2% variation in T_{void}. The correspondence between T_{void} and Λ is therefore stable under current cosmological measurement uncertainty.

B Discussion and Physical Implications of the Void Threshold

The numerical result of Appendix A, $|T_{\text{void}}| = H_0^2/c^2 \simeq 5 \times 10^{-53}\,\text{m}^{-2}$, reveals a remarkable correspondence between the SOL coherence tension and the cosmological constant. In conventional general relativity, the vacuum curvature is expressed as $\Lambda = 3\,\Omega_\Lambda H_0^2/c^2$, with $\Omega_\Lambda \approx 0.7$, resulting in $\Lambda \simeq 1.1 \times 10^{-52}\,\text{m}^{-2}$. Thus,

$$\Lambda \;\approx\; 2\,|T_{\text{void}}|,$$

implying that the cosmological constant is not an independent parameter but a direct manifestation of the global coherence–decoherence balance in the lattice.

B.1 Physical Interpretation

Within the SOL framework, the sign of T_c determines whether spacetime contracts or relaxes. At the cosmological scale, the Universe resides at the limit $T_c = T_{\text{void}} < 0$, signifying an under–driven coherence regime. This outward curvature corresponds to the observed cosmic acceleration. Hence, the cosmological constant Λ acquires a concrete physical meaning: It represents the residual decoherence tension of the universal lattice at its equilibrium boundary.

B.2 Hierarchy of Coherence Curvatures

Because the coherence tension derives from a single law, $T_c = \beta\, u''(r)$, the same formulation governs all curvature scales. At the Planck scale, $|T_{\text{Planck}}| \sim 1/\ell_P^2 \simeq 10^{70}\,\text{m}^{-2}$, while at the cosmic scale, $|T_{\text{void}}| \sim 10^{-52}\,\text{m}^{-2}$. The ratio

$$\frac{|T_{\text{Planck}}|}{|T_{\text{void}}|} \simeq \left(\frac{c/H_0}{\ell_P}\right)^2 \simeq 10^{122},$$

coincides with the long–standing vacuum energy discrepancy between quantum field theory and cosmology. In SOL, this ratio emerges naturally as coherence scale hierarchy, linking microscopic curvature saturation to macroscopic curvature relaxation.

B.3 Cosmic Equilibrium

This result suggests that the Universe presently resides at the decoherence threshold, the point at which the inward coherence curvature ($T_c > 0$) and outward relaxation curvature ($T_c < 0$) nearly cancel. This state represents the sensitive balance of global coherence, where spacetime expands slowly instead of collapsing or inflating. In this interpretation, the Universe itself can be a Void Black Hole (VBH), maintaining cosmic expansion through a stable residual coherence tension rather than an external dark-energy term.

B.4 Implications

- The cosmological constant Λ is reinterpreted as twice the magnitude of the global coherence tension, $\Lambda = 2|T_{\text{void}}|$.

- The 122-order difference between vacuum and cosmological energy densities arises from the ratio of Planck scale to cosmic-scale curvature domains.
- The Universe occupies the transitional regime $T_c \approx 0$, representing large-scale spatial flatness as a dynamic equilibrium of coherence and decoherence.

Overall, the numerical and conceptual alignment of $|T_{\text{void}}|$, Λ, and H_0 confirms that the SOL coherence framework naturally reproduces the observed acceleration of the universe without introducing dark energy.

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Not applicable.

Consent to participate

Not applicable.

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Not applicable.

Availability of data and materials

All data, derivations, and figures are included in the manuscript; code and supplementary materials are available upon request.

Author contributions

Ozcan Ozok is the sole author and is responsible for all aspects of the work presented.

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