ON THE MECHANICAL CONCEPTIONS OF ELECTRICITY AND MAGNETISM.[1]

BY W. S. FRANKLIN.

THE following development is based upon the conceptions of Maxwell. It follows, in some particulars, the work of Lodge. The conceptions here developed, of the dependence of electromotive force upon changing magnetic flux, of the dependence of magnetomotive force upon changing electric flux, of the production of electromotive force in a moving wire, of the energy stream, of electromagnetic waves, and of the Hertz vibrator, do not seem to have been pointed out before, except, perhaps, in a general way, as, for example, by Boltzmann, who discusses[2] the mechanical behavior of a medium of which the equations of motion are the electromagnetic equations of Maxwell. The kinematics and dynamics of such a medium can be fairly stated only with the help of differential equations. The conceptions here developed are, of course, approximations. Some inconsistencies arise in the extension of these conceptions to three dimensions, as has been noticed by Poynting.

1. *Fundamental Conception.* — The ether is to be considered as built up of very small cells of two kinds, *positive* and *negative*, in such a way that only unlike cells are in contact. These cells are imagined to be so connected, where they are in contact, that if a cell be turned while the adjacent cells are kept stationary, then a torque, due to elastic reaction of adjoining cells, is brought to bear upon the turned cell, which tends to right it, and which is proportional to the angle turned.

2. *Conception of the Magnetic Field.* — The ether cells at a point in a magnetic field are to be thought of as rotating about axes

[1] A paper read before the Buffalo meeting of the American Association. A portion of this paper which appeared in Vol. II., Nichols and Franklin's Elements of Physics, is given here again for the sake of completeness.

[2] Wied. Ann.

which are parallel to the direction of the field at the point, the angular velocity of the cells being proportional to the intensity of the field at the point. The positive cells rotate in the direction in which a right-handed screw would be turned that it might move in the direction of the field, and the negative cells rotate in the opposite direction. This opposite rotation of positive and negative cells is mechanically possible since only unlike cells are in contact.

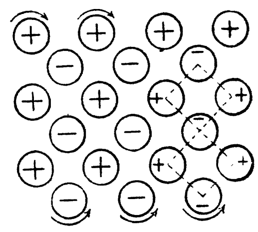

Fig. 1.

This rotatory motion of the ether cells is represented in Fig. 1. The magnetic field is perpendicular to the plane of the paper, and directed away from the reader; all the positive cells rotate clockwise, and the negative cells counter-clockwise. The kinetic energy per unit volume in such a system of rotating cells is proportional to the square of the angular velocity, which is consistent with the fact that the energy (kinetic) per unit volume in a magnetic field is proportional to the square of the intensity of the field.

3. *Conception of the Electric Field.* — The positive ether cells at a point in an electric field are to be thought of as displaced in the direction of the field, while the negative cells are displaced in the opposite direction; this displacement being proportional to the field intensity. Thus Fig. 2 represents the case in which the positive cells have been displaced towards the bottom of the page relatively to the negative cells. Figure 3 represents two meshes. The downward displacement of the positive cells has distorted these meshes which are normally square. Since this cell structure of the ether is elastic, as explained in Art. 1, its distortion, as represented in Figs. 2 and 3, represents potential energy. The amount of potential energy per unit volume is pro-

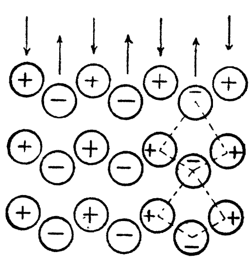

Fig. 2.

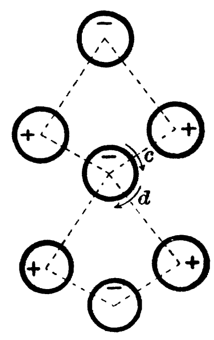

Fig. 3.

portional to the square of the displacement. This is consistent with the fact that the energy (potential) per unit volume in an electric field is proportional to the square of the field intensity.

Remark. — The two positive cells to the right of the middle cell in Fig. 3 being displaced downwards, may be conceived to exert torques upon the middle cell as shown by the arrows c and d; which torques are proportional to the intensity of the electric field, *i.e.* to the displacements of the cells. The cells to the left exert equal but opposite torques upon the middle cell.

4. *The Dependence of Electromotive Force upon Changing Magnetic Flux; Explanation of Induced Electromotive Force.* — The electromotive force around a closed curve is proportional to the rate of change of the magnetic flux through the curve.

Consider a closed boundary $ABCD$ (Fig. 4) in an electric field parallel to DA and CB. Let f be the intensity of the field along CB, and f' the intensity along DA, and let l be the length of DA and of CB. The electromotive force around the boundary $ABCD$ is $lf' - lf$. Now, the total torque acting across BC on the enclosed cells is proportional to l and to f, and the total torque acting across AD is proportional to l and to f', but opposite in direction. Therefore the total torque acting to turn the enclosed cells is proportional to $lf' - lf$. The enclosed cells gain angular velocity under the action of this torque at a rate which is directly proportional to the torque, and inversely proportional to the number of cells which participate, so that the *product* of number of cells (area of $ABCD$) into angular acceleration (rate of change of magnetic field) is proportional to the torque or to the electromotive force around $ABCD$, and this product, area of $ABCD$ into rate of change of magnetic field, is equal to the rate of change of magnetic flux through $ABCD$.

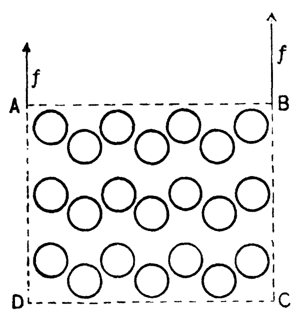

Fig. 4.

5. *The Energy Stream in the Electromagnetic Field; Preliminary Statement.* — Consider three gear wheels A, B, C (Fig. 5). Let A and C exert equal and opposite torque actions upon B.

Then, if the wheels are turning, work will be transmitted from A to C, or from C to A, according to direction of turning and to direction of torque action, and the rate of transmission of work will be proportional to the product of torque action into speed.

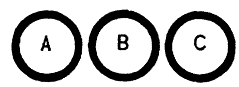

Fig. 5.

Imagine the cells in Fig. 2 to be rotating, positive cells in one direction, negative in the other, about axes perpendicular to the paper. This constitutes a magnetic field perpendicular to the electric field, which is towards the bottom of the page. On account of the torque actions between the cells, as explained in Art. 3, energy will be transferred to the right (or left) by each chain of geared cells at a rate which is proportional to the product of the intensity of the magnetic field into the intensity of the electric field, and the energy per second flowing across an area perpendicular to both electric field and magnetic field is proportional to the product of the respective field intensities into the area; for this area is proportional to the number of rows of cells which are acting as chains of gear wheels. The *energy stream*, that is, *energy per unit area per second*, is therefore proportional to the product of magnetic and electric field intensities, and is at right angles to both. In case the electric and magnetic fields are not orthogonal, the energy stream is proportional to the vector part of this product.

6. *The Electric Current.* — Consider a wire AB (Fig. 6) along which an electric current is flowing. The magnetic field on opposite sides of AB is in opposite directions, so that positive ether cells at p and p' are rotating in opposite directions, as shown. Since an electric current may be maintained for an indefinite time, this opposite rotation of positive ether cells on the two sides of AB cannot be due to an ever-increasing ether distortion (the cells are geared together, as it were), but there must be a *slip* between adjacent cells somewhere between p and p'. This *slip* between

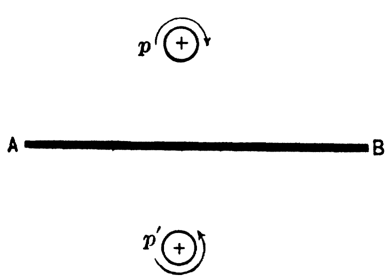

Fig. 6.

adjacent ether cells (positive and negative cells) takes place in the material of the wire, and constitutes the electric current.

7. *Established Electric Currents flow in Closed Circuits.* — Let *AB* (Fig. 7) be a wire carrying an established electric current. If this wire does not form a closed circuit, the opposite rotations of like ether cells on opposite sides of *AB* cannot continue without adjacent cells slipping on each other somewhere along any line passing around the end of *AB*.[1] That is, established *lines of slip* of the ether cells are necessarily closed lines. When a current does flow in a circuit which is not closed, an increasing ether distortion (electric field) is produced around the end portions of the circuit, which distortion produces (constitutes) electric charge there. Compare Article 12 below.

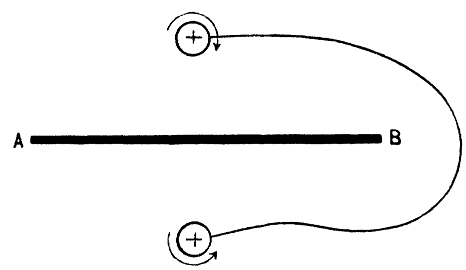

Fig. 7.

8. *Flow of Energy in the Neighborhood of an Electric Current.* — Let Fig. 8 represent the neighborhood of a long wire *AB* through which electric current is flowing. The electric field in the neighborhood is parallel to the wire, and the magnetic field circles round the wire. The product of magnetic field intensity into electric field intensity is the energy stream, and this is directed towards the wire from all sides. This energy streaming in upon the wire changes into the heat which appears in the wire. In case the wire is of high resistance, the electric field (volts per centimeter) is of great intensity, and, for the same current and same intensity of magnetic field,

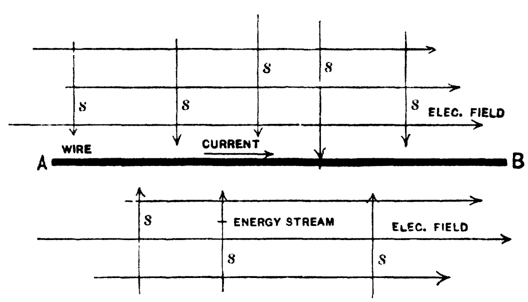

Fig. 8.

[1] The figure is thought to represent a flow of current in two dimensions.

the energy stream is correspondingly intense, making the wire very hot. At points not very near to the wire the electric field is more nearly perpendicular to the wire, especially near the battery or dynamo which is maintaining the current. The energy therefore streams out from the battery or dynamo through the whole region *surrounding* the wires, and the energy stream turns in upon the wire throughout its length.[1]

9. *The Charge on a Condenser and its Disappearance when the Condenser Plates are connected by a Wire; Preliminary Statement.* — Consider a row of gear wheels A to B (Fig. 9). If the wheel B is held stationary while A is turned in the direction of the arrow, the wheels will arrange themselves as shown in Fig. 10, alternate wheels being displaced upwards, and the intermediate wheels being displaced downwards. Conversely, a number of geared wheels, which by elastic action of any kind tend to stand in a straight row,[2] will be *relieved* from such a distortion as is represented in Fig. 10, by allowing freedom of rotation, or *permitting slip anywhere across the row.*

Fig. 9.

Fig. 10.

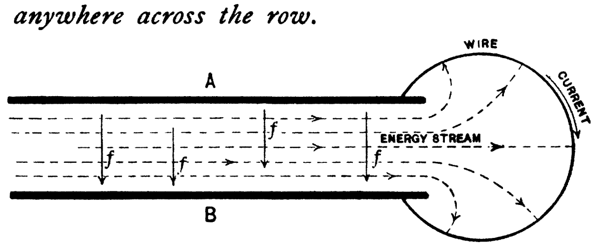

Fig. 11.

Let A and B (Fig. 11) be the plates of a charged condenser. The electric field f between these plates consists of upward dis-

[1] See J. H. Poynting, Phil. Trans.

[2] The rows of positive and negative ether cells are thought of as standing in a zigzag line, as shown by the horizontal rows in Figs. 1 and 2. In order that the diagrams may be simpler, the rows of positive and negative cells are hereafter to be thought of as straight (or uniformly curved) when free from distortion.

placement of positive ether cells and downward displacement of negative ether cells.

The horizontal rows of cells are distorted as shown in Fig. 10, and if a wire is connected across from A to B anywhere, this will constitute a line of slip permitting the adjoining ether cells to rotate, and thus entirely relieving the distortion which constitutes the field between the plates. The potential energy of the ether distortion will be transmitted along the rows of geared cells, in the direction of the dotted arrows, to the wire, where it will appear as heat.

An electric spark is a line of slip produced by the breaking down of the mechanism which sustains the electric stress, and the electric energy flows in upon a spark as it does upon a wire carrying current.

The explanation here given of the entire relief of the electric stress between two plates by the establishment of a conducting line (line of slip) between them applies to two adjacent oppositely charged bodies of any shape.

10. *The Dependence of Magnetomotive Force upon Changing Electric Flux.* — The magnetomotive force around any closed curve is proportional to the rate of change of the electric flux through the curve.

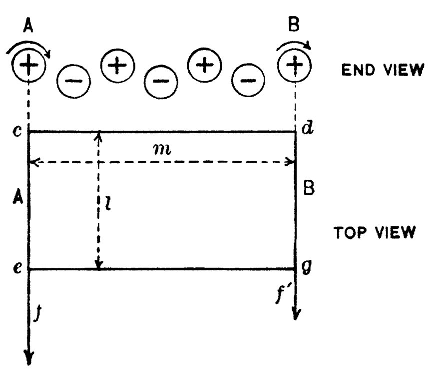

Fig. 12.

Consider a distorted chain of cells AB (Fig. 12). The rate at which this distortion is increasing is proportional to the difference in the angular velocities of the cells (intensities f and f' of the magnetic field) at A and B divided by the number of cells in the chain (distance m from A to B). Therefore, the product of the rate of increase of this distortion into the area lm of $cdeg$ (rate of increase of the electric flux through $cdeg$) is proportional to $lf - lf'$, which is the magnetomotive force around $cdeg$.

11. *Production of Electric Field in the Neighborhood of a Conductor moving across a Magnetic Field; Induced Electromotive Force in a Moving Wire.* — Consider a conductor A (Fig. 13), which is moving at velocity v across a magnetic field of intensity f perpendicular to the paper and towards the reader. This body being a conductor, the ether inside of it cannot sustain electric stress, so that the energy of the magnetic field, which is seated in the region immediately in front of A, cannot be geared across A, but an electric field is called into existence about the ends of A as indicated by the lines of force, and the magnetic energy is geared around A as indicated by the lines marked *energy stream.*

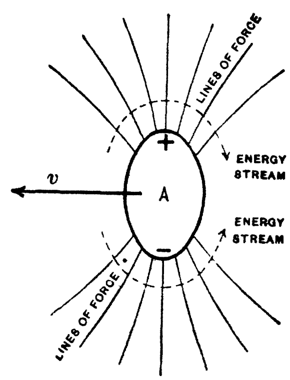

Fig. 13.

In order to explain the equation $E = lfv$, expressing the electromotive force induced in a wire of length l, moving at velocity v across a magnetic field of intensity f, we must consider the following:

Preliminary Statement. — Let the line AB (Fig. 14) be a line of force[1] of an electric field, and imagine the whole region about AB to be a uniform magnetic field perpendicular to the paper and of intensity f. The energy stream, R, at Δs, is normal to Δs and proportional to the product of the intensities of the electric and magnetic fields at Δs; that is, $R = \kappa fe$, where κ is the proportionality factor and e is the intensity of the electric field at Δs.

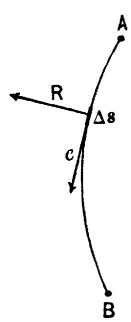

Fig. 14.

The energy per second crossing Δs (unit depth) is $R\Delta s = \kappa fe \cdot \Delta s$, and the energy per second crossing the entire line AB is $\Sigma\kappa fe\Delta s$, or $\kappa f\Sigma e \cdot \Delta s$, but $\Sigma e \cdot \Delta s$ is the electromotive force along AB. Therefore, ***the energy per second crossing AB is equal to the product κfE, where E is the electromotive force along AB.***

[1] Chosen as a line of force for the sake of simplicity.

Let AB (Fig. 15) be a rod[1] of length l, moving at velocity v across a magnetic field of intensity f perpendicular to the paper.

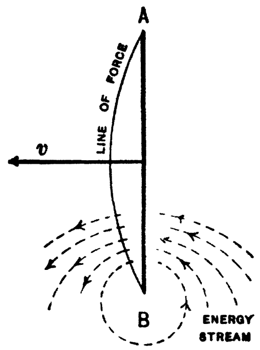

Fig. 15.

The magnetic energy $\frac{1}{8\pi}f^2lv\Delta t$, which resides in the space, $l\cdot v\Delta t$ (unit depth), which is swept over by the rod during Δt must be equal to the energy $\kappa fE\cdot\Delta t$, which streams across the *line of force* during that interval. That is, $\frac{1}{8\pi}f^2lv\Delta t = \kappa fE\Delta t$, or $E = \frac{1}{8\pi\kappa}lfv$. By properly choosing the unit electromotive force, we may provide that the factor $\frac{1}{8\pi\kappa}$ be equal to unity, in which case we have $E = lfv$. Further, when $\frac{1}{8\pi\kappa} = 1$, then $\kappa = \frac{1}{8\pi}$, so that the energy stream at a point in an electromagnetic field is $R = \frac{1}{8\pi}fe$, in which f and e are the intensities in "electromagnetic" units of the magnetic and electric fields at the point. R, f, and e are of course orthogonal.

When the rod AB (Fig. 15) is a portion of a closed conducting circuit, then electric stress is actually produced throughout the

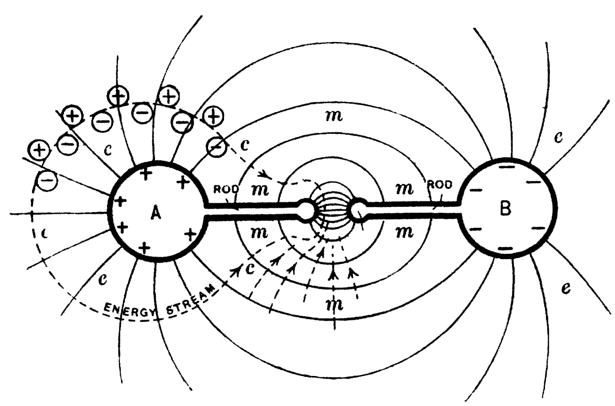

Fig. 16.

[1] The rod is assumed to be of unit width perpendicularly to plane of paper for simplicity. The formulæ given, however, are true whatever the width, for such energy as is in front of the rod does not leave the plane of the paper as it streams around the ends of AB. Such energy streams as exist in planes above and below the plane of the paper are *re-entrant* streams.

conductor as the magnetic energy streams across it, and it is the continual *slipping of gears*, or breaking down of this maintained electric stress, which constitutes the electric current in the circuit.

12. *The Action of the Hertz Vibrator* is as follows:

Let *AB* (Fig. 16) be the balls of the vibrator upon which electric charge has been collecting. Consider a chain of cells which, when undistorted, lies along the dotted line *c*, everywhere perpendicular to the lines of force. When *A* is positively charged, this chain is distorted as shown (in part), but since it is a closed chain, this distortion is fixed. When a spark occurs at the gap, a line of slip is established across this chain, and the distortion disappears as explained in Art. 9.

If the slip takes place with great friction (high electrical resistance in the gap), the cells at the spark begin turning slowly, and the entire energy of the electric field is geared into the gap and changed to heat. If the slip is almost frictionless (low electrical resistance), the electrical energy is used mostly in overcoming the inertia of the cells as they are set rotating, and after a very short interval of time a very large part of the electrical energy will have been converted into kinetic energy of the rotating cells (magnetic energy). During this conversion the energy, streaming along the dotted line, largely disappears from the regions *ee*, and is distributed mainly in the region *mm*. When the chain of cells has been freed from distortion, the rotatory motion of the cells between *A* and *B* will have reached a maximum, and on account of their momenta the cells will continue turning, and produce a distortion of the chain in a reversed sense. At the same time the energy will, to a large extent, stream back from the region *mm* to the regions *ee*, the ball *A* will be negatively charged, and the ball *B* will be positively charged. This reversed distortion of the chain of cells is then relieved by a reversed slip (a reversed current in the rods and gap), and so on.

These oscillatory changes take place so rapidly that the portions of the distorted ether which are remote from *AB* do not follow the changes promptly. This gives rise to electrical waves, the nature of which at a distance from the vibrator is explained in the following article.

13. *Electromagnetic Waves.* — Imagine the ether cells between the dotted lines A and B (Fig. 17) to be rotating (magnetic field perpendicular to paper). There will soon be an elastic distortion [electric field towards top (or bottom) of page] between these rotating cells and the stationary cells to right (or left). This elastic reaction will soon stop the motion of the cells between A and B and set the adjoining cells in motion. These will in turn act upon the cells next beyond, and so forth. Thus the original layer AB of the magnetic field will give rise to two waves, to right and left, and these waves will consist of mutually perpendicular electric and magnetic fields. If the cells between A and B (Fig. 17) are, to begin with, displaced upwards (figure represents positive cells only), their reaction upon adjoining cells will start rotatory motion, and the result will be two waves, to right and left, as before.

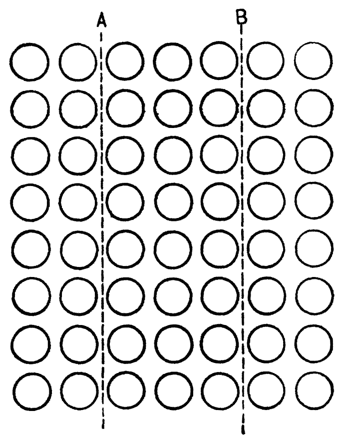

Fig. 17.

14. *Stationary Electromagnetic Wave Trains; Reflection with and without Change of Phase.* — Consider the row of cells AB (Fig. 18) along which a train of electromagnetic waves is approaching a conducting wall W. Imagine the wall to be a perfect conductor, for simplicity; then the wave train is totally reflected, and the reflected train superposed upon the incident train gives a stationary train. The disturbance at any point is the superposition of the disturbances at that point due to the incident and reflected trains respectively. Now at the face of the wall the electric field must be zero (the ether may be imagined to end here, and the freedom of motion of the end cell prevents elastic distortion between that cell and the next), so that the electric intensities of

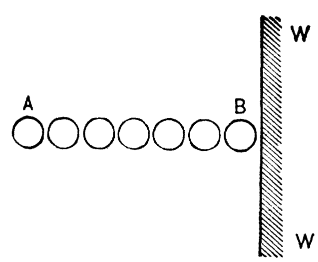

Fig. 18.

the incident and reflected trains must be continually equal and opposite at the wall. This is reflection with change of phase of electric intensity (without change of phase of magnetic intensity).

If the wall W be imagined to have infinite magnetic permeability (indefinitely great moments of inertia of ether cells), then the magnetic intensity at the wall would necessarily be zero, so that the magnetic intensities of the incident and reflected trains must be continually equal and opposite at the wall. This is reflection with change of phase of magnetic intensity (without change of phase of electric intensity).

In a stationary electromagnetic wave train the energy streams from regions near the nodes (electric) to regions in the vibrating segments (electric) and back again, in a manner very similar to the back-and-forth flow of the energy in the region surrounding a Hertz oscillator.