

Structure of SO(3) hypothesis mixing matrix

CP-conserving SO(3) parameterization of the neutrino mixing matrix

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SO3 Generators

$$\text{In}[\bullet] := \mathbf{G}_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}; \mathbf{G}_y = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}; \mathbf{G}_z = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

Rotation vector and matrix exponent

$$\text{In}[\bullet] := \mathbf{A} = -\theta_x * \mathbf{G}_x - \theta_y * \mathbf{G}_y - \theta_z * \mathbf{G}_z;$$

A // MatrixForm

$$\text{UExpA} = \text{FullSimplify}[\text{MatrixExp}[\mathbf{A}] /. \{(\theta_x^2 + \theta_y^2 + \theta_z^2) \rightarrow \theta^2, (\theta_x)^2 + (\theta_y)^2 + (\theta_z)^2 > 1\};$$

Out[\bullet] // MatrixForm =

$$\begin{pmatrix} 0 & \theta_z & -\theta_y \\ -\theta_z & 0 & \theta_x \\ \theta_y & -\theta_x & 0 \end{pmatrix}$$

SO(3) mixing matrix

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In[ ]:= US03 = MatrixForm[{{ UExpA[[1, 1]] /.  $\sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2} \rightarrow \theta$ ,
  UExpA[[1, 2]] /.  $\sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2} \rightarrow \theta$ , UExpA[[1, 3]] /.  $\sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2} \rightarrow \theta$ },
 { UExpA[[2, 1]] /.  $\sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2} \rightarrow \theta$ , UExpA[[2, 2]] /.  $\sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2} \rightarrow \theta$ ,
  UExpA[[2, 3]] /.  $\sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2} \rightarrow \theta$ }, { UExpA[[3, 1]] /.  $\sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2} \rightarrow \theta$ ,
  UExpA[[3, 2]] /.  $\sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2} \rightarrow \theta$ , UExpA[[3, 3]] /.  $\sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2} \rightarrow \theta$ }}];
```

US03 // MatrixForm

Out[]//MatrixForm=

$$\begin{pmatrix} \frac{\theta_x^2 + \cos[\theta] (\theta_y^2 + \theta_z^2)}{\theta^2} & \frac{-((-1 + \cos[\theta]) \theta_x \theta_y) + \theta \sin[\theta] \theta_z}{\theta^2} & -\frac{\theta \sin[\theta] \theta_y + (-1 + \cos[\theta]) \theta_x \theta_z}{\theta^2} \\ -\frac{(-1 + \cos[\theta]) \theta_x \theta_y + \theta \sin[\theta] \theta_z}{\theta^2} & \frac{\theta_y^2 + \cos[\theta] (\theta_x^2 + \theta_z^2)}{\theta^2} & \frac{\theta \sin[\theta] \theta_x - (-1 + \cos[\theta]) \theta_y \theta_z}{\theta^2} \\ \frac{\theta \sin[\theta] \theta_y - (-1 + \cos[\theta]) \theta_x \theta_z}{\theta^2} & -\frac{\theta \sin[\theta] \theta_x + (-1 + \cos[\theta]) \theta_y \theta_z}{\theta^2} & \frac{\cos[\theta] (\theta_x^2 + \theta_y^2) + \theta_z^2}{\theta^2} \end{pmatrix}$$