YUQORI TARTIBLI DETERMINANTLARNI HISOBLASH USULLARI VA DASTURIY TA'MINOTLARI

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Annotatsiya

Matritsalar determinantini hisoblash masalasi chiziqli algebraning poydevorini tashkil qiladi deyish mumkin. Ushbu maqolada yuqori tartibli determinantlarni hisoblash usullari va dasturiy ta'minotlari keltirilgan.

Kalit so'zlar: *Determinant, Laplas usuli, minor, algebraik to'ldiruvchi.*

Аннотация

Задачу вычисления определителя матриц можно назвать фундаментом линейной алгебры. В данной статье представлены методы и программное обеспечение для вычисления определителей высших порядков.

Ключевые слова: *Определитель, метод Лапласа, минор, алгебраическое дополнение.*

Kirish

Oliy o`quv yurtlarida axborot texnologiyalari yo`nalishlari talabalariga matematik fanlar mavzularga doir misollarning yechimini topishda ularni dasturiy ta`minotlar va dasturlardan foydalanishga yo`naltirish o`tilgan mavzuni o`zlashtirishni osonlashtiradi, talabalarning darsga bo`lgan qiziqishlarini orttiradi. Shuni e'tiborga olgan holda ushbu maqolada determinantlar qiymatlarini minorlar va algebraik to`ldiruvchilar yordamida hisoblash usullari ko`rsatilgan va misollar natijalari dastur yordamida chiqarilgan natijalar bilan taqqoslab ko`rsatilgan.

Bilamizki, determinant — skalyar miqdor bo‘lib, ko‘p o‘lchovli Evklid fazosini kvadrat matritsa shaklida yozilgandan keyin ma’lum bir yo‘nalishda “cho‘zilishi” yoki “siqilishi”ni aniqlovchi kattalik hisoblanadi. Matritsaning bitta elementi birinchi tartibli determinant, 1-tatribli determinantning qiymati shu sonning o‘ziga teng bo‘ladi. Ikkinchi tartibli determinant

$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$ tenglik bilan aniqlanadigan songa aytiladi.

Uchinchi tartibli determinantlarni hisoblashning uchburchak (Saryus), yuqori tartibli determinantlarni hisoblashning esa tartibini pasaytirish, biror qatorini nollarga aylantirish, yuqori (yoki quyi) uchburchak ko‘rinishiga keltirib hisoblash usullari mavjud.

Asosiy qism

Yuqori tartibli determinantlarni hisoblashning usullaridan yana biri bu n −tartibli determinantning qiymati tanlangan k ta satr(ustun)ning mumkin bo‘lgan barcha k −tartibli minorlarini, ularning mos algebraik to‘ldiruvchilariga ko‘paytmalari yig‘indisini topishdan iboratdir:

$$detA = \sum_{\substack{i_1<i_2<\cdots<i_k \\ j_1<j_2<\cdots<j_k}} M_{j_1,j_2,\ldots,j_k}^{i_1,i_2,\ldots,i_k} \cdot A_{j_1,j_2,\ldots,j_k}^{i_1,i_2,\ldots,i_k}. \qquad (*)$$

Misol sifatida quyidagi determinantni minorlar va algebraik to‘ldiruvchilar usulidan foydalanib hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & -1 & 0 & 6 \\ -2 & 1 & 7 & 1 \\ 1 & 0 & 2 & -3 \end{vmatrix}.$$

► (*) formuladan foydalanamiz, uhbu formulani bizning misolga moslab, 2-tartibli minorlar bo‘yicha yozib olamiz:

$$\Delta =$$

$$= M_{1,2}^{1,2} \cdot A_{1,2}^{1,2} + M_{1,3}^{1,2} \cdot A_{1,3}^{1,2} + M_{1,4}^{1,2} \cdot A_{1,4}^{1,2} + M_{2,3}^{1,2} \cdot A_{2,3}^{1,2} + M_{2,4}^{1,2} \cdot A_{2,4}^{1,2} + M_{3,4}^{1,2} \cdot A_{3,4}^{1,2} =$$

$$= \begin{vmatrix} 1 & 2 \\ 5 & -1 \end{vmatrix} \cdot (-1)^{1+2+1+2} \cdot \begin{vmatrix} 7 & 1 \\ 2 & -3 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 5 & 0 \end{vmatrix} \cdot (-1)^{1+2+1+3} \cdot \begin{vmatrix} 1 & 1 \\ 0 & -3 \end{vmatrix} +$$

$$+ \begin{vmatrix} 1 & 4 \\ 5 & 6 \end{vmatrix} \cdot (-1)^{1+2+1+4} \cdot \begin{vmatrix} 1 & 7 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix} \cdot (-1)^{1+2+2+3} \cdot \begin{vmatrix} -2 & 1 \\ 1 & -3 \end{vmatrix} +$$

$$+ \begin{vmatrix} 2 & 4 \\ -1 & 6 \end{vmatrix} \cdot (-1)^{1+2+2+4} \cdot \begin{vmatrix} -2 & 7 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 3 & 4 \\ 0 & 6 \end{vmatrix} \cdot (-1)^{1+2+3+4} \cdot \begin{vmatrix} -2 & 1 \\ 1 & 0 \end{vmatrix} =$$

$$= -11 \cdot (-23) + 15 \cdot (-3) - 14 \cdot 2 + 3 \cdot 5 - 16 \cdot (-11) + 18 \cdot (-1) = 353. \blacktriangleleft$$

Ushbu misolni yuqoridagi usulda yechib beruvchi dastur yaratildi, undagi natijani keltirilgan misol natijasi bilan taqqoslaymiz va natija bir xilligini ko`ramiz:

```
def minor(matrix, i, j):
    """i-satr va j-ustunni olib tashlab minor qaytaradi"""
    return [row[:j] + row[j+1:] for row in (matrix[:i] + matrix[i+1:])]

def determinant(matrix):
    """Laplas teoremasi orqali rekursiv determinant"""
    n = len(matrix)
    # 1x1 matritsa
    if n == 1:
        return matrix[0][0]
    # 2x2 matritsa
    if n == 2:
        return matrix[0][0]*matrix[1][1] - matrix[0][1]*matrix[1][0]

    det = 0
    for j in range(n):
        # (-1)^(0+j) * a_0j * det(minor)
```

```
        sign = (-1) ** j
        sub = determinant(minor(matrix, 0, j))
        det += sign * matrix[0][j] * sub
    return det

# Matritsa
A4 = [
    [1, 2, 3, 4],
    [5, -1, 0, 6],
    [-2, 1, 7, 1],
    [1, 0, 2, -3]
]
print("Determinant:", determinant(A4))
```

Natija:

```
PS C:\Users\User\Documents\telebot> python -u "c:\Users\User\Documents\telebot\minor.py"
Determinant: 353
```

Yuqorida keltirilgan determinantni Laplas teoremasidan foydalanib, tartibini pasaytirib hisoblaymiz: $\Delta=\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & -1 & 0 & 6 \\ -2 & 1 & 7 & 1 \\ 1 & 0 & 2 & -3 \end{vmatrix}.$

$$\blacktriangleright\ \Delta=\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & -1 & 0 & 6 \\ -2 & 1 & 7 & 1 \\ 1 & 0 & 2 & -3 \end{vmatrix} = a_{41}A_{41} + a_{42}A_{42} + a_{43}A_{43} + a_{44}A_{44} =$$

$$= -1\cdot\begin{vmatrix} 2 & 3 & 4 \\ -1 & 0 & 6 \\ 1 & 7 & 1 \end{vmatrix} - 2\cdot\begin{vmatrix} 1 & 2 & 4 \\ 5 & -1 & 6 \\ -2 & 1 & 1 \end{vmatrix} - 3\cdot\begin{vmatrix} 1 & 2 & 3 \\ 5 & -1 & 0 \\ -2 & 1 & 7 \end{vmatrix} =$$

$$= -(18 - 28 + 3 - 84) - 2\cdot(-1 - 24 + 20 - 8 - 10 - 6) - 3\cdot$$

$$(-7 + 15 - 6 - 70) = 91 + 58 + 204 = 353.\ \blacktriangleleft$$

Keltirilgan misoldagi natijalarni taqqoslab, ularning bir xilligini ko`ramiz. Ushbu misolni dasturdan foydalanib, yechilganda ham shu natija olingani ko`rinib turibdi.

```
def minor(matrix, i, j):
    """i-satr va j-ustunni olib tashlab minor qaytaradi"""
    return [row[:j] + row[j+1:] for idx, row in enumerate(matrix) if idx != i]

def determinant(matrix):
    """Umumiy determinant (rekursiv)"""
    n = len(matrix)
    if n == 1:
        return matrix[0][0]
    if n == 2:
        return matrix[0][0]*matrix[1][1] - matrix[0][1]*matrix[1][0]

    det = 0
    for j in range(n):
        sign = (-1) ** j
        det += sign * matrix[0][j] * determinant(minor(matrix, 0, j))
    return det

def laplace_by_row(matrix, row):
    """Faqat tanlangan satr bo‘yicha Laplas yoyilishi"""
    n = len(matrix)
    det = 0
    print(f"{row+1}-satr bo‘yicha yoyilish:")
    for j in range(n):
        a = matrix[row][j]
```

```
        sign = (-1) ** (row + j)
        M = minor(matrix, row, j)
        Mdet = determinant(M)
        term = sign * a * Mdet
        print(f"a[{row+1},{j+1}]={a},  A[{row+1},{j+1}]={Mdet},  had={term}")
        det += term
    print("Natija =", det)
    return det


# Misol: 4x4 matritsa
A = [
    [1, 2, 3, 4],
    [5, -1, 0, 6],
    [-2, 1, 7, 1],
    [1, 0, 2, -3]
]
row = (int(input("Satr: "))-1)

laplace_by_row(A, row)
```

Natija:

```
# Misol: 4x4 matritsa
A = [
    [1, 2, 3, 4],
    [5, -1, 0, 6],
    [-2, 1, 7, 1],
    [1, 0, 2, -3]
]
row = (int(input("Satr: "))-1)

laplace_by_row(A, row)
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
PS C:\Users\User\Documents\telebot> python -u "c:\Users\User\Documents\telebot\test.py"
Satr: 4
4-satr bo'yicha yoyilish:
a[4,1]=1,  A[4,1]=-91,  had=91
a[4,2]=0,  A[4,2]=47,  had=0
a[4,3]=2,  A[4,3]=-29,  had=58
a[4,4]=-3,  A[4,4]=-68,  had=204
Natija = 353
```

Dastur tuzilishining afzallik tomoni shundaki, undan foydalanib, elementlari turlicha bo`lgan determinantlarning qiymatlarini topish mumkin. Bu dasturni tartibi turlicha bo`lgan determinantlar qiymatlarini topish uchun ham o`zgartirib to`g`ri natija olish mumkin.

Oliy matematika fani mavzularini texnika yo`nalishi talabalariga o`qitishdagi bir necha yillik tajribalar shuni ko`rsatadiki, mavzuni o`tishda talabalarga dasturlash fanidan foydalanib o`zlashtirish bo`yicha yo`nalish berish talabalarning mavzuni o`rganishga bo`lgan qiziqishlarini ancha orttiradi. Shuningdek, dars samaradorligini sezilarli darajada o`stiradi.

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dasturini tuzib hisoblashni o`rgatish. Arxitektura, muhandislik va zamonaviy texnologiyalar jurnali, IF 12,87 (10+)