

We start from Eq. B6 of the paper and write $F = \frac{2^{|l|-1}}{|l|!f^2} \tilde{I}_1$, with

$$\begin{aligned} \tilde{I}_1 = \int_0^{\pi/2} d\theta \sin \theta \exp\left(-\frac{\sin^2 \theta}{2f^2}\right) \left(\frac{\sin \theta}{2f}\right)^{2|l|} (\cos^2 \theta + 1) \stackrel{t=\cos \theta}{=} \int_0^1 dt \exp\left(-\frac{1-t^2}{2f^2}\right) \left(\frac{1-t^2}{4f^2}\right)^{|l|} (t^2 + 1) = \\ \exp\left(-\frac{1}{2f^2}\right) \int_0^1 dt \exp\left(\frac{t^2}{2f^2}\right) \left(\frac{1-t^2}{4f^2}\right)^{|l|} (t^2 + 1) \stackrel{u=\frac{t}{f\sqrt{2}}}{=} \\ f\sqrt{2} \exp\left(-\frac{1}{2f^2}\right) \int_0^{\frac{1}{f\sqrt{2}}} du \exp(u^2) \left(\frac{1}{2}\right)^{|l|} \left(\frac{1}{2f^2} - u^2\right)^{|l|} (2f^2 u^2 + 1) \quad (1) \end{aligned}$$

Function **FullIntExpression** expresses F in terms of f and the Dawson function $D_+(\frac{1}{f\sqrt{2}})$. To do it, the polynomial inside the integral in (1) is expanded. Then, for each term of the polynomial, the integral is recursively rewritten by integration by parts. This is what is done by function **ExpandInt**. In particular **ExpandInt** expands

$$\begin{aligned} \exp\left(-\frac{1}{2f^2}\right) \int_0^{\frac{1}{f\sqrt{2}}} u^n e^{u^2} du = \exp\left(-\frac{1}{2f^2}\right) \left(\frac{u^{n-1}}{2} e^{u^2} \Big|_0^{\frac{1}{f\sqrt{2}}} - \frac{n-1}{2} \int_0^{\frac{1}{f\sqrt{2}}} u^{n-2} e^{u^2} du \right) = \\ \frac{1}{2} \left(\frac{1}{f\sqrt{2}} \right)^{n-1} - \frac{n-1}{2} \exp\left(-\frac{1}{2f^2}\right) \int_0^{\frac{1}{f\sqrt{2}}} u^{n-2} e^{u^2} du \quad (2) \end{aligned}$$

and repeats the transformation until the power of u in the integral becomes 0 and the exponential times integral becomes equal to the Dawson function.

After applying **FullIntExpression**, we have a symbolic expression for F , to which we substitute numerical values with **EvaluateInt**.