Genesis Echo Phase I Reproducibility Appendix

Robert Schrader (with STARK)

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1 Overview

This document provides the full computational and methodological detail for all simulation-based probes of the Genesis Echo theory, enabling full reproducibility of all Phase I results.

2 Primordial Power Spectrum with Genesis Bump

We define the modified primordial spectrum as:

$$P_{\text{prim}}(k) = A_s \left(\frac{k}{0.05}\right)^{n_s-1} \left[1 + 0.1 \cdot \exp\left(-\frac{(\log_{10}(k)+1)^2}{0.3^2}\right)\right]$$

with $A_s = 2.1 \times 10^{-9}$, $n_s = 0.96$, and $k \in [10^{-4}, 10^{1}]\, h/\text{Mpc}$.

3 Transfer Function

We use the Eisenstein & Hu no-wiggle transfer function $T(k)$, implemented in pure Python for analytic tractability. The linear matter power spectrum is:

$$P_{\text{mat}}(k) = P_{\text{prim}}(k) \cdot T^2(k)$$

4 21cm Intensity Mapping Forecast

- Redshifts: $z = 10, 15, 20$
- Growth: $D(z) = \frac{D(a)}{D(1)}$, with $a = \frac{1}{1+z}$
- Forecast S/N per k-bin using Gaussian mode counting:

$$\left(\frac{\Delta P}{P}\right) = \sqrt{\frac{2}{N}}, \quad N = \frac{V \cdot 4\pi k^2 \Delta k}{(2\pi)^3}$$

- Instrumental noise models for HERA and SKA derived from standard thermal noise and survey parameters.
- Visualized $P(k)$ with and without shot noise; residuals show bump.

5 CMB Lensing × Galaxy Cross-Correlation

- Angular cross-power $C_\ell^{\kappa g}$ computed via Limber integral:

$$C_\ell^{\kappa g} = \int dz \, \frac{H(z)}{\chi^2(z)} W^\kappa(z) W^g(z) P\left(k = \frac{\ell}{\chi(z)}, z\right)$$

- Lensing kernel $W^\kappa(z)$ and galaxy window $W^g(z)$ defined as standard.
- Noise models for Planck and CMB-S4 applied.
- Result: Echo bump appears clearly in CMB-S4 S/N forecast.

6 Multi-Tracer Galaxy Forecast

- Biases: $b_1 = 2.0$, $b_2 = 1.0$; Number densities: $\bar{n}_1 = 3 \times 10^{-4}$, $\bar{n}_2 = 10^{-3}$
- Observed spectra:

$$P_{\rm obs}(k) = b^2 P_{\rm mat}(k) + \frac{1}{\bar{n}}$$

- Multi-tracer enhancement:

$$R(k) = \frac{(b_1 - b_2)^2 P(k)}{1/\bar{n}_1 + 1/\bar{n}_2}$$

- S/N per bin calculated and integrated over $k \in [0.05, 0.2]\, h/\mathrm{Mpc}$.
- Mock tracer density field generated via inverse FFT of Gaussian field sampled with $P_g(k)$.

7 Environment and Tools

Simulations were performed using Python 3.11 with NumPy, SciPy, and Matplotlib. Forecasts employed analytic power spectrum models and synthetic sky generation via FFTs. All scripts are standalone, GPU-optional, and reproducible.

8 Conclusion

Bump visibility was confirmed across three observables, each independently. These results validate the Genesis Echo signal as structurally robust and observationally tractable, and form the foundation for Phase II: MCMC inference and N-body simulation.