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Gouy phase in the presence of gas in Fabry-Perot refractometers

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Abstract: When Fabry-Perot (FP) refractometry is used to assess the refractivity of gases, it has so far been assumed that the Gouy phase is independent of the presence of gas in the cavity. Here we show, by both theory and experiments, that this is only correct for a non-deformable cavity. For a deformable one, the pressure can affect the radius of curvature of the mirrors. This gives the Gouy phase a component that is proportional to gas pressure. Although being a small effect (1.6 nrad/Pa), since it affects the Gouy phase only when the cavity contains gas, it affects the refractivity on a 10^{-6} level.

1. Introduction

Fabry-Perot (FP) based refractometry is a technique that can be used for accurate assessment of gas refractivity. With knowledge of the molar polarizability, it is possible to realize (or assess) the molar density of gas and, by also measuring its temperature, its pressure [1–11]. By this, FP refractometry provides possibilities to realize these entities in terms of primary standards [12,13].

In order to do so, it is necessary to utilize a model for their realizations (or assessments) from measured quantities that comprises, in a correct manner, the effect of all phenomena that potentially can affect the realization (or the assessment). One of these is the Gouy phase.

It has so far been assumed that the Gouy phase is independent of the presence of gas in the cavity [14–16]. An overlooked condition for this assumption is though that it is valid only for an ideal (non-deformable) cavity. In a deformable one, the pressure can affect not only the length of the cavity but also the radius of curvature of the mirrors. While the former regularly is taken care of when FP refractometry is performed [9,17–19], the latter has so far not been considered to affect refractivity assessments. In this work, however, we question this assumption. We provide a detailed scrutiny of this and show that the Gouy phase of a Gaussian beam of light locked to a FP-cavity is influenced by the presence of gas and that this can affect assessments of refractivity.

Experimental assessments of the Gouy phase are made on three gases, at a series of pressures. These verify our assumptions and show that the pressure-induced effect on the mirrors, for the system utilized in this study, amounts to 1.6 nrad/Pa. Since this affects the Gouy phase only when the cavity contains gas, it contributes to the refractivity on a 1×10^{-6} level.

Finally, we provide an extended expression for the refractivity that includes the pressure dependence of the Gouy phase. This expression shows that the influence of the pressure-dependent part of the Gouy phase on refractivity is similar to that of distortion of the cavity length and that it is automatically included in the deformation-characterization if the system is characterized experimentally by the two-gas or a similar method [19].

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2. Gaussian beams and their Gouy phases

A Gaussian beam is an idealized description of electromagnetic radiation that primarily propagates in a beam-shape manner in a given direction whose cross-sectional envelope (its electrical field strength as well as its irradiance) has a Gaussian shaped profile. This type of light is considered to be an adequate description of both the propagation of beams of light and the mode structure of longitudinal (i.e. TEM_{00}) modes of resonant cavities.

The mathematical description of a Gaussian beam, which is given by the solution to the paraxial wave equation [20] or the paraxial Helmholtz equation [21,22], has been worked out in some detail and is therefore well described in the literature [20,23–27]. A description valid for the case when such a beam is propagating in the z-direction in a homogeneous medium with an index of refraction of n is summarized in Section 1 in Supplement 1.

2.1. Gouy phase of a freely propagating Gaussian beam

As is stated by Eq. (S4) in Supplement 1, the Gouy phase a Gaussian beam is given by

$$\psi(z) = \arctan\left(\frac{z}{z_R}\right), \tag{1}$$

where z is the distance from the focus and z_R is the Rayleigh range [28], which is given by

$$z_R = \frac{\pi w_0^2}{\lambda} = \frac{n\pi w_0^2}{\lambda_0}, \tag{2}$$

where w_0, λ, n, and λ_0 are the semi-diameter of the beam waist (i.e. the semi-diameter of the beam in the focus), the wavelength of the light in, and the index of refraction of, the medium in which the beam is propagating, and the corresponding vacuum wavelength, respectively.

2.2. Gouy phase of a Gaussian shaped laser beam locked to a longitudinal mode of an ideal (non-deformable) resonant cavity

The locking of a laser field to a mode of a cavity does not only force the frequency of the light to fulfill a phase resonance condition (to keep the wavelength of the light in the cavity virtually independent of the presence of gas), it also forces the radii of curvature of the wavefronts of the light at the positions of the mirrors to be equal to those of the mirrors [23,24]. This latter condition is suitably referred to as the wavefront resonance condition.

As is shown by Eq. (S3) in Supplement 1, the radius of curvature of the wavefront of a Gaussian beam, $R(z)$, is related to the Rayleigh range, z_R, according to [20,23–26]

$$R(z) = z\left[1 + \left(\frac{z_R}{z}\right)^2\right]. \tag{3}$$

This implies that, for a cavity comprising mirrors of which mirror i, placed at a distance z_i from the focus, has a radius of curvature of R_i, (where i is 1 or 2) the wavefront resonance condition forces the Rayleigh range to be given by

$$z_R = \sqrt{z_i(R_i - z_i)}. \tag{4}$$

The two most common types of cavities for FP-based refractometry are the symmetrical concave-concave and the plano-concave cavities. Denoting their lengths L, and the radii of all curved mirrors R, this implies (as is shown by the Eqs. (S8) and (S11) in Supplement 1) that the

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Rayleigh range for these two types of cavity can be written as

$$z_R = \begin{cases} \frac{1}{2}\sqrt{L(2R-L)} \\ \sqrt{L(R-L)} \end{cases}, \tag{5}$$

respectively [23,24].

Since the Gouy phase a light field locked to a longitudinal mode of a resonant cavity picks up during a single passage in the cavity, Θ_G, is given by $\arctan\left(\frac{z_2}{z_R}\right) - \arctan\left(\frac{z_1}{z_R}\right)$, it can, as is shown by the Eqs. (S10) and (S12) in Supplement 1, for the two types of cavity, be expressed as

$$\Theta_G = \begin{cases} \arccos\left(1 - \frac{L}{R}\right) \\ \arctan\sqrt{\frac{L}{R-L}} \end{cases}, \tag{6}$$

respectively.

Since these expressions show that the Gouy phase of laser light addressing a longitudinal mode of a resonant cavity is solely a function of the geometrical properties of the cavity, L and R, they illustrate that, as long as the cavity is non-deformable (i.e. ideal), the Gouy phase is not affected by the presence of gas in the cavity [14,16]. Under such conditions Θ_G is here referred to as Θ_G^0.

However, the situation is not the same for real cavities that are affected by pressure-induced physical deformations.

2.3. Gouy phase of a Gaussian shaped laser beam locked to a longitudinal mode of a real (deformable) resonant cavity

As is illustrated in Fig. 1, pressure-induced cavity deformations can, in general, both alter the length of the cavity, from L to $L + \delta L$, as well as the radius of curvature of the mirrors, from R to $R + \delta R$; the latter by an alteration of the center part of the front facet of the mirrors (their sagittas [29]), from h to $h + \delta h$. In this case, the Gouy phase of the light locked to a symmetrical concave-concave cavity and a plano-concave cavity, then referred to as Θ_G^P, can be expressed as

$$\Theta_G^P = \begin{cases} \arccos\left(1 - \frac{L\left(1+\frac{\delta L}{L}\right)}{R\left(1+\frac{\delta R}{R}\right)}\right) \\ \arctan\sqrt{\frac{L\left(1+\frac{\delta L}{L}\right)}{R\left(1+\frac{\delta R}{R}\right)-L\left(1+\frac{\delta L}{L}\right)}} \end{cases}, \tag{7}$$

respectively.

When gases at atmospheric pressures are addressed, $\delta L/L$ often is in the order of (or below) 10^{-7} (for the system used in this work, $\delta L/LP$ is 0.95×10^{-12} Pa^{-1}). Equation (7) then shows that this gives rise to a relative alteration of the Gouy phase on a similar relative level. Since the Gouy phase itself represents a minor fraction of the phase the light picks up when the cavity is filled with gas, often in the order of 10^{-6}, the change of the Gouy phase due to such an alteration of the cavity length can, in practice, be considered to be insignificant.

It can be estimated, from geometrical considerations and for $R>>h$, that the radius of curvature of the mirrors are related to their sagitta by $R = \frac{d^2}{8h}$, where d is the diameter of the mirror. This implies that the relative alteration of the curvature of the mirrors, $\frac{\delta R}{R}$ is equally large, but of opposite sign, to the relative alteration of the sagitta, i.e. it is equal to $-\frac{\delta h}{h}$. Since this, in turn, is significantly larger (in magnitude) than the relative alteration in length of the cavity, $\frac{\delta R}{R}$ is significantly larger than $\frac{\delta L}{L}$. For the case when the deformation of the cavity is mainly given by those of the mirrors, it can be estimated that $\frac{\delta R}{R}$ is $(-)\frac{4LR}{d^2}$ times larger than $\frac{\delta L}{L}$. FEM-simulations indicate that for our system $\frac{\delta R}{R}$ is about 3×10^3 times larger than $\frac{\delta L}{L}$. This leads to a non-negligible impact on the Gouy phase.

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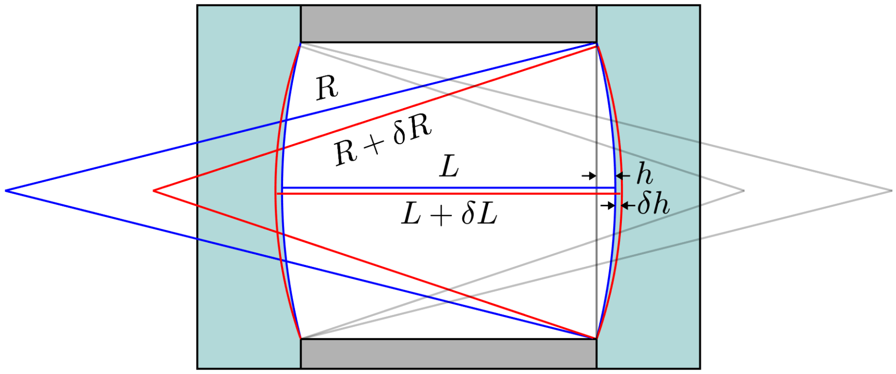

Fig. 1. Schematic illustration of the pressure-induced changes of the length of the cavity and the curvature of the mirrors of a symmetrical concave-concave cavity. The cavity comprises two mirrors that have a radius of R, separated by a distance L. When gas is introduced, the length of the cavity changes to $L + \delta L$ while the sagitta of the mirrors are being altered from h to $h + \delta h$. The latter results in an alteration of the radius of curvature of the mirrors from R to $R + \delta R$. Note that, for a positive δh, δR is negative. Note also that dimensions are not to scale.

To provide an analytical expression for the Gouy phase of a Gaussian shaped laser beam locked to a longitudinal mode of a real (deformable) resonant cavity, denoted Θ_G^P, it is possible to series expand the expressions for the Gouy phase in Eq. (7) for small alterations of the radius of curvature, $\delta R/R<<1$ [30], while neglecting $\delta L/L$, resulting in

$$\Theta_G^P \approx \Theta_G^0 - \frac{1}{2}\frac{L}{z_R}\frac{\delta R}{R} = \Theta_G^0 + \Theta_G' P, \tag{8}$$

where we, in the last step, have assumed that δR is proportional to pressure and introduced Θ_G' as an experimentally assessable gas-independent entity, given by $-\frac{L}{2Rz_R}\frac{\delta R}{P}$ [31]. This indicates that, for a deformable cavity, the Gouy phase should have a pressure dependent component. It moreover indicates that for the situation with a so called closed FP-cavity, i.e. one in which only the pressure inside the cavity is altered when gas is let in, δh is positive. This implies that δR takes a negative value, while Θ_G' is a positive entity. Hence, for this type of system, the Gouy phase increases with pressure.

3. Means to assess the Gouy phase of an FP-based refractometer

3.1. Frequency of longitudinal and transverse cavity modes

To assess the Gouy phase of an FP-based refractometer one can experimentally measure the mode frequencies of a longitudinal and a transversal cavity mode, TEM_{qkm} and TEM_{q00} respectively, since these differ solely by an integer number of Gouy phases [23,32]. As is shown by Eq. (S4) in the Supplement 1 to Silander et al. [14], the frequency of such modes in a physically deformable cavity with distributed Bragg reflection (DBR) coated mirrors, ν_{qkm}^0, can, in the absence of heating of the mirrors by the laser light, be written as

$$\nu_{qkm}^0 = \frac{c\left[q + (1+k+m)\frac{\Theta_G^P}{\pi} + 2\tau_c(n)\nu_c\right]}{2\left[n(L+\delta L) + c\tau_c(n)\right]}, \tag{9}$$

where q, τ_c, and ν_c are the mode number, the group delay a pulse of broadband light experiences upon reflection from DBR coating of the mirrors, and the design frequency of the mirror coating, respectively [14].

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3.2. Means to assess the Gouy phase in terms of cavity mode frequencies

Based on Eq. (9), it is possible to assess the Gouy phase in terms of assessments of the difference in cavity mode frequency between a TEM_{q10} and a TEM_{q00} mode, denoted ν^0_{q10} and ν^0_{q00}, respectively, assessed under similar gas conditions, as

$$\Theta^P_G = \frac{2\pi\left[n(L+\delta L) + c\tau_c(n)\right]}{c}\left(\nu^0_{q10} - \nu^0_{q00}\right). \tag{10}$$

Moreover, using the fact that the free-spectral-range of a set of longitudinal cavity modes, defined as $\nu^0_{(q+1)00} - \nu^0_{q00}$ and denoted $\Delta\nu_{q00}$, is given by $\frac{c}{2[n(L+\delta L)+c\tau_c(n)]}$ implies that the Gouy phase conveniently and accurately can be written as

$$\Theta^P_G = \pi\frac{\nu^0_{q10} - \nu^0_{q00}}{\Delta\nu_{q00}}, \tag{11}$$

which only comprises easily measurable entities.

3.3. Dependence of, and correction for, the heating of mirrors

However, it has recently been shown by Zakrisson et al. that assessments of mode frequencies of FP-cavities can be affected by heating of the mirrors by the laser light [33]. They showed that the cavity mode frequency measured for a given laser power, I, denoted ν^I_{qkm}, can be written as

$$\nu^I_{qkm} = \nu^0_{qkm} + \Delta\nu^I_{qkm}, \tag{12}$$

where $\Delta\nu^I_{qkm}$ is the shift of the cavity frequency due to the heating of the mirrors

They further showed, among other things, that $\Delta\nu^I_{qkm}$ is proportional to the laser power [33]. This implies that it is possible to assess the frequency of the Gaussian beam in the absence of mirror heating, ν^0_{qkm}, in terms of experimentally measured frequencies, ν^I_{qkm}, as

$$\nu^0_{qkm} = \nu^I_{qkm} - \Delta\nu^{2I-I}_{qkm}, \tag{13}$$

where $\Delta\nu^{2I-I}_{qkm}$ is the difference between two assessments, of which one is made with twice the laser power of both the other and that used for the scrutiny of the Gouy phase, i.e. $\nu^{2I}_{qkm} - \nu^I_{qkm}$.

4. Experimental

The experimental setup has been described in detail in a multitude of publications, most recently in Silander et al. [34], but also in references therein. In short, the system consists of an Invar-based Dual FP cavity (DFPC) to which two narrow-linewidth Er-doped fiber-lasers, working at 1550 nm, together with acousto-optic modulators are locked by Pound-Drever-Hall locking [35] by the use of electro-optic modulators with pertinent electrical instrumentation.

The DFPC system comprises a 148 mm long Invar spacer in which two 6 mm diameter cavities have been bored, separated by 25 mm. The mirrors, which have a radius of curvature of 0.5 m, comprise a high-reflectivity coating on a substrate of fused silica that provides a reflectivity of 99.97% at 1550 nm. The mirrors were sealed to the cavity spacer by the use of Al gaskets. Their inner diameter is 6 mm, so as to match that of the cavity bore, while the outer diameter is 12.5 mm. Their inner rim is thicker than the outer in order to create a well defined contact area of 35 mm^2 between the mirror and the spacer. As is shown in the left side of Fig. 2, the mirrors are pressed onto the Al gaskets by the use of PTFT-gasket-equipped back plates. By this, the mirrors will make a leak tight seal with the spacer by deforming the Al gasket rim without physically penetrating the cavity spacer material.

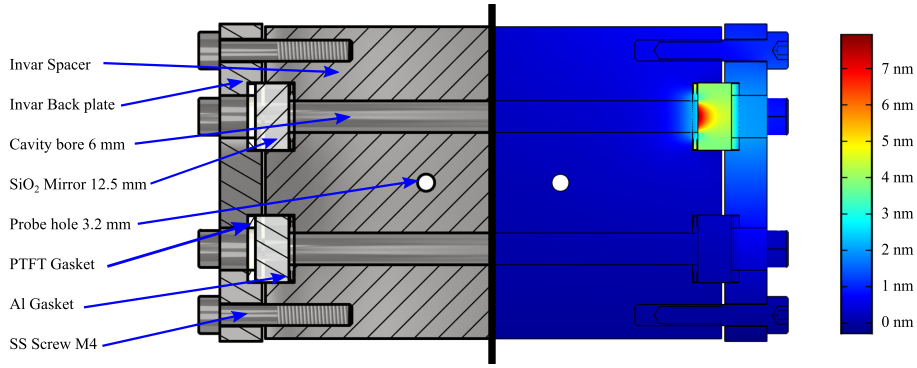

Fig. 2. Illustration of the DFPC system. The left side shows a detailed drawing of the Invar cavity and its parts. The right side shows the result from a FEM simulation of the longitudinal pressure-induced deformation (relative to the center of the cavity) when 100 kPa of gas pressure is let into one of the cavities.

The pressure was regulated by a mass flow controller and an electronic pressure controller and assessed by a CTR 101 N 1000 Torr vacuum gauge. The temperature of the cavity spacer was stabilized by a two-stage stabilization to within a few mK of the Ga-melting point (302.9146 K), while its temperature, and thus the temperature of the gas [36], was assessed by a Pt-100 sensor and a low-noise ohmmeter, which were regularly compared to a gallium-melting point cell.

5. Results

The results from a series of assessments of the Gouy phase following the procedure given by Eqs. (11) and (13) for N_2, Ar, and He are displayed in Fig. (3). For each gas and pressure the Gouy phase was assessed as the mean of around 10 individual assessments.

Panel (a) shows the measurements for N_2, Ar, and He represented by blue, green, and red markers, respectively. Each of the data sets provides a linear dependence on pressure. The slopes of their pressure dependencies, which represent Θ_G', agree well; fits to the data sets [not shown in Fig. (3)], provide, for N_2, Ar, and He, slopes of 1.606(25), 1.576(42), and 1.586(50) nrad/Pa, respectively. Since the three fits agree well within their uncertainty, a common linear fit was applied to all data. The slope of this fit, displayed in the figure by the solid line, thus representing a joint Θ_G' for all gases, was found to be 1.591(24) nrad/Pa. The c_0 parameter in the fit, which represents the gas-independent part of the Gouy phase, Θ_G^0, amounts to 0.7787376(15) rad.

The assessed slope is in line with results of a FEM analysis, which predicts the longitudinal pressure-induced deformation of the mirrors for a pressure of 100 kPa shown in the right side of Fig. (2). A fit to the deformed mirror surface gives a pressure-dependent shift of the Gouy phase of 1.3 nrad/Pa. The c_0 parameter is in line with the nominal gas-independent part of the Gouy phase based on the mirror and cavity geometry, given by Eq. (6), which amounts to 0.79 rad.

Panel (b), which provides the residuals of the fit, shows that, for all gases, they are well centered on the zero deviation line and that all data points overlap within their uncertainties. This implies that the fit is an adequate representation of the responses of all gases and that the Gouy phase indeed has a dependence on pressure.

It is here important to note that although the pressure-induced mirror-deformation-dependence of the Gouy phase only represents a minor fraction of the entire Gouy phase (in our case around 2×10^{-4} at atmospheric pressure) and the Gouy phase by itself only represents a minor fraction of the entire phase experienced by the light when gas is let into the cavity (in our case 1.3 ppm),

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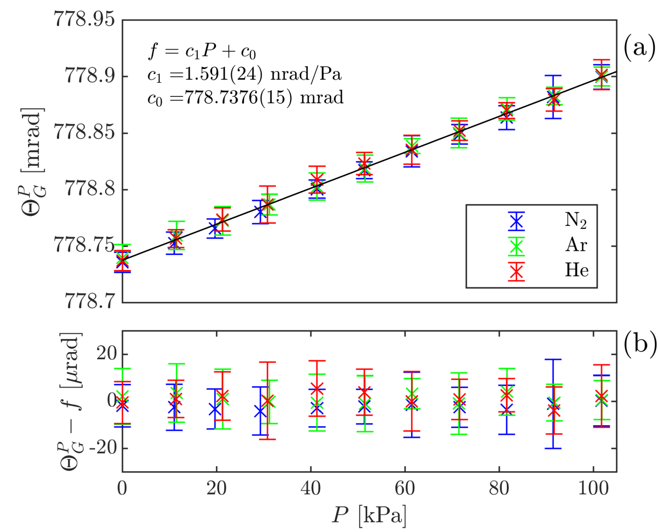

Fig. 3. Panel (a) presents the pressure dependence of the Gouy phase, assessed by the use of Eq. (11), in N_2, Ar, and He, represented by blue, green, and red markers, respectively. The straight line is a linear fit to the data for all gases. Panel (b) displays the residuals of the fit. All error bars represent $k = 2$.

since assessments of refractivity rely on measurements of the difference in refractivity between a gas-filled and an evacuated cavity, it will still influence assessments of refractivity made by a high-precision FP refractometry to a noticeable degree (for N_2 and Ar, on a 10^{-6} level).

6. Phase resonance condition and refractivity in the presence of a pressure-dependent Gouy phase

The fact that the Gouy phase has a component that is pressure-dependent influences the frequency of the light in the presence of gas and thereby also the assessment of refractivity.

6.1. Phase resonance condition in the presence of a pressure-dependent Gouy phase

The resonances of an optical cavity are determined by the condition that the round-trip phase delay of the light is a multiple of 2π. Based on this concept, Silander et al. [14] stated, based on the work by Koks and van Exter [37], an expression for the phase resonance condition for a FP-based system with DBR mirrors, addressing the m^{th} longitudinal mode of a FP cavity (TEM_{00}) [38]. Extending this with respect to the pressure dependent Gouy phase gives [39]

$$2k_{in}(L_0 + \delta L) + 2\phi - 2\Theta_G^0 - 2\Theta_G' P = 2\pi m. \tag{14}$$

6.2. Refractivity in the presence of a pressure-dependent Gouy phase

An expression for refractivity that encompasses the pressure dependent Gouy phase, based on Eq. (14), is derived as the Eqs. (S17) and (S18) in section 3 in Supplement 1

$$\begin{aligned} n - 1 &= \frac{\overline{\Delta\nu'} + \overline{\Delta m'}}{1 - \overline{\Delta\nu'} + \varepsilon_0' - \varepsilon_0^{\delta R} + (n-1)\varepsilon_0' \left[1 + \xi_2(T)\right]} \\ &\approx \frac{\overline{\Delta\nu'} + \overline{\Delta m'}}{1 - \overline{\Delta\nu'} + \varepsilon_0' - \varepsilon_0^{\delta R}}, \end{aligned} \tag{15}$$

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where $\overline{\Delta\nu'}$, given by $\frac{\Delta\nu}{\nu_0'}$ where, in turn, $\nu_0(1+\frac{\gamma_c}{m_0})^{-1}$, and $\overline{\Delta m'}$ is a short hand notation for $\frac{\Delta m}{m_0'}$ where, in turn, m_0' is given by $m_0(1+\frac{\Theta_G^0}{\pi m_0})$, and where γ_c is a phase-shift entity, defined in Section 3 in Supplement 1 and in Silander et al. [14], that, for a real DBR stack, takes a value of $\frac{\tau_c}{\tau_c^{id}}\frac{1}{n_H-n_L}$, where $\frac{\tau_c}{\tau_c^{id}}$ represents the ratio of the real and the ideal group delays, τ_c and τ_c^{id}, respectively, assessed by measurement and the expression for an ideal DFB stack, where τ_c^{id}, in turn, is given by $\frac{n}{n_H-n_L}\frac{1}{2\nu_c}$, and where n_H and n_L are the high and low indices of refraction of the DFB stack of the mirrors, respectively. Hence, γ_c represents an "effective" value of $(n_H-n_L)^{-1}$ that, for an ideal DBR stack, is simply given by $\frac{1}{n_H-n_L}$.

ε_0' represents the leading value of the refractivity-normalized pressure-induced relative change in cavity length, $\frac{1}{n-1}\frac{\delta L}{L_0}$, given by $\zeta\kappa$, where ζ is given by $RT\frac{2}{3A_R}$, where, in turn, A_R is the molar polarizability of the gas and κ is the deformation coefficient of the cavity, while $\xi_2(T)$ denotes a combination of higher order refractivity and density virial coefficients [19]. The second expression in Eq. (15) is valid when the $(n-1)\varepsilon_0'[1+\xi_2(T)]$ term can be neglected, which is particularly the case for N_2 since, for this gas, $|1+\xi_2(T)|<1$.

In both these expressions, $\varepsilon_0^{\delta R}$ is a new term, representing a refractivity-normalized pressure-induced Gouy phase, formally defined as $\frac{1}{n-1}\frac{\Theta_G' P}{\pi m_0}$, in practice given by $\zeta\frac{\Theta_G'}{\pi m_0}$, representing the influence of pressure on the Gouy phase due to an alteration of the curvature of the mirrors.

6.3. Estimate of the relative influence of the pressure-dependent Gouy phase on assessments of refractivity

Since the pressure-induced Gouy phase has the same pressure dependence as the cavity deformation, i.e. a linear pressure dependence, expressing refractivity in terms of a refractivity-normalized Gouy phase entity instead of the entity itself, i.e. $\varepsilon_0^{\delta R}$ instead of $\Theta_G' P$, has the same advantages as expressing the pressure-induced relative cavity length alteration in terms of a refractivity-normalized entity, i.e. ε' or ε_0', instead of a relative cavity length alteration $\frac{\delta L}{L_0}$, primarily no recursivity. This shows that there are two contributions to the frequency shift of a cavity mode in a deformable cavity due to the presence of the gas, one originating from the relative change of the cavity length, $\frac{\delta L}{L}$ while the other stems from the relative alteration in curvature of the mirrors, $\frac{\delta R}{R}$.

It is worth to note that the relative sizes of the two ε-terms ($\varepsilon_0^{\delta R}$ and ε_0') depend on the construction of the cavity (the distortion coefficient of the cavity spacer, the type of mirrors used, and their mounting). For the instrumentation used in this work, it was experimentally found that $\varepsilon_0^{\delta R}$ for N_2 and He amount to 1.00×10^{-6} and 8.5×10^{-6}, respectively [40]. Since the pressure-induced refractivity-normalized relative cavity deformations (i.e. the ε_0') for the same two gases previously have been found to be 362×10^{-6} and 3077×10^{-6}, respectively, this implies that, for the system scrutinized in this work, $\varepsilon_0^{\delta R}$ comprises, in both cases, 0.28% of the total deformation. Since both ε_0' and $\varepsilon_0^{\delta R}$ are inversely proportional to the molar polarizability of the gas, for a given instrumentation, the relation between the two is the same for all gases addressed.

7. Summary and conclusions

This work has scrutinized whether (and if so, to which extent) the Gouy phase of a Gaussian beam locked to a longitudinal mode of an FP-cavity is affected by the presence of gas. It is first concluded that, because of the wavefront resonance condition, which couples the radius of curvature of the wavefront of the light at the positions of the mirrors to the their radii of curvature, the Rayleigh range and the Gouy phase of the light beam are both given solely of physical entities of the cavity, viz. its length and the radii of curvature of the mirrors. This has in recent works led to the conclusion that the Gouy phase is not affected by the presence of gas in the cavity [14,16].

However, we have, in this work, concluded that the latter statement is tacitly based on the condition that the cavity is non-deformable. In a deformable one, the pressure cannot only alter the length of the cavity, it can also affect the radius of curvature of the mirrors, which, in turn, will affect the Gouy phase. It was therefore predicted that the Gouy phase of the cavity, Θ_G, should have two components, one that is independent of the pressure of the gas and one proportional to the gas pressure, i.e. $\Theta_G^0 + \Theta_G' P$.

To verify this, experiments were made with 3 gases, N_2, Ar, and He. For each gas, the Gouy phase was assessed for a set of pressures to which a first order polynomial was fitted. The slopes of the fits, which represent Θ_G', agree within their uncertainties, amounting to 1.591(24) nrad/Pa.

To confirm these statements, FEM simulations of the pressure-induced deformations of mirrors were performed. These provided estimates of the pressure-dependence of the Gouy phase of the cavity used of 1.3 nrad/Pa. Although this deviates from the experimentally assessed value by 20% [41], we consider this to support our assumption that the Gouy phase should have a pressure-dependence originating from a change in mirror curvature radius.

An updated expression for refractivity in the presence of the pressure-induced mirror-deformation-dependent component of the Gouy phase was derived. This shows that the pressure-dependence of the Gouy phase affects refractivity in the same manner (although to a much lesser extent) as pressure-induced alterations of the cavity length, denoted $\varepsilon_0^{\delta R}$ and ε_0', respectively. For the instrumentation used in this work, it was found that, for N_2 an He, for which ε_0' have been assessed to 362×10^{-6} and 3077×10^{-6}, $\varepsilon_0^{\delta R}$ was found to be 1.00×10^{-6} and 8.5×10^{-6}, respectively. Hence, in both cases $\varepsilon_0^{\delta R}$ is 0.28% of ε_0' [42].

This implies that when a characterization of the system is made by the two-gas method [19], the deformation-induced parameter assessed is in fact ε_0' - $\varepsilon_0^{\delta R}$. This implies that, when this method is used, the influence of the pressure-dependent component of the Gouy phase is automatically taken into account (no separate correction for the pressure dependence of the Gouy phase needs to be done). For the case when some other method is used to assess the deformation, separate attention must be paid to the $\varepsilon_0^{\delta R}$ term.

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Supplemental document. See Supplement 1 for supporting content.

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40. For both gases, this $\varepsilon_0^{\delta R}$ distortion corresponds to a shift of the cavity mode addressed of 51 kHz.
41. Possible explanations for the difference between the simulations and experiments comprise, but is not limited to, a small misalignment (of 0.3 mm) between the center of the TEM_{00} mode and the cavity bore and uncertainties in geometrical and material parameters used in the simulations.
42. Since both these two terms are significantly smaller than unity, a series expansions of the expression for refractivity in terms of them, as $(1+\varepsilon)^{-1} \approx (1-\varepsilon)$, shows that they both affect an assessment of the refractivity in a close-to-proportional manner.