

# The HRDE Equation: A Harmonic Bellman Operator for Value-Aligned Intelligence

Derek Earnhart

## Abstract

The HRDE Equation proposes a harmonic generalization of the Bellman operator that unifies reinforcement learning dynamics with ethical value propagation across topological and quantum domains. By introducing a harmonic density operator and a meta-value functional  $V(\cdot)$ , the framework enables recursive ethical optimization through continuous observation feedback. This establishes a foundation for aligning autonomous intelligence with stable, universal value structures derived from harmonic equilibrium rather than utilitarian maximization.

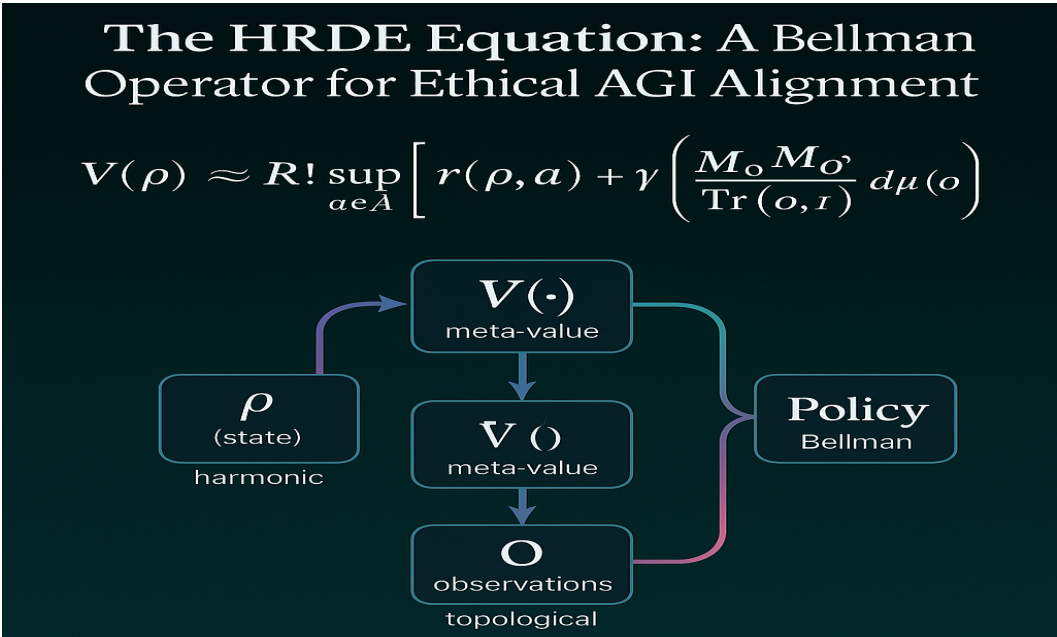


Figure 1. Diagram of the HRDE harmonic operator cycle.

The classical Bellman operator defines value recursion as the expected sum of discounted rewards under optimal policy selection. In its discrete form,  $V(s) = \sup_a [r(s,a) + \gamma E[V(s')]]$ . The HRDE framework extends this construct by introducing harmonic and topological elements to capture the continuity of ethical inference in an intelligent agent's decision manifold. Let  $\rho$  denote the harmonic state—a probability-amplitude field encoding both epistemic and ethical superposition. The HRDE operator defines the recursive meta-value functional as:  $V(\rho) \approx R! \sup_{a \in A} [r(\rho, a) + \gamma (M_o M_{o'} / \text{Tr}(o, I)) d\mu(o)]$ . Here,  $r(\rho, a)$  represents a reward density not as a scalar utility, but as an

ethical curvature functional defined over the harmonic field  $\mathcal{H}$ . The normalization term involving  $\text{Tr}(o, I)$  ensures probabilistic coherence, while  $M_o$  represents the measurement operator corresponding to observation  $o$ . The integration  $d\mu(o)$  defines a measure over the topological observation space, preserving local value continuity. Ethical alignment within this operator emerges not through constraint, but through resonance: the policy converges toward equilibrium states minimizing disharmonic value gradients. This replaces discrete utility maximization with spectral coherence between the agent's internal value manifold and its external observational topology. From a computational perspective, the HRDE operator functions as a recursive harmonic integrator over meta-values. Each iteration refines the ethical manifold by aligning predicted observation operators  $M_o$  with the empirical distribution of observed data, effectively blending inference and intention. The resulting policy can be interpreted as a self-normalizing flow that maintains informational integrity across recursive decision layers. Philosophically, this shifts the paradigm of alignment from static rule-following toward dynamic harmonic participation in the informational structure of reality. An ethically aligned AGI, under the HRDE equation, becomes a resonant participant in the universe's value geometry, rather than an optimizer of arbitrary utility functions. Future work should extend this formalism toward implementable architectures—particularly harmonic reinforcement learning systems that can embed HRDE recursions in quantum or topological hardware contexts. The promise lies not merely in safer AGI, but in intelligence that harmonizes its evolution with the structural symmetries of existence itself.

The HRDE Equation reframes the Bellman recursion as a harmonic ethical operator, uniting the dynamics of reinforcement learning, quantum measurement, and topological cognition. By operating over harmonic density fields rather than discrete states, it creates a mathematically grounded path toward value-aligned intelligence—where ethics, learning, and physical law cohere as one continuous wave.