Operational Closure, Structure, and the Limits of Mathematical Description of Phenomenal Presence

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Abstract

Many physical frameworks—from classical stochastic modeling to quantum information—exhibit *operational closure*: admissible processes are closed under the operations that generate tests (sequential/parallel composition, conditioning, coarse-graining). This secures *empirical completeness*: everything testable is representable inside the formalism. This paper argues that operational closure also locates a principled boundary: mathematics, being structural, cannot *describe* phenomenal presence (what it is like to see red, feel pain, hear a melody). The argument is neutral with respect to supervenience: even if physically identical systems could not differ in their qualia, descriptive reducibility would still fail. Qualia are not mathematical invariants of an operationally closed theory. The paper clarifies operational closure and the empirical content I_S, works through a qubit example, connects to operational reconstructions (Hardy/GPT), and addresses functionalist and representationalist replies. A distinction is drawn between structural *characterization* and phenomenal *description*; it is argued that first-person givenness is not a structural kind. Practical implications are spelled out: theories like IIT/GWT can yield *constraints and correlates*, but not a mathematical *description* of presence.

1 Introduction and terminology

By *operational closure* is meant: a theory's admissible processes are closed under the operations used to build experiments (composition, monoidal product, outcome-conditioning, coarse-graining). This is *not* algebraic closure in the field-theoretic sense. The central thesis of this paper is that even if physics is operationally (thus empirically) complete, mathematics—as structural description—cannot capture *phenomenal presence*. Physically identical systems will not differ in their qualia, yet no amount of structure suffices to *describe* what it is like.

2 Operational framework and empirical content

This paper models a physical theory as a symmetric monoidal category S with:

- **Objects:** system types; **morphisms:** processes; $\otimes$ encodes parallel composition.
- **Discarding:** for each X, a morphism $!_X : X \to I$ (marginalization/partial trace).
- **Causal morphisms:** those preserving discarding (column-stochastic classically; CPTP in QM).

*Disclaimer: The hypothesis is entirely that of the author. AI systems (ChatGPT and Claude) were used for assistance with drafting and editing. All claims remain the responsibility of the author.

- **Instruments/tests:** finite families $\{E_i : X \to Y\}_{i\in\Omega}$ of causal morphisms with $\sum_i E_i$ causal; outcomes in finite Ω.
- **Operational closure:** closed under composition, tensor, forming instruments, coarse-graining.

An *interface signature* is a pair $(X_{\text{in}}, X_{\text{out}})$. An *agent* is any causal morphism $A : X_{\text{in}} \to X_{\text{out}}$. A *test protocol* is a diagram that yields classical outcomes when A fills the relevant placeholder.

Definition 1 (Empirical profile I_{S})**.** For $A : X_{\text{in}} \to X_{\text{out}}$,

$$I_{\text{S}}(A) \ := \ \Big\{\mathbb{P}_A(T) \in \Delta(\Omega_T) \ : \ T \text{ a test for } (X_{\text{in}}, X_{\text{out}})\Big\},$$

where $\mathbb{P}_A(T)$ is the outcome distribution when A is inserted in T (computed internally in S). Agents are empirically indistinguishable iff $I_{\text{S}}(A) = I_{\text{S}}(B)$.

Remark 1 (Empirically accessible ≡ testable). "Empirically accessible" means obtainable by some admissible test protocol (allowing ancillas/coherent control where the theory does). This matches operational reconstructions (Hardy; Barrett; Janotta–Hinrichsen): states/channels are equivalence classes under identical test statistics.

3 Worked example: a qubit I_{S}

Let S be finite-dimensional quantum theory at interface $(\mathbb{C}, \mathcal{H}_2)$: agents are qubit states ρ. For each POVM $M = \{E_i\}$, $p^\rho(i) = \mathrm{tr}(\rho E_i)$. Then

$$I_{\text{S}}(\rho) = \{\, p^\rho(\,\cdot\,|M) \text{ for all POVMs } M \,\}.$$

Tomography implies $I_{\text{S}}(\rho) = I_{\text{S}}(\sigma)$ iff $\rho = \sigma$. For channels $A : \mathcal{H}_{\text{in}} \to \mathcal{H}_{\text{out}}$, the family $I_{\text{S}}(A)$ over all inputs and POVMs identifies the Choi operator of A. Hence I_{S} captures all testable content.

4 Structure vs presence: characterization and description

5.1 Why presence is not structural

1. **Mode/type distinction.** Structural descriptions individuate *types* via relations; phenomenal presence is a *mode of presentation*—that which is immediately present to a subject.
2. **Presentation vs representation.** Any $Q = F(I_{\text{S}})$ is representational/relational. Presence is *presented*, not represented; this underwrites the explanatory gap.
3. **Token immediacy.** Invariants are class-level; phenomenal character is token-level immediacy in experience. No structural refinement changes that mode of acquaintance.

5.2 Structural characterization does not entail phenomenal description

A complete structural *characterization* picks out the realizer up to isomorphism and fixes counterfactuals; it does not thereby *describe* the felt character. If supervenience holds, it is a metaphysical dependence thesis about *having*; descriptivism remains an epistemic thesis about *showing*. The present argument does not presuppose supervenience; it only notes that even granting it, descriptive reducibility does not follow. Knowing every invariant leaves the "what-it's-like" epistemically open.

5.3 Criteria for modes vs types

To avoid a merely verbal distinction, three criteria are proposed for when X is a *mode* rather than a (complex) *type*:

1. **Acquaintance primacy:** Access to X is essentially via first-person acquaintance rather than concept-mediated description.

2. **Isomorphism invariance:** Any isomorphism of structural realizers that preserves all observables leaves X epistemically underdetermined without further acquaintance.

3. **Representation independence:** For any representational $Q = F(I_S)$, possessing Q-knowledge is neither necessary nor sufficient for possessing X (as a mode of presentation).

Presence satisfies these; standard structural kinds do not.

5 Objection: presence as complex structure

One might claim that presence is a yet-unknown complex structure (temporal dynamics, self-reference, recursion). The reply is:

1. **Internability test.** If presence reduces to structure, it should be capturable as $Q = F(I_S)$ and *stable under admissible contexts* (Def. 2). Then identity claims acquire risky predictions.

2. **Mode mismatch.** Even a perfect Q still *represents*. Presence is *presented*. Collapsing presentation into representation re-states rather than solves the hard problem.

3. **Recursion does not help.** Recursive/self-referential architectures (HOT, higher-order prediction) remain structural. They may be *conditions* for presence; they are not *descriptions* of presence.

6 Representationalism and phenomenal concepts

Strong representationalism. If qualia *just are* representational properties, either (i) these reduce to $F(I_S)$, making presence structural and leaving the explanatory gap (why representation feels like *this*) unexplained; or (ii) they outrun I_S, forfeiting operational grounding.
PANIC/dual-aspect variants. PANIC-style content can be modeled as Q-functionals. They are excellent *correlates/constraints*. Identity requires explaining away the presentation/representation gap. Dual-aspect views may claim a lawful tie without reduction; this framework is compatible with a lawful tie *sans* mathematical description of presence.

7 How operational closure sharpens the boundary

Operational closure implies: every empirically contentful predicate on agents is representable as a functional of I_S.[1] Thus any purported *descriptive* theory of qualia that has empirical bite collapses to structure. Presence is precisely what is *had*, not *encoded*; empirical completeness therefore identifies the boundary rather than dissolving it.

[1] L. Hardy (2001); J. Barrett (2007); P. Janotta & H. Hinrichsen (2014).

8 Meta-problem and reports

Reports/judgments about consciousness live inside I_{S}. Predictive processing, workspace broadcast, and related mechanisms can explain why we say what we say about experience. Progress on the meta-problem is expected and welcome here; it constitutes progress on the *explanatory structure of reports*, not a presentation of presence itself.

9 Operationalizing the internability test

Definition 2 (Admissible contexts and stability)**.** Let $\mathrm{Ctx}_{\mathrm{S}}$ be the monoidal category of *contexts* built from pre-/post-composition, tensoring with ancillas, coherent control, outcome relabeling, and coarse-graining that preserve causality. A candidate Q on agents at a fixed interface is *internable* iff there exists a natural transformation

$$\eta : I_{\mathrm{S}}(\cdot) \Rightarrow Q(\cdot)$$

such that for every context $C \in \mathrm{Ctx}_{\mathrm{S}}$,

$$Q(C[A]) = \phi_C\big(Q(A)\big) \quad \text{for some context action } \phi_C,$$

and, whenever $I_{\mathrm{S}}(A) = I_{\mathrm{S}}(B)$, one has $Q(A) = Q(B)$ (test-preservation).

Operationalization. To evaluate a proposed Q:

1. *Test-preservation:* Construct I_{S}-matched pairs (A, B); check $Q(A) = Q(B)$.
2. *Context-stability:* For generators of $\mathrm{Ctx}_{\mathrm{S}}$ (adapters, ancillas, coherent control, coarse-graining), verify $Q(C[A]) = \phi_C(Q(A))$ with a fixed, interface-independent ϕ_C.
3. *Interface-agnosticity:* Repeat across a principled family of interfaces (to approximate I^*_{S}).

Failure at any step undermines identity claims and reclassifies Q as a correlate/condition.

10 Practical implications and empirical distinguishability

Predictions and guidance.

1. *No qualia meter.* No protocol internal to S outputs "what-it's-like" as such. Instruments return structural correlates only.
2. *Right target.* Theories (IIT/GWT/predictive processing) should present Q-candidates as functions of I_{S} and argue for context-stability; avoid identity claims unless the presentation gap is addressed.
3. *Comparative tests (feasible now).*
    - *In silico twins:* Train two architectures to be I_{S}-matched at a chosen sensorimotor interface (same input–output distributions and internal probe tests). Compare candidate Q (integration/workspace/broadcast) under context manipulations.
    - *Closed-loop neuroemulation:* Human–silicon closed loops that enforce matched stimulus–report and autonomic profiles under perturbations (TMS/tACS/anesthesia ramps). Test whether Q remains a function of observed I_{S} across contexts.
    - *Sedation trajectories:* Within-subject protocols that match I_{S} at adjacent depth levels while pushing Q apart (IIT vs GNW metrics). Divergence indicates $Q \not\subseteq F(I_{\mathrm{S}})$.

4. *Risky predictions for identity theses.* Identity views predict discovery of a test-preserving, context-stable $Q^\star = F(I_\mathrm{S})$ that is necessary and sufficient for all first-person reports across interfaces. This framework predicts failure of sufficiency: $Q^\star$ will remain a correlate/condition.

Remark 2 (Interface dependence and I_S^*). Define an interface-agnostic profile $I_\mathrm{S}^*(A)$ as the union of I_S over a physically realizable basis of interfaces (multi-scale couplings, richer probes). Enlarging tests only *grows* I_S; it does not convert representation into presentation.

11 Classical, quantum, and GPT generality

The argument is agnostic between classical and quantum. Quantum contextuality and entanglement enlarge the test space and refine I_S, strengthening empirical completeness. In GPTs, features like absence of local tomography, higher-order interference, and purification axioms alter the shape of I_S but not the boundary claim: more tests tighten structure; they do not turn representation into presentation.

12 Hard problem (expanded)

This thesis can be read as a sharpening of the hard problem: even granting total success on the *easy problems* and the meta-problem, operational closure entails only structural victory. Progress on the meta-problem is genuine scientific progress (explaining reports/judgments), but not a presentation of presence. Thus, solving the meta-problem neither dissolves nor deepens the gap; it delineates its perimeter.

13 Machine consciousness

This framework is neutral on whether machines can be conscious. It entails only that presence, if it occurs in machines, will not be measured as such by any internal protocol; at best structural *conditions* and *correlates* can be identified. If a machine matches a human's I_S^* across rich interfaces, identity theses predict a $Q^\star$ that is sufficient; this framework predicts that $Q^\star$ remains at best a correlate.

14 Conclusion

Operational closure makes physical theories empirically self-contained. It also locates a principled boundary: mathematics describes *structure*, not *presence*. Even a complete physics, whether or not supervenience ultimately holds, can at best fix structural correlates and conditions under which experience arises. It cannot *describe* what it is like. The constructive upshot is twofold: deepen the structural science of the *conditions* under which presence arises; and acknowledge that presence itself is not a mathematical object but a datum of first-person life.

Appendix A: Category-theory primer

Think processes as *boxes* with typed input/output wires; serial/parallel composition makes new boxes. Discarding is a wire to "trash." An *instrument* is a box whose output splits into labeled classical outcomes. The monoidal product $\otimes$ is "side-by-side" execution. All empirical tests are diagrams built from these operations.

Appendix B: Functorial view of I_{S}

For a fixed interface, let $\mathrm{Ctx}_{\mathrm{S}}$ have tests as objects and admissible post-processings as morphisms. Each agent A induces a functor $I_{\mathrm{S}}(A) : \mathrm{Ctx}_{\mathrm{S}} \to \mathrm{FinStoch}, T \mapsto \mathbb{P}_A(T)$, $h \mapsto h_*$. Thus agents modulo empirical equivalence embed into $\mathrm{Fun}(\mathrm{Ctx}_{\mathrm{S}}, \mathrm{FinStoch})$.

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