FRC 566.001 — Entropy–Coherence Reciprocity and UCC (Scientific Draft)

October 2025

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1. Introduction

We formalize the FRC 566 reciprocity between entropy S and coherence C and the associated flow equation (UCC). The goal is to provide unit–consistent definitions, thermodynamic projections, and reproducible validations without any extra metaphors or layer taxonomies.

2. Definitions and Units

Entropy S is measured in nats (information layers) or J/K (thermodynamic layers). Coherence C is a dimensionless scalar gauge. We adopt two conventions for the coherence constant k_*:

- information/cognition layers: $k_* = 1$ (nats),
- thermo/physical layers: $k_* = k_B$ (Boltzmann constant).

Regularization for numerical work uses $C_\varepsilon = 1/(Z^2 + \varepsilon)$ with small $\varepsilon > 0$.

3. Reciprocity Law

The core relationship is

$$dS + k_* \, d \ln C = 0, \qquad \Rightarrow \qquad S + k_* \ln C = \text{const.} \tag{1}$$

In information form, with a distribution p, define $C[p] = \exp[-H(p)/k_*]$ where H is Shannon entropy (nats). Then KL divergence and mutual information yield coherence ratios and coupling penalties:

$$\mathrm{RER}(p \to q) = \frac{C[q]}{C[p]} = \exp\big[- D_{\mathrm{KL}}(p \| q)/k_* \big], \tag{2}$$

$$I(X;Y) = D_{\mathrm{KL}}(p_{XY} \| p_X p_Y) \quad \Rightarrow \quad C_{XY} = \exp[-I/k_*]. \tag{3}$$

Thermodynamic projection (isothermal) gives the unit–consistent free–energy relation

$$\Delta G = - k_* T \, \Delta \ln C. \tag{4}$$

4. Universal Coherence Condition (UCC)

We define the local flow form

$$\partial_t \ln C = -\nabla \cdot J_C + S_C, \qquad J_C = -D_C \, \nabla \ln C, \tag{5}$$

with $D_C > 0$. An energy–like dissipation follows under suitable boundary conditions (Neumann/Dirichlet):

$$\sigma(t) \equiv k_* D_C \int \|\nabla \ln C\|^2 \, dV \;\geq\; 0. \tag{6}$$

Linear well–posedness holds for the diffusion–reaction form; nonlocal kernels (fractional Laplacian class) can be admitted if the operator remains dissipative.

5. Worked Examples (Reproducible)

(A) Chemical toy (isothermal). Small dataset fit of ΔG vs $\Delta \ln C$ validates slope $-k_B T$.
(B) Stochastic field (OU + coherence drift). Add drift $\propto \nabla \ln C$; demonstrate stationary $\ln C$ profile and nonnegative $\sigma(t)$.
(C) Learning system (classifier). Coherence–regularized training: $\mathcal{L} = \mathrm{CE} + \lambda H(Z)/k_*$ where $H(Z)$ is Gaussian entropy of features; empirically fit $\Delta S \approx -k_* \Delta \ln C$ and report calibration/OOD deltas.

6. Validation

We compute slopes and R^2 for ΔS vs $-k_* \Delta \ln C$ across examples; verify UCC dissipation; include KS tests against nulls for coherence concentration.

7. Limits and Identifiability

Discuss gauge freedom in C, calibration of k_*, and sensitivity to regularization ε, grid, and boundary conditions.

8. Reproducibility

Code lives under `code/566/` with a single command to regenerate all figures and pack artifacts. A GitHub Release triggers Zenodo archiving.