

Deduction of the Fine-Structure Constant and Quantum–Atomic–Relativistic Unification through the Constant $A = 0.86$

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Abstract. Modern physics faces a fundamental dichotomy between quantum randomness and the determinism of its laws. This work postulates that chance is intrinsically bounded and that the stability of a system emerges from the entanglement of the cumulative probability functions of two complementary events. This principle forms the basis of the Law of Probability Equilibrium (LPE). The LPE reveals that energy accumulation, represented by photon frequency, maps onto the domain of probability measured in cycles, thereby establishing a formal equivalence between the energy of a photon ($h\nu$) and the probabilistic structure of light. The model postulates a duality between probability and information, where the equilibrium of a system also reflects the amount of information required to maintain its stability. The model introduces the unification constant ($A \approx 0.86$), a value directly derived from the mathematical foundations of the theory and aligned with the relativistic velocity limit of the electron in Oganesson ($Z = 118$). It is demonstrated that the fine-structure constant (α) is the intersection between two universal limits: the probability accumulation limit (γ) and the material limit of the proton number ($Z = 118$). The manifestation of the LPE at the escape-probability limit of a black hole suggests a deep connection with General Relativity, where the limiting potential (γ) constrains geometry, and the constant A governs the informational stability of that boundary. Taken together, the LPE establishes a theoretical framework that unifies atomic, quantum, and relativistic physics.

Keywords: Probability Equilibrium Law (PEL); bounded randomness; Euler–Mascheroni convergence.

1. Introduction

Since the dawn of quantum physics, with foundational work on the quantization of energy [1], science has faced an inherent tension between the apparent randomness of nature and the determinism of its fundamental laws. This debate intensified with the development of quantum mechanics by [2], [3], and [4], who proposed that randomness is an intrinsic and irreducible feature of reality through the Copenhagen Interpretation. [5] introduced the probabilistic interpretation of the wave function, consolidating the role of chance within the quantum formalism. On the opposite side, [6] defended a predictable universe, and decades later, [7] demonstrated that no local hidden-variable theory could reproduce all the predictions of quantum mechanics, deepening the conflict between determinism and nonlocality.

Beyond philosophical debate, this dichotomy has persisted at the heart of physics, manifesting in the nature of universal constants. The fine-structure constant (α), in particular, has resisted any compelling theoretical justification—leading [8] to describe it as one of the greatest mysteries in physics. Its value, which governs the electromagnetic interaction, seems arbitrary, hinting at a deeper lack of understanding of the principles shaping our universe.

From a statistical perspective, [9] pioneered the connection between entropy and probability, laying the groundwork for a statistical interpretation of physical disorder. Later, [10] information theory established that uncertainty can be quantified, while [11] fractal geometry revealed underlying order within seemingly chaotic phenomena.

In this context, the present work seeks to explore whether, within the domain of probability, a hidden principle of certainty may exist within randomness [12], and whether it is possible to construct a unified model providing an explanatory framework for random phenomena in matter and energy, applicable at the atomic, quantum, and relativistic levels.

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2. Methodology

2.1 Law of Probability Equilibrium (LPE)

2.1.1 Statement. Let Ω be a sample space satisfying axioms [13], and let A and A' be two mutually exclusive events with strictly positive probabilities such that:

$$P(A) = 0 < p < 1$$

$$P(A') = 0 < 1 - p < 1$$

The LPE postulates that, instead of the cumulative distribution function (CDF) of a single event ($F_A(n)$) converging to unity, total certainty is distributed between two complementary events (A y A'). This distribution is achieved through an interlaced cumulative distribution function ($F_{A \oplus A'}(n)$), defined as the sum of the cumulative functions of each event:

$$F_{\{A \oplus A'\}(n)} = F_{A(n)} + F_{A'(n)} \quad (1)$$

This framework establishes a state of probabilistic equilibrium and entanglement in which each individual function converges to 1/2 at infinity, while their sum reaches unity, as required by the axioms of probability:

$$\lim_{n \rightarrow \infty} F_{A(n)} = \lim_{n \rightarrow \infty} F_{A'(n)} = 1/2 \Rightarrow F_{\{A \oplus A'\}(n)} = 1$$

2.1.2 Demonstration. Consider a sequence of cycles (discrete steps) in which, in each trial, event A occurs with probability p and event A' with probability $(1-p)$. Define the random variables representing the waiting times until the first occurrence:

- N : number of cycles required for A to occur.
- N' : number of cycles required for A' to occur.

The corresponding cumulative distribution functions (CDFs) for n cycles are:

$$\begin{aligned} F_{A(n)} &= P(N \leq n) = 1 - (1 - p)^n \\ F_{A'(n)} &= P(N' \leq n) = 1 - p^n \end{aligned} \quad n = 1, 2, \dots$$

The random system is in equilibrium if and only if the medians of the waiting times (median delays) for the occurrence of A and A' are equal ($m = m'$):

$$m = m' \Leftrightarrow p = 1/2$$

The median of N (number of trials until A occurs) satisfies:

$$1 - (1 - p)^m = \frac{1}{2}$$

Solving for m :

$$m = \frac{\ln\left(\frac{1}{2}\right)}{\ln(1 - p)}$$

Similarly, for N' :

$$1 - p^{m'} = 1/2$$

$$m' = \frac{\ln\left(\frac{1}{2}\right)}{\ln(p)}$$

At equilibrium, $m = m'$, so:

$$\ln(1 - p) = \ln(p) \Rightarrow 1 - p = p \Rightarrow 1 = 2p$$

$$p = 1/2$$

2.2 Interlaced Probability and Certainty of Occurrence

As a consequence, instead of the cumulative probability of a single event converging to unity, total certainty is evenly distributed between two complementary events (A and A'), provided their probabilities are strictly positive. Probabilistic entanglement ensures that the sum of their cumulative distribution functions always tends to unity ($F_A(n) + F_{A'}(n) = 1$) as $n \rightarrow \infty$. When one cumulative function increases (accumulative behavior), its complement necessarily decreases (de-accumulative behavior). This establishes an analogy with Uncertainty Principle [14] and Complementarity Principle [15]. Since both functions converge to the equilibrium value $1/2$ at infinity, the certainty of occurrence is shared between events, eliminating the possibility of an infinite number of failures for either event.

2.3 Validation of the LPE in Discrete Random Variables

The LPE is validated for discrete binomial variables. Define $X \sim B(n, h_i)$ and $Y \sim B(n, 1-h_i)$, where h_i is the relative frequency. The cumulative distribution functions for a large number of trials ($n \rightarrow \infty$) are:

$$F(X)_{(k)} = P(X \leq k) = \sum_{i=0}^k \binom{n}{i} \cdot (h_i)^i \cdot (1 - h_i)^{n-i}$$

$$F(Y)_{(n-k)} = P(Y \leq n - k) = \sum_{i=0}^{n-k} \binom{n}{i} \cdot (1 - h_i)^i \cdot (h_i)^{n-i}$$

The symmetry of complementary binomial distributions allows their probability functions to be equalized in the limit:

$$\lim_{n \rightarrow \infty} F(X)_{(k)} = \lim_{n \rightarrow \infty} F(Y)_{(n-k)}$$

The only solution satisfying probabilistic equilibrium is $h_i = 1 - h_i = 1/2$. The LPE confirms that the case of consecutive failures for X and successes for Y cannot persist indefinitely, demonstrating that both distributions converge to the equilibrium value of $1/2$ as $n \rightarrow \infty$.

2.4 Validation of the LPE in Continuous Variables

The validation of the LPE in the continuous domain is achieved through the Central Limit Theorem. When the number of trials is large, the relative frequency h_i converges to the theoretical probability (p), and the standardized normal variable Z_x is distributed as:

$$Z_{(x)} = \frac{h_i - p}{\sqrt{p(1-p)/n}} \sim N(0,1)$$

As $n \rightarrow \infty$, the variable Z_x tends to zero, which is its expected value. The cumulative probability of Z_x at this point is $1/2$ due to the symmetry of the normal distribution:

$$F(Z_{(x)} \leq 0) = \int_{-\infty}^0 f(z) dz = \frac{1}{2}$$

Similarly, for the complementary variable Z_Y :

$$Z_{(Y)} = \frac{(1 - h_i) - (1 - p)}{\sqrt{p(1-p)/n}} = \frac{p - h_i}{\sqrt{p(1-p)/n}} = -Z_{(x)}$$

Therefore, Z_Y is also standard normal, and by symmetry:

$$F(Z_{(Y)} \leq 0) = F(Z_{(x)} \geq 0) = \int_0^{\infty} f(z) dz = \frac{1}{2}$$

These equations confirm that the cumulative probability distributions for complementary events converge to $1/2$ at equilibrium, demonstrating the validity of the LPE in the continuous limit.

2.5 Probability Potential $|\psi_p|$

The probability potential $|\psi_p|$ is defined as the absolute difference between the cumulative probability functions of both events, $F_A(n)$ and $F_{A'}(n)$. Given the entanglement postulate $F_{A'}(n) = 1 - F_A(n)$, the potential in the fundamental cycle ($n = I$) is expressed as:

$$|\psi_p| = |F_{A(n)} - F_{A'(n)}| = |2 \cdot F_{A(n)} - 1|$$

For $n=I$:

$$|\psi_p| = |2p - 1|$$

2.5.1 Case $|\psi_p| = 0$. Symmetry Equilibrium Point

The probability potential is zero when the cumulative probability functions of the complementary events are equal, $F_A(n) = F_{A'}(n)$. Since, in the interlaced distribution framework, their sum is $F_A(n) + F_{A'}(n) = 1$, it follows that each must equal $1/2$.

The symmetry equilibrium point ($|\psi_p| = 0$) represents the central equilibrium of the system—its probabilistic symmetry—which occurs at the median delay of event A , where 50% of the success possibilities are accumulated.

2.5.2 Case $/\psi_p/ = 1/3$. Potential Equilibrium Point

The LPE postulates that the Potential Equilibrium Point occurs under the fundamental condition in which the probability potential equals the improbability of the complementary event:

$$|\psi_p| = F_{A'(n)}$$

Since $F_A(n) + F_{A'}(n) = 1$, this condition necessarily implies the following relationship between accumulated probabilities:

$$F_{A'}(n) = F_A(n) - F_{A'}(n) \implies F_A(n) = 2 \cdot F_{A'}(n)$$

Solving the system to maintain unity, the accumulated probabilities are:

$$F_{A'}(n) = \frac{1}{3} \implies F_A(n) = \frac{2}{3}$$

Therefore, the potential at this point is $/\psi_p/ = 1/3$. This point represents the average delay of event A, where 66.67% of the success possibilities are accumulated.

2.5.3 Case $/\psi_p/ = \gamma$. Theorem of Probabilistic Stability Limit (γ)

Convergence to the Euler–Mascheroni constant ($\gamma \approx 0.5772$) occurs at the limit where the system maintains equilibrium under the condition of maximum tolerable imbalance. This stability threshold emerges internally from the interlaced structure of the cumulative distribution functions (CDFs) of the LPE. Hence, there exists a unique critical probability value $p^* \in (0, 1)$, such that the difference between the cumulative sum of the Basic Potential $/\psi_p/^{*}(k)$ and the natural logarithm of the number of cycles $\ln(n)$ converges, in the asymptotic limit, to the Euler–Mascheroni constant (γ):

$$\gamma = \lim_{n \rightarrow \infty} \left(\sum_{k=1}^n |F_A k - F_{A'} k| - \ln n \right)$$

$$\gamma = \lim_{n \rightarrow \infty} \left(\sum_{k=1}^n (| (1 - p^*)^k - (p^*)^k | - \ln n) \right)$$

For the series $\sum / \psi_p /^{*}(k)$ to grow asymptotically like the harmonic series ($\sum 1/k$), the system's initial imbalance must match the structural imbalance defined by γ . This is established by the Condition of Maximum Stable Disequilibrium (MDE):

$$|\psi_p| = \gamma$$

$$|1 - 2p| = \gamma$$

Since p must be greater than $1/2$ for proper convergence, the solution for the critical probability is:

$$p^* = \frac{1+\gamma}{2} \approx 0.7886$$

Thus, convergence to γ arises as a structural consequence of the summed differences between the interlaced cumulative probability functions of the LPE, linking the inherent probability p^* to the constant γ itself. Total certainty (1) is conserved and distributed as a sum of powers of the two fundamental equilibrium states of the LPE ($1/2$ and $1/3$), connected through a universal exponent p :

$$1 = \left(\frac{1}{2}\right)^p + \left(\frac{1}{3}\right)^p$$

The numerical solution to this nonlinear equation is $p \approx 0.7886$. Substituting this critical value into the potential expression for the fundamental cycle ($n = 1$):

$$|\psi p|_{n=1} = |2p - 1|$$

gives the value that governs the maximum tolerable imbalance:

$$\lim_{n \rightarrow \infty} |\psi p| = |2(0.7886) - 1| = 0.5772 \approx \gamma$$

This value is numerically identical to the Euler–Mascheroni constant ($\gamma \approx 0.5772$), thus postulating γ as the stability threshold of the LPE (Figure 1).

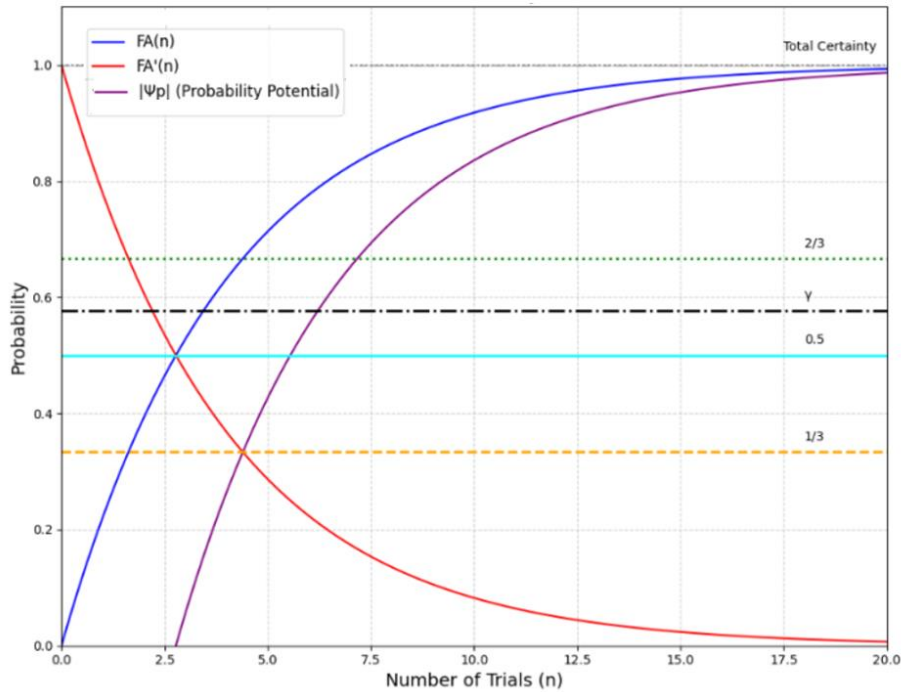


Figure 1. Entangled Probability Distribution Functions: Cumulative and Deaccumulative.

2.5.4 Potential Limit and the Euler–Mascheroni Identity

The previous derivation establishes a Limit Isomorphism that formally validates the numerical equivalence between the potential $|\psi p|$ and γ .

i) Potential as Accumulation Excess ($|\psi p|$): The potential measures the accumulated difference between the CDFs:

$$|\psi p(n)| = |F_A(n) - F_{A'}(n)| = |(1 - p)^n - p^n|$$

ii) Structural Equivalence ($p \approx 1/2$): The limiting value of this process is analogous to the classical definition of γ . The theorem's expression shows that the discrete accumulation excess between $F_A(n)$ and $F_A(n)$ (governed by p^*) reproduces the same limit structure (γ) as the harmonic–logarithmic difference:

$$\lim_{n \rightarrow \infty} |\psi p^*(n)|_{cumulative} - \ln(n) = \lim_{n \rightarrow \infty} (H_n - \ln(n)) = \gamma$$

The LPE interprets γ not as an analytical anomaly, but as an emergent structural property that quantifies the discrete imbalance excess of any complementary probabilistic system relative to continuous growth. The numerical convergence of the Power Equilibrium Axiom implies that the potential $|\psi_p|$ obeys this same universal limit law, since both processes express the quantification of the excess between discrete and continuous accumulation (Figure 2).

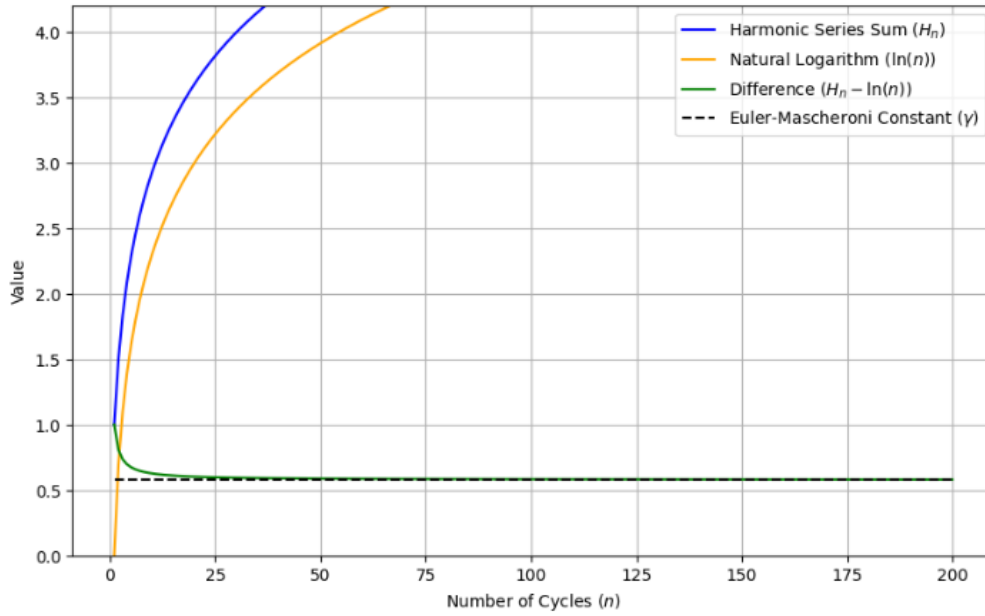


Figure 2. Convergence of the Probabilistic Potential Limit to γ .

2.5.5 Asymptotic Exponential Modeling of the Potential $|\psi_p|$

In the limit of a large number of trials (n), the potential $|\psi_p|$ can be modeled as an exponential form encapsulating the constant (γ). By defining $x = \gamma n$, the decay law is approximated as:

$$|\psi p| \approx e^{-x} \Rightarrow |\psi p| \approx e^{-\gamma n}$$

This expression indicates that $|\psi_p|$ is bounded by an exponential decay process, describing how the cumulative probability approaches stability.

3. Results and Discussion

3.1 Probabilistic Unification Constant $A \approx 0.86$

We begin with the system's critical condition, where $|\psi_p|$ reaches its limiting value:

$$|\psi_p| = \gamma$$

This potential can also be expressed as a cumulative exponential form:

$$|\psi_p| = 1 - e^{-p}$$

By equating both expressions, the relationship between the critical probability p and γ is obtained:

$$\gamma = 1 - e^{-p}$$

Solving for p gives:

$$e^{-p} = 1 - \gamma \Rightarrow p = -\ln(1 - \gamma)$$

The value obtained for p characterizes the exact point of maximum stable disequilibrium within the system. Under this critical condition, and using the $|\psi_p|$ equation, we can postulate a factor A , which is the mathematical manifestation of this fundamental principle:

$$A = -\ln(1 - \gamma)$$

$$A \approx -\ln(1 - 0.5772) \approx 0.8609$$

The LPE proposes that every dynamic system tends toward a probabilistic equilibrium point characterized by a critical probability $\gamma \approx 0.5772$. This constant A is not an axiom but a natural consequence of the random system's behavior around equilibrium. It represents the minimum amount of information (or “surprise”) associated with the non-equilibrium event, whose probability is $1 - \gamma \approx 0.423$.

In terms of Information Theory [10], this “surprise” is defined as:

$$I(x) = -\log(x)$$

where less probable events contain more information. Thus, A represents the self-information associated with the non-equilibrium state, revealing how much internal structure a system requires to remain stable at (γ) .

The LPE introduces a fundamental duality between probability and information, postulating that equilibrium is achieved through the proportional relationship between both. It is proposed that this law can be applied to any system governed by probabilistic dynamics, and that A is an inherent unifying constant of equilibrium throughout the universe.

In the results section, it will be shown that the unification factor derived from the LPE ($A \approx 0.8609$) and the Euler–Mascheroni constant ($\gamma \approx 0.5772$) manifest across the three physical levels (Table 1).

Table 1. Convergence of constants (γ) and A across different levels of Physics

Physical Level	Main Variable	Relation to LPE	Value Convergence
Probabilistic	n (Number of cycles)	$F_A = 1 - (1-p)^n$	The value of F_A passes through universal equilibrium points: 1/3, 1/2, 2/3, γ .
Quantum	N (Number of photons)	$N = e^{\gamma n}$	The process of photon accumulation (N) is governed by (γ).
Atomic	Z (Atomic number)	$Z\alpha$ (Coupling probability)	The coupling probability $Z\alpha$ converges to the constant A (≈ 0.86) in Oganesson ($Z=118$), indicating the limit of probabilistic stability.
Relativistic	r (Orbital radius)	L_γ (Gravitational limit $\propto 1/\gamma$)	The limiting $\hbar\psi_p/(\gamma)$ bound the black hole geometry, and A (non-equilibrium information) determines the threshold for this boundary.

3.2 Probabilistic Cycle at the Quantum Level

3.2.1 Unified Principle Between Probabilistic Equilibrium and the Fundamental Cycle

The Euler–Mascheroni constant (γ) is introduced here as an empirical scaling factor that statistically approximates the photon emission per cycle, without implying a direct physical causation:

- Principle LPE: Following the asymptotic exponential modeling of $\hbar\psi_p/$ (Section 2.5.5.), the number of photons (N) per cycle (n) follows an exponential process characterized by the Euler–Mascheroni constant γ :

$$N = e^{\gamma n} \quad (2)$$

- Principle of the Distribution Constant (δ): It states that the universal value δ remains constant for the distribution of a photon and is defined as the product of the Minimum Action Probability (p_h)—the dimensionless probability defining the unit of action in the fundamental cycle—and the distance traveled by light in one second (d_c).

The circular distance $d_c = 3 \times 10^8$ m, corresponding to one full cycle, defines a reference wavelength, with reference frequency $\nu = 1$ Hz and unitary power. This means that in one second, that distance is covered in one cycle where the number of photons (N) that emerge is $N = 1.509 \times 10^{33}$. Therefore, the probability of each photon is $p = 1/1.509 \times 10^{33}$, and the number of photons in the reference distance is:

$$N = \frac{1}{p_h}$$

Here, p_h the probability of photon emergence in the normalized reference cycle, $p_h \approx 6.626 \times 10^{-34}$, derived from the unitary reference cycle as the probability of photon emergence, dimensionless but numerically coinciding with Planck’s constant. The expected value $E(X)$ of the cycle length associated with the emergence of a photon in that cycle is:

$$E(X) = n \cdot p = d_c \cdot p_h$$

where:

d_c : total length corresponding to each photon in one cycle ($n = 1$) (m)

p_h : probability of successful photon emission

From the product above, the delta constant (δ) is obtained numerically as:

$$\delta = p_h \cdot d_c = 6.626 \times 10^{-34} \cdot 3 \times 10^8 \text{ m} \approx 1.986 \times 10^{-25} \text{ m}$$

This constant value represents the fraction of a meter corresponding to each of the 1.509×10^{33} photons emitted along the distance traveled by light in one second. The energy is normalized in the [0–1] space and is equivalent to probability. Thus, energy accumulation is analogous to probability accumulation (F_A).

3.2.2 Quantum Physics Relationship

Starting from the cumulative probability function:

$$F_{A(n)} = 1 - (1 - p)^n$$

If p is defined as the probability of photon emission per cycle, it can be expressed in two equivalent ways.

$$p = \frac{d_c \cdot p_h}{\lambda} = \frac{1}{N}$$

Then the cumulative probability function becomes:

$$F_{A(n)} = 1 - \left(1 - \frac{d_c \cdot p_h}{\lambda}\right)^n = 1 - \left(1 - \frac{1}{N}\right)^{AN}$$

As an example, for unitary power and red light with $\lambda = 7.00 \times 10^{-7}$ and $N = 3.52 \times 10^{18}$:

$$F_{A(n)} = 1 - \left(1 - \frac{1}{3.52 \times 10^{18}}\right)^{(0.8609) \cdot (3.52 \times 10^{18})} \approx 0.5772$$

For $N \rightarrow \infty$:

$$F_{A(n)} = 1 - (e)^{-0.8609} \approx 1 - 0.4228 \approx \gamma$$

It is verified that the maximum accumulated probability is γ . The number of photons emitted per second with unitary power is defined as the ratio between the wavelength (λ) in meters and the expected delta (δ) in meters:

$$N = \frac{\lambda}{\delta} \quad (3)$$

By equating the two expressions for N (LPE and Quantum Physics), the connection between both domains is established:

$$e^{\gamma n} = N = \frac{\lambda}{\delta} \quad (4)$$

Rearranging gives the expression that links the distribution constant (δ) with the wavelength (λ) and the probabilistic equilibrium (γ):

$$\delta = \frac{\lambda}{e^{\gamma n}}$$

This expression reveals that δ is not an independent value but the direct result of a cyclic probabilistic process governed by the Euler–Mascheroni constant and a fundamental number of cycles (n). Thus, for any wavelength λ , the total number of photons it contains can be directly calculated as $N = \lambda/\delta$. For red light ($\lambda = 7.00 \times 10^{-7}$ m), the number of photons for unitary power is:

$$N = \frac{7.00 \times 10^{-7} \text{ m}}{1.986 \times 10^{-25} \text{ m}} \approx 3.52 \times 10^{18} \text{ photons/s}$$

By the traditional method:

$$E = h\nu \Rightarrow N = \frac{1}{E} = \frac{1}{2.84 \times 10^{-19}} \approx 3.52 \times 10^{18} \text{ photons/s}$$

This relationship shows that the quantization of energy ($E = h\nu$) corresponds to the equilibrium value of probabilistic accumulation governed by γ . It ensures that the emission probability is quantized and that energy and accumulated probability are conceptually and mathematically equivalent in the fundamental cycle.

3.2.3 Cycle Variation Across the Electromagnetic Spectrum

For the theory to hold, the number of cycles (n) must be shown to vary as a measurable property of electromagnetic waves. From the unified equation (4), the number of cycles can be derived. Applying the natural logarithm yields:

$$n = \frac{\ln(N)}{\gamma} = \frac{\ln(\frac{\lambda}{\delta})}{\gamma}$$

For different wave frequencies across the spectrum, the expression $n = 1/\gamma \cdot \ln(N)$ allows us to obtain the number of cycles required to emit one photon, while $N = e^{\gamma n}$ gives the photon count consistent with conventional physics (Table 2). The LPE model is fully consistent with the quantum energy formula $E = h\nu$. By combining the probabilistic equilibrium principle with physical laws, it is shown that energy results directly from the number of cycles (n) and the frequency (ν).

Table 2. Number of cycles (n) for different types of radiation, assuming 1 W of power.

Radiation	λ [m]	ν [Hz]	Photon Energy [J]	Photons [s^{-1}] λ/δ	Cycles $n = 1/\gamma \ln(N)$	N: photons/cycle $e^{\gamma n}$
Radio	1.00×10^3	3.00×10^5	1.99×10^{-28}	5.02×10^{27}	110.49	5.02×10^{27}
Red Light	7.00×10^{-7}	4.29×10^{14}	2.84×10^{-19}	3.52×10^{18}	73.97	3.52×10^{18}
Green Light	5.50×10^{-7}	5.45×10^{14}	3.61×10^{-19}	2.77×10^{18}	73.56	2.77×10^{18}
Blue Light	4.50×10^{-7}	6.67×10^{14}	4.41×10^{-19}	2.27×10^{18}	73.22	2.27×10^{18}
Gamma	1.00×10^{-12}	3.00×10^{20}	1.99×10^{-13}	5.02×10^{12}	50.67	5.02×10^{12}

The number of cycles (n) varies across the electromagnetic spectrum but remains consistent in adjacent segments, indicating that the LPE governs photon emission at all frequencies. This explains why (n) for visible light is nearly the same between red and blue: they are relatively close frequencies in the vast spectrum. The LPE thus describes a universal accumulation process that, while changing with wave energy, maintains local consistency.

The number of cycles (n) represents the number of discrete probabilistic steps required for a photon to emerge. The formula $n = 1/\gamma \cdot \ln(N)$ uses the Euler–Mascheroni constant (γ) to model the exponential growth of the photon count N . The continuous accumulation of probability in the cumulative distribution function leads to the success of the probabilistic event.

The LPE explains the wave–particle duality [16] as the manifestation of this principle. The wave represents the continuous accumulation process of energy. The particle (photon) represents the discrete success or “firing” event when the system reaches the equilibrium threshold. The LPE shows that the product between Planck’s probability (p_h) and the light distance (d_c) is not arbitrary but a direct consequence of the wave’s geometry (λ) and the probabilistic equilibrium principle (e^{γ}), which is dimensionless.

As an analogy, the Planck probability (p_h) represents the energy of the “reference wheel” of unitary frequency. Electromagnetic waves are other wheels that, having different sizes (λ), must complete more or fewer cycles to travel the same distance. A smaller wheel (blue light) spins faster, meaning a photon takes fewer cycles to emerge; conversely, a larger wheel (red light) spins more slowly and requires more cycles. This explains why red light compensates for its lower individual photon energy with a higher emission frequency, while blue light compensates for its lower emission rate with higher energy per event. Human vision in the visible spectrum occurs around 73 cycles, confirming the LPE’s predictive consistency at the quantum level.

3.3 Probabilistic Cycle at the Atomic Level

3.3.1 Deduction of the Fine-Structure Constant (α)

The central hypothesis postulates that the fine-structure constant (α), interpreted as the probability of electromagnetic coupling (p), emerges from the deep relationship between the probabilistic stability limit (γ) and the atomic limit number ($Z = 118$) of the element Oganesson (Og). This proposal addresses the long-standing enigma of α [8], [17] while pursuing conceptual economy without adjustable parameters [18].

Starting from the discrete form of the LPE (cumulative distribution function):

$$F_{A(n)} = 1 - (1 - p)^n$$

By assuming that the stability limit corresponds to the maximum accumulated probability ($F_A(Z) \approx \gamma$), and that the number of cycles (n) equals the atomic number limit ($Z = 118$), the expression becomes:

$$\gamma \approx 1 - (1 - \alpha)^Z$$

Since α is very small ($\alpha \ll 1$), the logarithmic approximation can be used:

$$F_{A(n)} = 1 - (e)^{-\alpha Z}$$

Solving for α in this exponential relation yields the expression linking the fine-structure constant α with γ [19] and Z [20]:

$$\alpha \approx \frac{-\ln(1-\gamma)}{Z} \quad (5)$$

This deduction aligns with the Unification Constant (A), defined as $A \approx -\ln(1-\gamma)$. Assuming that A represents the relativistic velocity limit of the electron in Og ($v/c \approx Z\alpha$), the unified relation can be written as:

$$Z\alpha \approx A \Rightarrow \alpha \approx \frac{A}{Z}$$

This dual formulation demonstrates that α arises from the balance between quantum probability (γ) and relativistic stability (Z). Substituting the model values ($A \approx 0.8609$ and $Z = 118$) gives by direct mathematical deduction:

$$\alpha \approx 0.00729$$

Which agrees with the experimental value [21]. This implies that the maximum velocity of electrons in matter is a direct manifestation of the amount of information (A) required for the universe to remain in its probabilistic equilibrium state. Thus α arises from the balance between quantum probability and relativistic stability [22]. The fleeting existence of the most stable isotope of Oganesson (Og-294), with a half-life near 0.89 milliseconds [23], corroborates its role as an empirical marker at the relativistic threshold ($v/c \approx A \approx 0.86$). As Z increases, the electron velocity approaches the speed of light, and the classical Bohr model ceases to be valid. In the framework of relativistic atomic physics, $v/c \approx Z\alpha$ describes the 1s electron velocity. This value places Og in an extreme coupling regime, near the limit where $v \rightarrow c$, marking an observable physical threshold of atomic stability.

3.3.2 Convergence Points in the Periodic Table

The theoretical predictions of the LPE are consistent with the physical properties and positions of key elements in the periodic table. Modeling the atomic number (Z) as the number of cycles (n), the cumulative distribution function ($F_A(Z)$) reveals that the stability of elements is governed by the universal constants of the LPE. The convergence points demonstrate how the equilibrium of atomic systems follows probabilistic constants. From the accumulated probability equation derived from the LPE, F_A is calculated for selected elements (Table 3):

$$F_A(Z) = 1 - (1 - \alpha)^Z$$

Table 3. Chemical Elements and Cumulative Probability by Atomic Number (Z).

Chemical Element	Z	$F_A(Z)$	Reference Points	$v/c = Z \cdot \alpha$
Hydrogen	1	0.00727	$\approx \alpha$	0.00729
Helium	2	0.01449		0.01454
Lithium	3	0.02165		0.02181
Beryllium	4	0.02876		0.02908
Boron	5	0.03582		0.03635
Xenón	54	0.32562		0.39253
Cesium	55	0.33053	$\approx \psi_p = 1/3$	0.39980
Barium	56	0.33539		0.40707
Plutonium	94	0.49631		0.68330
Americium	95	0.49997	$\approx \psi_p = 0$	0.69057
Curium	96	0.50361		0.69784
Tennessine	117	0.57412		0.85049
Oganesson	118	0.57722	$\approx \psi_p = \gamma$	0.85766 $\approx A$
...				
Theoretical	137	0.63194	$\approx \psi_p = 1 - 1/e$	0.99587

3.3.3 The Schrödinger Equation and the Coupling Potential

The LPE suggests a possible correspondence between probabilistic equilibrium and atomic interaction potentials. Within this framework, the Unification Constant (A) can be interpreted as a structural factor modulating system stability.

A Probabilistic Coupling Potential (V_C) is conceptually introduced, formally analogous to the Coulomb potential, and expressed as:

$$V_C = \frac{k' \cdot AZ}{r}$$

This formulation preserves the same radial dependence ($\propto 1/r$) and allows exploring an equivalence between the classical electromagnetic coupling and one governed by probabilistic equilibrium. The normalization constant $k' \approx 2.68 \times 10^{-28} \text{ J} \cdot \text{m}$ converts the dimensionless parameter A into the energy domain. Without altering the formal structure of the Schrödinger Equation, this approach aims to show that discrete energy levels may also be understood as manifestations of probabilistic equilibrium.

Hydrogen ($Z=1$): The cumulative probability F_A coincides with the fine-structure constant α , interpreted as the a priori probability of coupling, $p(A) \approx 0.00729$.

Cesium ($Z=55$). F_A approaches the equilibrium point of the potential, where the absolute potential value is $|\psi_p| \approx 1/3$. Cesium, known for its high reactivity, corresponds to a transition point in the probabilistic system.

Americium ($Z=95$): It approaches the point of symmetry equilibrium where $F_A \approx 1/2$ and $|\psi_p|=0$

Oganesson ($Z=118$): F_A reaches the probabilistic stability limit $|\psi_p| = \gamma \approx 0.5772$. Simultaneously, the relativistic velocity of the electron ($Z\alpha$) converges to the Unification Constant $A \approx 0.85766$ in the heaviest known element.

Theoretical Limit ($Z=137$): At this theoretical point, F_A approaches the relativistic limit where the cumulative probability tends to $1 - 1/e \approx 0.632$. The theoretical inner electron velocity would equal that of the photon ($Z\alpha = 1$), implying a fundamental transition in physical laws. The LPE establishes that the equilibrium of the random system occurs at $|\psi_p|=0$, where the probabilities of two complementary events are identical ($P(A) = P(A') = 1/2$).

The model postulates that the Unification Constant A (≈ 0.8609) represents a velocity threshold defining the onset of significant relativistic effects, linking system equilibrium to a universal constant. The convergence of two numerical values—arising from distinct fields of study—reveals a fundamental equilibrium constant. As demonstrated, the dimensionless equilibrium factor $A \approx 0.86$ emerges from mathematical foundations of the LPE, not from empirical fitting, yet coincides visibly with the relativistic velocity limit (Figure 3).

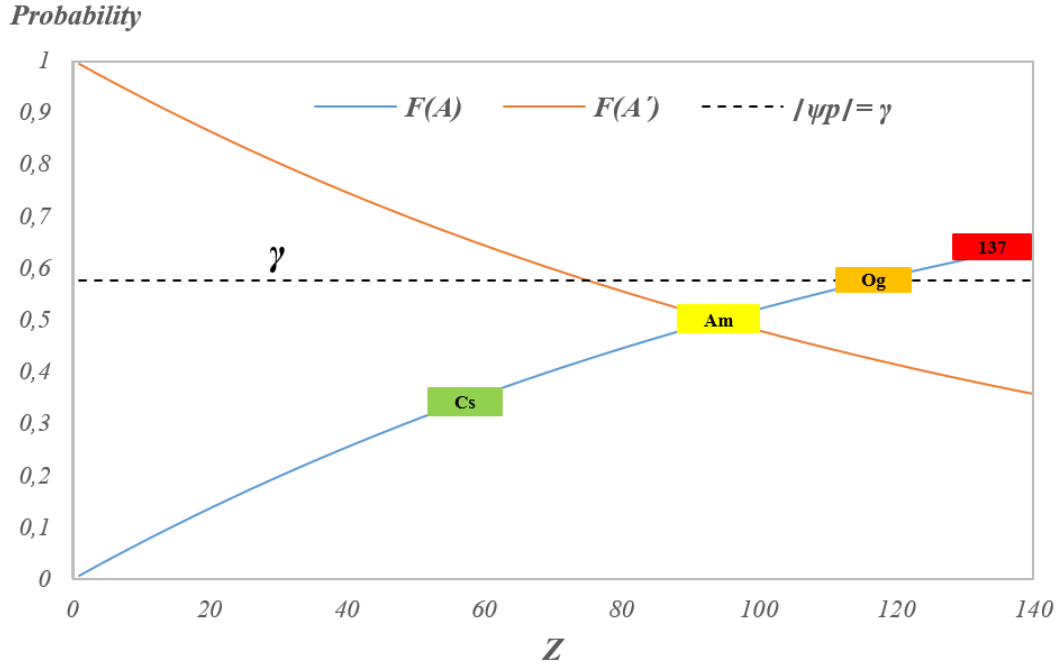


Figure 3. Cumulative Probability Distribution Functions of F_A and $F_{A'}$ by Atomic Number (Z).

3.3.4 Connection with the Lorentz Factor

The velocity corresponding to the threshold of space-time distortion is linked to the constant $A = v/c$. Since the Lorentz factor $\gamma_L \geq 1$, its inverse $(1/\gamma_L) \leq 1$ lies in the range $[0, 1]$, and can be interpreted as a probability.

Substituting the relation $v/c = Z\alpha$ derived from the LPE gives a new expression connecting the relativistic effects of each element with its position in the periodic table:

$$p = \frac{1}{\gamma_L} = \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - (Z\alpha)^2}$$

Substituting $v/c \approx A \approx 0.8609$ gives:

$$p = \sqrt{1 - (0.8609)^2} = \sqrt{1 - 0.7411} = \sqrt{0.2589} \approx 0.5088$$

This probability (p) converges to the median equilibrium point $| \psi_p | \approx 0$ and $F_A = 1/2$ predicted by the LPE. For Og ($Z = 118$), with $Z\alpha \approx 0.861$, $\gamma_L \approx 1.96$, demonstrating that relativistic effects are very pronounced in heavy elements. However, the most relevant case arises at the theoretical element $Z = 137$, where:

$$Z\alpha = 1 \Rightarrow \gamma_L = \frac{1}{\sqrt{1 - (1)^2}} = \frac{1}{0} \rightarrow \infty$$

This result validates a fundamental theoretical limit: at this velocity, the electron would move at the speed of light, surpassing the limits of classical physics where the Bohr model ceases to apply.

Since γ_L is always ≥ 1 , its inverse $1/\gamma_L$ lies in the range $[0, 1]$. As velocity increases, γ_L increases and (p) decreases; therefore, the probability of distortion $P = 1 - p$ always grows with velocity. This is consistent with physical limits: for a system at rest ($v = 0$), $\gamma_L = 1$, giving $P = 1 - 1 = 0$ (no relativistic effects). As $v \rightarrow c$, $\gamma_L \rightarrow \infty$, so $P \rightarrow 1 - 0 = 1$, meaning relativistic effects fully

manifest. Therefore, the LPE expression for distortion probability can be rewritten in Lorentz terms as:

$$P = 1 - \frac{1}{\gamma_L}$$

Replacing the Lorentz term:

$$P = 1 - \sqrt{1 - \frac{v^2}{c^2}} = 1 - \sqrt{1 - (Z\alpha)^2}$$

This final equation shows that the probability of experiencing relativistic effects is a direct function of the atomic number (Z) and the fine-structure constant (α). Hence, the probabilistic behavior is directly tied to the structure of elements in the periodic table. The analysis distinguishes between two types of probability. Individual probability (p and P), derived from the Lorentz factor; and process probability (F_A). In the first, the maximum is $|\psi_p| = \gamma$; in the second (process), it is at the median ($\approx 1/2$). Both correspond to equilibrium reference points within the LPE.

The final coherence of the model is confirmed by a key numerical result: the probability of non-distortion (p), derived from the atomic number of Og ($Z = 118$), is (0.5085) — almost identical to (0.5088) obtained independently from the Unification Constant (A) using the relation $(1-A^2)^{1/2}$. This value ($p \approx 1/2$) confirms the Individual Equilibrium predicted by the LPE, where the two complementary probabilities (p and P) are essentially equal (Table 4). The corresponding probability potential in absolute value is $|\psi_p| = 0$. The LPE Process Reference Points (F_A), showing convergence to γ and $1-1/e$, are presented in Table 3.

Table 4. Elements and Cumulative Probability of Lorentz Distortion by Atomic Number (Z).

Chemical Element	Z	p (No Distortion)	P (Distortion)
Hydrogen	1	0.999973	0.000027
Cesium	55	0.91592	0.08408
Oganesson	118	0.50846	0.49154
Theoretical	137	0.00000	1.00000

The LPE model demonstrates consistency by adhering to the relativistic limit. At $Z \approx 137$ ($Z\alpha \rightarrow 1$), the electron velocity approaches c . When the boundary is exceeded ($Z\alpha > 1$), such as $Z = 173$ [24], the LPE formula produces a physically meaningless negative square root. This mathematical breakdown correctly reflects the physical reality: the end of stable atoms and the onset of spontaneous electron-positron pair creation [21]. Therefore, matter's equilibrium is governed by the LPE.

3.4 Cycle at the Relativistic and Gravitational Levels

The manifestation of the LPE in the photon orbit of a black hole—a structure theoretically described by [25] from Einstein's field equations—suggests a profound connection between probabilistic equilibrium and General Relativity. The parallel indicates that the same principle governing the stability of matter in the microcosm is essential to understanding the geometry of spacetime in the macrocosm.

The smallest measurable time is the Planck time (t_p) [26]. Its quantization manifests in the photon orbit, where the orbital period (T) is an integer multiple (n) of this fundamental unit:

$$T = n \cdot tp$$

The derivation of the radius (r_n) combines this principle with the circular motion of the photon:

$$r_n = c \cdot T$$

Substituting T gives the discrete radius, and the numerical verification for $n=1$

$$r_n = n \cdot lp \Rightarrow r_1 \approx 4.845 \times 10^{-35} m$$

This value is of the same order of magnitude as the Planck length ($lp \approx 1.616 \times 10^{-35}$ m, the smallest and most fundamental scale in physics. The LPE model predicts for the minimum radius:

$$r_m = \frac{1}{2\pi} \cdot lp$$

A key aspect of the LPE model is the emergence of (γ) in extreme gravitational contexts. The LPE postulates that the relative probability of a photon (p), defined as the ratio of its quantum energy to its relativistic energy, is governed by the equilibrium limit $p \approx \gamma$. In the orbit of the photon sphere, r is the orbital radius corresponding to the distance from the black hole's center, and R_s is the Schwarzschild radius corresponding to the point of no return ($R_s = 2GM/c^2$). Given that the relation between both radii is $r = 1.5 \cdot R_s$, the gravitational factor (γ_G) becomes:

$$\gamma_G = \sqrt{1 - \frac{2GM}{rc^2}} = \sqrt{1 - \frac{2GM}{rc^2}} = \sqrt{\frac{R_s}{r} \cdot \frac{1}{3}} = \sqrt{\frac{R_s}{1.5 \cdot R_s} \cdot \frac{1}{3}} = \frac{1}{\sqrt{3}} \approx 0.5772 \approx \gamma$$

This result unifies the quantum and gravitational domains. The LPE model shows that the constant (γ) emerges directly from spacetime geometry by demonstrating its equivalence with the time dilation factor at the photon sphere. This alignment establishes a deep symmetry: (γ)—the factor governing probabilistic equilibrium—is nearly identical to the factor governing geometry in the photon orbit, independently of mass.

This reveals that (γ) acts as a structural invariant encoding the fundamental coupling between spacetime curvature and quantum processes. When a photon orbits a compact mass, its circular orbit lies in the photon sphere, at a radius 1.5 times the Schwarzschild radius (R_s), and its energy is modulated by the gravitational factor [27], [28]. Therefore, the photon orbital radius (r_{ph}) is:

$$r_{ph} = 1.5 \cdot R_s$$

By equating both radii (r_n) and (r_{ph}), one obtains:

$$n \cdot lp = 1.5 \cdot R_s$$

In the photon sphere, the LPE model can predict the number of cycles (n), understood as the number of fundamental Planck units composing the quantized orbit. The value of (n) is therefore directly proportional to the size of the black hole:

$$n = \frac{1.5 \cdot R_s}{lp} \tag{6}$$

The appearance of (γ) as a probability limit and its equivalence to the gravitational factor of the photon sphere (where $\hbar\psi_p/\gamma \approx \gamma_G$) represents the deepest unification of the LPE model. This relation establishes a coherent framework in which quantum probabilistic equilibrium governs spacetime geometry at the relativistic level.

4. Conclusions

This work proposes a coherent theoretical framework in which fundamental constants, such as the fine-structure constant (α) and the Unification Constant (A), can be interpreted as emerging from a principle of probabilistic equilibrium (LPE).

The LPE suggests that randomness operates within bounded limits, with system stability arising from a probabilistic balance that converges toward $1/2$. From this principle, the unification constant $A \approx 0.86$ emerges as a reference for the minimal information required to sustain equilibrium, linking energy accumulation ($E = \hbar\nu$) with quantum-level probabilistic accumulation.

At the atomic–relativistic scale, the framework provides a probabilistic interpretation of α ($\alpha \approx A/Z$), and identifies Oganesson ($Z = 118$) as an empirical marker of the stability threshold where $\nu/c \approx Z\alpha \approx A$.

The introduction of a Probabilistic Potential (V_C) in the Schrödinger equation offers a conceptual link between energy quantization and equilibrium, while the LPE parameters appear consistently in extreme relativistic and gravitational contexts, such as the photon sphere of black holes.

Overall, the results suggest that a principle of probabilistic equilibrium may provide a unifying perspective connecting quantum, atomic, relativistic, and gravitational phenomena, offering a compact interpretative framework for fundamental constants and the stability of physical systems.

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