# GOLS Lab 10 — User Manual (V2025)

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## Preface

GOLS Lab 10 demonstrates the unification of the discrete and the continuous — a central idea of the Coupled Order Lattice System (COLS). Through this page, users can observe how modular arithmetic, a discrete structure, becomes a continuous geometry when represented as the GOLS triangle. Each exponent n acts as a resonance point where number theory meets geometry. This interactive system allows users to explore, visually and numerically, the bridge between discrete arithmetic logic and continuous geometric order.

## 1 Scope & Purpose

This manual describes GOLS Lab 10, an interactive laboratory for visualising modular geometry and symmetry breaking in semiprimes M=pq. It renders the Coupled Order Lattice System (COLS) through geometric and arithmetic observables A(n), h\_p(n), h\_q(n), E\_uni(n), and E\_bi(n). The Lab is designed for both mathematicians exploring modular orders and users interested in visual patterns in number theory.

## 2 Mathematical Model (overview)

Input: composite M=pq, base a.  
Geometry: for exponent n, define the GOLS triangle S0=(1,0), SN=(-1,0), Sn=(cos(2πn/M),sin(2πn/M)).  
Asymmetry: by Thales, γ(n)=90°, and A(n)=|α(n)−90°|=arcsin(|sin(πn/M)|)∈[0°,90°].  
Modular proximities: h\_p and h\_q measure closeness to ±1 mod p and mod q.  
Energies: E\_uni=(A/90)(1−h\_min), E\_bi=(A/90)(1−h\_max), where h\_min=min(h\_p,h\_q), h\_max=max(h\_p,h\_q).  
Effective exponent: halving n→n/2 until gcd(a^{n/2}±1,M) non-trivial gives n\_eff.

## 3 Interface Overview

Left panel: inputs for M, a, CRT and animation controls.  
Central panel: GOLS triangle, live bar chart, CRT torus.  
Right panel: numeric KPIs, mini-triangle at n\_eff, and factor history.  
Footer: copyright and version.

## 4 Controls and Parameters

4.1 Module M – defines the working semiprime. Changing M resets graphs.  
4.2 Base a – multiplicative base. Typical values {2,3,5,7}.  
4.3 Exponent n – manual slider or Play mode.  
4.4 CRT (p,q) – manual or automatic. Auto activates when a factor is found.  
4.5 Energy Display – legend toggles curves (A,h\_min,h\_max,E\_uni,E\_bi).  
4.6 Trace Options – ε torus sets CRT band width; Trace On activates tracking.  
4.7 Exports – PNG (combined view) and CSV (factor log).

## 5 Displayed Quantities (KPI zone)

θ = 2πn/M ; A = Asymmetry [0–90°] ; h\_g,h\_p,h\_q ∈ [0,1]; E (global/uni/bi) ∈ [0,1]; gcd± values; n\_eff; v2 depth.

## 6 Geometric Visualisations

Triangle GOLS: halo intensity ∝ E.  
Bar Chart: concurrent A, h\_min, h\_max, E\_uni, E\_bi.  
CRT Torus: residues (x\_p/p,x\_q/q); colour code for unilateral/bilateral events.  
Mini-Triangle: displayed when factor found (n\_eff).

## 7 Factor Detection and Auto-CRT

For each even n, compute gcd(a^{n/2}±1,M). On first non-trivial gcd, p,q auto-fill, CRT activates, mini-triangle appears, and the factor is logged.

## 8 Operational Tips

Observe geometry only: leave p,q empty.  
Full arithmetic mode: let auto-CRT trigger.  
Energy studies: keep E\_uni,E\_bi visible.  
Play mode: continuous n increment.  
Compare bases: vary a.  
Export findings: use CSV.

## 9 Performance & Limits

Works smoothly for M≤2^16. Trace length 500 points. Memory limited by browser; no external dependencies.

## 10 Reproducibility

Deterministic BigInt arithmetic ensures identical results on any modern browser. To reproduce: record M,a,ε and export CSV logs.

## 11 Scientific Positioning

GOLS Lab implements COLS as an interactive visual system. It demonstrates the geometric coupling between modular orders — a bridge between discrete and continuous domains.

## 12 Troubleshooting

Nothing moves → enter valid M>1.  
n slider inactive → refresh.  
No bars → enable series in legend.  
CRT grey → factors not yet found.  
Mini-triangle absent → no factor.  
Lag → reduce M or disable Trace.