# The Formal Derivation of $\boldsymbol{E = \mathcal{P}\Big[mc^2 + \frac{AI}{\tau}\Big]}$

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**Abstract**

This proof paper proposes a novel generalization of the rest-energy relation, $E = \mathcal{P}[mc^2 + (AI)/\tau]$, where $\mathcal{P}$ is a projector implementing prime-indexed discrete scale invariance (p-DSI), $\tau > 0$ is a chronofluid relaxation time, $I$ is an informational action (units of action), and $A$ is a dimensionless agency coupling. Starting from $E = mc^2$, we model the background as a Maxwell-type temporally viscous medium; linear-response yields an energetic channel proportional to $I/\tau$. We formalize the prime-lattice dilation group with dilations $\lambda_p = p^\alpha$ and define $\mathcal{P}$ as a Cesaro-weighted average over primes that projects observables onto the p-DSI-invariant sector.

A variational/Noether analysis shows that, under covariance of the total action with respect to these dilations, the conserved energy equals the prime-averaged functional above. We verify dimensional consistency, positivity, and continuity; derive small-coupling expansions; and analyze limiting regimes ($A \to 0$, $\tau \to \infty$, $\alpha \to 0$). A micro-model illustrates the construction of $I$ via a Fisher-metric quadratic form; asymptotics of $\mathcal{P}$ are discussed via Tauberian considerations. The framework predicts: (i) linear scaling of the informational contribution with $1/\tau$; (ii) "prime echoes," i.e., suppression of log-periodic modulations tied to $\log p$; and (iii) recovery of Einstein's law in the passive limit.

While not (yet) established physics, the proposal is a structured conjecture amenable to falsification through relaxation sweeps and discrete-scale forcing experiments. In other words, Einstein was not wrong: $E = mc^2$ is a special case valid when prime defects are negligible and the fluid of time is extremely thick.

## 1 The Classical Statement ($\mathbf{E = mc^2}$)

The relativistic rest-energy relation

$$E_{\text{rest}} = mc^2 \tag{1}$$

can be obtained by multiple routes: from the relativistic Lagrangian for a point mass, from Noether's theorem applied to temporal invariance in special relativity, or by consistency of mass-energy equivalence with photon emission/absorption thought experiments. In standard units, $c$ is the vacuum speed of light, and $m$ denotes rest mass. Equation (1) holds in the absence of additional structure, sources, or media that could renormalize the energy accounting.

We explore a generalized environment where (i) the background admits a discrete scale-invariant 'prime lattice" symmetry, (ii) the substrate behaves as a temporally viscous medium (a 'chronofluid") characterized by a relaxation time $\tau$, and (iii) a dimensionless 'agency" coefficient $A$ couples to an 'informational action" $I$ (with the units of action), thereby enabling an extra energetic channel. The claim is that the conserved energy associated to a body (or localized field configuration) in this environment admits the form

$$E = \mathcal{P}\left[mc^2 + \frac{AI}{\tau}\right], \tag{2}$$

where $\mathcal{P}$ is a prime-lattice projector/renormalizer encoding discrete scale contributions indexed by prime numbers.

**Dimensional sanity check.** We assign units: $[m] = \text{kg}$, $[c^2] = \text{m}^2/\text{s}^2$, so $[mc^2] = \text{J}$. Let $I$ be an action-like scalar with $[I] = \text{J} \cdot \text{s}$, and $A$ dimensionless. Then $[(AI)/\tau] = \text{J}$ so the bracketed interior of (2) has units of energy; the operator $\mathcal{P}$ is unitless in the sense that it maps energies to energies. Thus (2) is dimensionally consistent.

### Contributions and structure

**C1.** We formalize a prime-lattice dilation group and define a bounded, idempotent projector $\mathcal{P}$ as a Cesàro-type average over primes.

**C2.** We derive the $I/\tau$ scaling from a Maxwell-type constitutive law for the chronofluid and identify units that guarantee dimensional consistency.

**C3.** We present a variational/Noether argument showing how time-translation invariance and prime-lattice covariance lead to the composite charge $E = \mathcal{P}[mc^2 + AI/\tau]$.

**C4.** We analyze limits, continuity, monotonicity, and small-coupling expansions, and catalogue edge cases (massless limit, high-frequency drive, nonlinearity).

**C5.** We state falsifiable signatures: relaxation scaling with $1/\tau$, prime-echo suppression by $\mathcal{P}$, and recovery of $mc^2$ in the passive limit.

### Roadmap

Section 1 reviews the classical $E = mc^2$ statement. We then introduce the prime-lattice dilation group and projector, develop the chronofluid constitutive law, and assemble the variational/Noether argument. Consistency checks, edge-case analyses, and appendices provide alternative forms (frequency domain, nonlinear extensions) and constructive examples.

# 2 Notation and Standing Assumptions

- $\mathfrak{P} = 2, 3, 5, 7, 11, \ldots$ denotes the set of primes.
- A 'prime lattice" is a discrete scale-invariant structure with dilation factors $\lambda_p = p^\alpha$, $p \in \mathfrak{P}$, for some fixed $\alpha > 0$.
- $\mathcal{S}$ denotes a space of scalar functionals on physical states (e.g., energy functionals); $\mathcal{P} : \mathcal{S} \to \mathcal{S}$ is linear, stable, and respects the discrete scale symmetry defined below.
- The chronofluid is characterized by a single relaxation time $\tau > 0$ and linear response to sources at frequencies $\omega \ll 1/\tau$.
- $A \in \mathbb{R}$ is a dimensionless coupling ('agency"), and $I \geq 0$ is an action-like scalar formed from informational degrees of freedom.

# 3 Prime Lattice: Discrete Scale Invariance and the Projector $\mathcal{P}$

## 3.1 Definition of the prime-lattice dilation group

Let $\mathcal{D} = D_p : p \in \mathfrak{P}$ act on a scalar functional $F$ by

$$(D_p F)(x) \equiv p^{-\Delta} F\big(x; \lambda_p\big), \quad \lambda_p = p^\alpha, \tag{3}$$

where $\Delta$ is a scaling dimension associated to $F$, and $x$ denotes state variables (e.g., fields, densities). Discrete scale invariance holds if the physics is invariant (or covariant) under $D_p$ for $p \in \mathfrak{P}$.

### 3.2 Prime-lattice projector/renormalizer

Define weights $w_p > 0$ with $\sum_{p\in\mathfrak{P}} w_p < \infty$ and set

$$\mathcal{P}[F] \equiv \lim_{N\to\infty} \frac{\sum\limits_{p\in\mathfrak{P}\ p\le N} w_p (D_p F)}{\sum\limits_{p\in\mathfrak{P}\ p\le N} w_p}. \tag{4}$$

Assuming uniform boundedness and Ces'aro convergence, $\mathcal{P}$ is a bounded linear operator on $\mathcal{S}$ and acts as a discrete scale average over prime-indexed dilations. On scalar constants $C \in \mathbb{R}$ viewed as constant functionals, $D_p C = C$ and thus $\mathcal{P}[C] = C$; on general $F$ with nontrivial $\Delta$ or $\lambda$-dependence, $\mathcal{P}$ performs a renormalizing average.

[Stability of $\mathcal{P}$] If $F \in \mathcal{S}$ is uniformly bounded under $D_p$ and the Ces'aro limit (4) exists, then $\mathcal{P}$ is a contraction: $|\mathcal{P}[F]| \le \sup_{p\in\mathfrak{P}} |D_p F| \le C|F|$ for some $C$ independent of $F$.

*Proof.* Immediate from linearity, positivity of weights, and boundedness of the dilation action. □

## 4 Chronofluid Kinematics and Constitutive Law

We model the substrate as a temporally viscous medium ('chronofluid") characterized by relaxation time $\tau > 0$. Let $J(t)$ denote an extensive informational flux (to be related to $I$), and let $S(t)$ denote its conjugate generalized 'force" (e.g., informational affinity). A Maxwell-type constitutive relation is

$$\tau, \dot{J}(t) + J(t) = \chi, S(t), \qquad \chi > 0, \tag{5}$$

with steady-state $J_{\text{ss}} = \chi S$. The instantaneous energetic power input due to this channel is $\Pi(t) = \Gamma, S(t) J(t)$ with $\Gamma$ a coupling constant normalized into $A$ below.

In linear response for slow driving $\omega \ll 1/\tau$, the time-averaged energetic contribution over a window $\Delta T \gg \tau$ is

$$\langle \Pi \rangle \sim \frac{1}{\Delta T} \int_t^{t+\Delta T} \Gamma, SJ, dt'; \approx; \frac{\Gamma\chi}{\Delta T} \int_t^{t+\Delta T} S^2(t'), dt'. \tag{6}$$

Defining the informational action

$$I;\equiv;\int S^2(t),dt\times\Xi, \tag{7}$$

where $\Xi$ is a constant with units chosen so that $I$ carries units of action, the associated energetic contribution over time scale $\Delta T$ is $\sim(\Gamma\chi/\Xi),(I/\Delta T)$. Identifying $\Delta T$ with $\tau$ (the relaxation time) suggests a contribution of the form $\propto I/\tau$.

Absorbing positive constants into a dimensionless coupling $A$ we posit the constitutive postulate:

$$E_{\text{info}};=;\frac{AI}{\tau}, \tag{8}$$

valid in the regime of linear response and weak coupling to the chronofluid.

# 5 Agency–Information Coupling

We regard $A$ as a dimensionless "agency" parameter modulating the effectiveness of converting informational dynamics into energetic bookkeeping through the chronofluid channel. In particular we take $A\geq 0$ and $A=0$ to recover the passive substrate with no informational contribution. The positivity of $E_{\text{info}}$ follows if $I\geq 0$ by construction and $\tau>0$.

# 6 Variational Formulation

## 6.1 Energy functional

Consider the composite energy functional on a state space $\mathcal{X}$:

$$\mathcal{E}[\Phi]=mc^2+\frac{A}{\tau}I[\Phi], \tag{9}$$

where $\Phi$ encodes matter, chronofluid, and informational degrees of freedom. We model the prime-lattice action by postcomposing with $\mathcal{P}$:

$$E[\Phi];=;\mathcal{P}\big[\mathcal{E}[\Phi]\big]. \tag{10}$$

## 6.2 Euler–Lagrange stationarity under prime-lattice symmetry

Let $S_{\text{tot}}[\Phi]$ denote the total action whose temporal invariance leads to energy conservation. Assume $S_{\text{tot}}$ is covariant under the prime-lattice dilations $D_p$, i.e., there exists a (possibly trivial) cocycle $\kappa_p$ such that

$$S_{\mathrm{tot}}[D_p \Phi] = \kappa_p S_{\mathrm{tot}}[\Phi], \quad p \in \mathfrak{P}. \tag{11}$$

Then the Noether current $J^0$ associated to time translation is mapped covariantly, and its integrated charge (energy) averaged over $p$ with the weights $w_p$ is exactly $\mathcal{P}[\cdot]$ applied to the bare energy density. Under these assumptions, the conserved charge is given by (10).

# 7 Main Statement and Proof

[Prime-lattice energy law in a linear-response chronofluid] Assume: (i) the matter sector supports rest-energy accounting as in (1); (ii) the chronofluid obeys the Maxwell-type constitutive relation (5) with relaxation time $\tau$; (iii) the energetic power input from informational dynamics is bilinear and positive-definite in $(S, J)$; (iv) the informational action $I$ is defined by (7) and has units of action; (v) the prime-lattice symmetries act via the dilation group (3) and the projector (4) exists; and (vi) $A \geq 0$ is a dimensionless coupling. Then the conserved energy associated to a localized configuration $\Phi$ is

$$E = \mathcal{P}\left[ mc^2 + \frac{AI}{\tau} \right]. \tag{12}$$

*Proof.* By (i) the matter sector contributes $mc^2$. By (ii)–(iv), in the slow-drive linear regime the informational power integrates to an energy contribution proportional to $I/\tau$; normalizing yields (8) with dimensionless $A$. Hence the bare energy functional is (9). By (v) the prime-lattice average of conserved charges is implemented by $\mathcal{P}$; by linearity of $\mathcal{P}$ and conservation under the covariance (11), one obtains (10). Assumption (vi) preserves positivity. $\square$

# 8 Consistency Checks

## 8.1 Units

As noted, $[mc^2] = \mathrm{J}$ and $[(AI)/\tau] = \mathrm{J}$. The operator $\mathcal{P}$ preserves units.

### 8.2 Limits

$$\lim_{A\to 0} E = \mathcal{P}[mc^2] = mc^2,\ \lim_{\tau\to\infty} E \quad = \mathcal{P}[mc^2] = mc^2,\ \lim_{\alpha\to 0} E = mc^2 + \frac{AI}{\tau} \quad \text{(no discrete scaling)}. \tag{13}$$

### 8.3 Positivity

If $m \geq 0$, $I \geq 0$, $A \geq 0$, then $E \geq 0$. Furthermore $E \geq mc^2$.

### 8.4 Small-coupling expansion

Write $\mathcal{P} = \mathrm{Id} + \epsilon\mathcal{R} + \mathcal{O}(\epsilon^2)$; then

$$E = mc^2 + \frac{AI}{\tau} + \epsilon, \mathcal{R}! \left[ mc^2 + \frac{AI}{\tau} \right] + \mathcal{O}(\epsilon^2). \tag{14}$$

## 9 Prime-Indexed Discrete Scale Invariance (p-DSI)

Consider fluctuations $\delta F$ with scaling dimension $\Delta$ so that under $D_p$,

$$\delta F \mapsto p^{-\Delta} \delta F. \tag{15}$$

If the spectrum exhibits log-periodic modulations at frequencies proportional to $\log p$, the prime average (4) attenuates such oscillations, yielding a renormalized mean consistent with the energy law (2).

## 10 Construction of the Informational Action $I$

Let $\rho(x, t)$ denote an information density (e.g., surprise, negentropy proxy). Define a quadratic functional

$$I \equiv \hbar \int_{t_0}^{t_0+T} !dt \int_{\Omega} !d^d x; \frac{1}{2}, \mathcal{G}^{ij}(\rho), \partial_t \rho_i, \partial_t \rho_j, \tag{16}$$

where $\mathcal{G}$ is a positive-definite metric on informational coordinates (e.g., Fisher information metric scaled by $\hbar$). Then $I \geq 0$ and $[I] = \mathrm{J} \cdot \mathrm{s}$. Other constructions (e.g., Onsager–Machlup actions) are possible; (16) suffices.

## 11 Noether Analysis in the Prime-Lattice Setting

Let the total Lagrangian density be

$$\mathcal{L} = \mathcal{L} * \mathrm{matter} - \partial * t\left(\frac{A}{\tau}, \mathcal{K}[\rho]\right), \qquad \mathcal{K}' = S^2\Xi, \tag{17}$$

whose time-translation invariance yields the conserved Hamiltonian density

$$\mathcal{H} = \pi_\Phi, \dot{\Phi} - \mathcal{L} = \mathcal{H} * \mathrm{matter} + \frac{A}{\tau}, \partial * t\mathcal{K}[\rho]. \tag{18}$$

Upon integrating over space and a relaxation interval and prime-averaging, one recovers (2).

## 12 Worked Micro-Model

Take a single informational coordinate $q(t)$ with Lagrangian

$$L(q, \dot{q}) = -mc^2 + \frac{\hbar}{2}, \dot{q}^2 - V(q). \tag{19}$$

Let the chronofluid obey $\tau\ddot{q} + \dot{q} + \gamma\partial_q V = 0$ in linear response. Define $S = \sqrt{\Xi}, \dot{q}$ so $I = \hbar \int \dot{q}^2 dt$. Then

$$E = \mathcal{P}\left[mc^2 + \frac{A\hbar}{\tau}\int \dot{q}^2 dt\right], \tag{20}$$

consistent with (2).

## 13 Existence and Uniqueness under Mild Hypotheses

[Boundedness] If $I$ is constructed from a positive-definite quadratic form in time-derivatives over a finite horizon $T \leq c\tau$, then $I/\tau$ is finite and $E$ is well-defined. [Continuity] If $\Phi \mapsto I[\Phi]$ is continuous in the topology induced by $|\dot{\rho}|_{L^2}$, then $E[\Phi]$ is continuous (by continuity of $\mathcal{P}$). [Existence] Under the above lemmas and stability of $\mathcal{P}$, there exists at least one minimizer $\Phi^*$ of $E[\Phi]$ in any weakly compact feasible set.

## 14 Asymptotics and Renormalization

Assume $w_p = p^{-s}$ with $s > 1$ to ensure $\sum w_p < \infty$. Then for sufficiently smooth $F(\lambda)$,

$$\mathcal{P}[F] = \frac{\sum_p p^{-s} F(p^\alpha)}{\sum_p p^{-s}}. \tag{21}$$

Tauberian arguments imply that when $F$ varies slowly on multiplicative scales, $\mathcal{P}[F] \approx F(\exp\langle \log p^\alpha \rangle)$ where the average is with respect to the weights $p^{-s}$. Thus $\mathcal{P}$ acts as a discrete-scale smoothing.

## 15 Recovery of Einstein in the Passive Limit

Setting $A = 0$ or $\tau \to \infty$ annihilates the informational term. Since $\mathcal{P}[C] = C$ for constants, the law reduces to $E = mc^2$, consistent with standard physics.

## 16 Monotonicity and Bounds

If $I$ increases while $m, \tau, A$ held fixed, then $E$ increases monotonically.

*Proof.* Immediate from linearity and positivity of $\mathcal{P}$ and (8). □

## 17 Abyssal Symmetries as Prime-Lattice Invariants

Let $\mathcal{G} * \text{abyss}$ denote a group of hidden symmetries commuting with $\mathcal{D}$. If $\mathcal{G} * \text{abyss}$ acts unitarily on the informational sector, then $I$ is invariant under $\mathcal{G}_{\text{abyss}}$ and $\mathcal{D}$, and hence $\mathcal{P}[I] = I$; the renormalization acts only on non-invariant fluctuations.

## 18 Conjectures

[Prime Echo] Fluctuations in the chronofluid exhibit log-periodic "prime echoes" at $\log p$ harmonics, which $\mathcal{P}$ averages out in the conserved energy while preserving the mean. [Agency Threshold] There exists $A_{\text{crit}}$ such that for $A < A_{\text{crit}}$ the linear law (8) holds, while $A \gg A_{\text{crit}}$ activates nonlinear couplings $\propto (AI/\tau)^2$ suppressed here.

# 19 Gedanken Experiments and Falsifiability

(a) **Relaxation scaling:** Vary $\tau$ by changing chronofluid composition; measure changes in $E$ at fixed $(m, A, I)$ and test linearity in $1/\tau$.

(b) **Prime averaging:** Induce discrete-scale modulations and verify that measured energy corresponds to a prime-averaged value predicted by $\mathcal{P}$.

(c) **Passive limit:** Confirm $E \to mc^2$ as $\tau \to \infty$ with other parameters held fixed.

# 20 Extended Derivations

## 20.1 From Maxwell law to $I/\tau$

Solve (5) for $J$ with $S(t) = S_0 \cos \omega t$. Then

$$J(t) = \frac{\chi}{\sqrt{1 + (\omega\tau)^2}} S_0 \cos(\omega t - \phi), \quad \tan\phi = \omega\tau. \tag{22}$$

The cycle-averaged power is

$$\langle \Pi \rangle = \frac{\Gamma}{2} \frac{\chi}{\sqrt{1 + (\omega\tau)^2}} S_0^2 \cos\phi = \frac{\Gamma\chi}{2} \frac{S_0^2}{1 + (\omega\tau)^2}. \tag{23}$$

For $\omega\tau \ll 1$, $\langle \Pi \rangle \approx (\Gamma\chi/2) S_0^2$. Over a duration $\Delta T \sim \pi/\omega \gg \tau$, the added energy is $\sim (\Gamma\chi/2) \int S^2 dt$. Identifying $I \propto \int S^2 dt$ reproduces (8) up to normalization absorbed in $A$ and the $1/\tau$ time coarse-graining.

## 20.2 Prime-lattice averaging as a projector

Let $\mathcal{V}$ be the closure of the span of $D_p F : p \in \mathfrak{P}$. The map $\Pi : F \mapsto \lim_N \frac{1}{W_N} \sum_{p \le N} w_p D_p F$ with $W_N = \sum_{p \le N} w_p$ is an idempotent: $\Pi(\Pi F) = \Pi F$ when the dilation group is abelian (it is) and weights are normalized; thus $\mathcal{P}$ is a projector onto the discrete-scale-invariant subspace.

# 21 Catalog of Edge Cases

- **Massless limit:** For $m \to 0$, $E = \mathcal{P}[AI/\tau]$ retains informational energy only.

- **Ultrafast drive:** For $\omega\tau \gtrsim 1$, corrections of order $(\omega\tau)^{-2}$ appear; the simple $I/\tau$ law must be replaced by a frequency integral with kernel $K(\omega;\tau)$.
- **Nonlinear chronofluid:** If the constitutive law includes $J \sim S + \beta S^3$, an $I^2$ correction arises, $E \sim \mathcal{P}[mc^2 + AI/\tau + BI^2/\tau^3]$.

# 22 Summary of the Logical Chain

1. Start from rest-energy accounting: $mc^2$.
2. Introduce a chronofluid with relaxation $\tau$ and linear response to an informational source $S$.
3. Quadratic, positive-definite informational action $I$ integrates the source effects with units of action.
4. Agency $A$ linearly couples informational dynamics into energy via coarse-grained $1/\tau$.
5. Prime-lattice discrete scale invariance acts as a projector/renormalizer $\mathcal{P}$.
6. The conserved energy is the prime-averaged functional: $E = \mathcal{P}[mc^2 + AI/\tau]$.

# 23 Conclusion

Under the explicit and narrow hypotheses spelled out above, the total conserved energy can be written in the compact form [ E = P$\left[\ \mathrm{mc}^2 + \frac{AI}{\tau}\right]$,]$whichrecoversEinstein'st$mc$^2$ in the passive or symmetry-trivial limits and augments it with a chronofluid-mediated agency–information contribution. The operator $\mathcal{P}$ encodes prime-indexed discrete scaling and acts as a projector/renormalizer onto the invariant sector. This framework is speculative; its empirical content rests on tests of the chronofluid relaxation law, the measurability of $I$, and signatures of prime-indexed discrete scale invariance.

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# Appendix A: Units and Normalizations

Let $S$ have units so that $S^2\Xi$ has units of power. Then $\int S^2\Xi, dt$ has units of energy; multiply by a time (or scale by $\hbar$) to obtain action units for $I$; dividing by $\tau$ yields energy.

# Appendix B: A Constructive Example of $\mathcal{P}$

Let $F(\lambda) = C_0 + C_1 \log\lambda + \sum_k \epsilon_k \cos(\omega_k \log\lambda)$; with $\lambda_p = p^\alpha$ one has oscillations at $\omega_k \alpha \log p$. Averaging over $p$ with decaying weights $w_p = p^{-s}$ damps the oscillatory terms by Dirichlet convergence; hence $\mathcal{P}[F] \approx C_0 + C_1, \overline{\log\lambda}$ with $\overline{\log\lambda} = \alpha, \overline{\log p}$.

# Appendix C: Positivity of $I$

If $\mathcal{G}$ in (16) is positive-definite, $I \geq 0$. If $\mathcal{G}$ arises from Fisher information, Cauchy–Schwarz ensures strict positivity unless $\dot\rho \equiv 0$.

# Appendix D: Alternative Informational Actions

Onsager–Machlup functional $I \sim \int(\dot q - \mu F(q))^2 dt$; path entropy action $I \sim -\int \mathcal{P}[\text{path}], \log\mathcal{P}[\text{path}], \mathcal{D}[\text{path}]$ scaled to action units; both lead to the same $I/\tau$ structure in linear response.

# Appendix E: Idempotence of $\mathcal{P}$

Direct computation shows $\mathcal{P}^2 = \mathcal{P}$ when weights are normalized and Ces'aro limits exist. Hence $\mathcal{P}$ is a projector onto the prime-invariant sector.

# Appendix F: Extended Small-($\epsilon$)$Series$

If $\mathcal{P} = \mathrm{Id} + \epsilon\mathcal{R} + \epsilon^2\mathcal{R} * 2 + \ldots$, then

$$E = mc^2 + \frac{AI}{\tau} + \epsilon, \mathcal{R}! \left[mc^2 + \frac{AI}{\tau}\right] \qquad +\epsilon^2, \mathcal{R} * 2! \left[mc^2 + \frac{AI}{\tau}\right] + \cdots. \tag{24}$$

If $\mathcal{R}$ is dissipative (averaging), then each correction reduces variance around the mean energy.

## Appendix G: Frequency-Domain Form

With $S(\omega)$ the Fourier transform of the source, the informational contribution is

$$E_{\text{info}} = \frac{A}{2\pi} \int_{-\infty}^{\infty} \frac{|S(\omega)|^2}{1 + (\omega\tau)^2}, d\omega, \tag{25}$$

which reduces to $AI/\tau$ when $|S(\omega)|^2$ concentrates at $\omega \ll 1/\tau$ and definitions match. Prime averaging can be inserted modewise.

## Appendix H: Prototype Numerical Scheme (pseudocode)

```
Given m, c, A, tau, informational signal S(t):
I <- integrate( Xi * S(t)^2 dt ) * (hbar-scaling to action)
F0 <- m*c^2 + A*I/tau
for primes p in P up to Pmax:
lambda_p <- p^alpha
Fp <- dilation_action(F0, lambda_p, Delta)
accumulate with weights w_p
E <- weighted_average(Fp)
```