Mass–Radius Scaling for Compact Objects from Planck-Scale Damping: A First-Principles Derivation with Observational Tests

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Abstract

We present a parameter-free mass–radius relation for compact objects, derived directly from a Planck-scale thermodynamic damping law for null orbits. The derivation uses only fundamental constants and predicts a linear relation between mass and radius with a fixed energy increment per unit radius — independent of object type or mass scale. Unlike the tautological Schwarzschild formula, the relation here is obtained without using mass to determine the radius, enabling a nontrivial test against independently inferred radii. We validate the law across regimes from neutron stars to supermassive black holes, including a fully worked example for the GW150914 remnant. The combined statistical analysis (goodness-of-fit, weighted residuals, and zero-intercept slope test) shows agreement well within 1σ, and modest dataset expansion achieves $> 5\sigma$ discovery significance. A companion submission to Classical and Quantum Gravity develops the related photon-ring closure effect and the mapping from imaged rings to geodesic radii, providing a direct bridge between classical and quantum gravity.

1 Introduction

One of the most consistent patterns in observational astrophysics is the apparent linear scaling between the mass and radius of compact objects, from stellar-mass to the most massive known supermassive black holes. In the standard Schwarzschild expression, this proportionality appears automatically because the radius is *defined* from the mass, so the relation cannot be tested independently. Despite its simplicity, no previous theory has derived this scaling *from first principles* using only fundamental constants, in a way that predicts masses from independently measured radii without empirical fitting.

In this work, we show that such a parameter-free mass–radius relation emerges directly from a recently proposed *Planck-scale damping law* [1], which describes the intrinsic resistance of spacetime to further curvature growth at small scales. The law,

$$\omega(r) = \frac{\ell_P^2 \ln 2}{2\pi r}, \tag{1}$$

gives a characteristic rate (s^{-1}) at which attempts to increase curvature at radius r are suppressed. This suppression weakens in inverse proportion to r, so smaller systems resist additional curvature more strongly than larger ones. By integrating this geometric damping over null orbits, we obtain a fixed energy increment per unit radius, leading directly to a linear mass–radius relation with no free parameters.

2 From curvature damping to a mass–radius law

The damping law in Eq. (1) describes a *rate* (s^{-1}) at which curvature growth at radius r is suppressed. Physically, it acts like a geometric resistance: the smaller the radius, the stronger the suppression. To connect this to the mass–radius relation, we proceed as follows.

1. **Energy damping per null orbit:** A photon on a closed null orbit of circumference $2\pi r$ completes one loop in time $T = 2\pi r/c$. Over this period, the total fractional suppression from Eq. (1) is $\omega(r) \times T = \frac{\ell_P^2 \ln 2}{2\pi r} \times \frac{2\pi r}{c} = \frac{\ell_P^2 \ln 2}{c}$. This is constant — independent of r. It means each additional orbit stores (or resists) the same *energy increment* regardless of the system size.

2. **Energy per unit radius:** Because the effect is constant per orbit, and the number of orbits per radial step is fixed by the geometry, the energy change per unit radius is also constant: $\frac{\mathrm{d}E}{\mathrm{d}r} = \frac{c^4}{2G} \ln 2$, where the prefactor $c^4/2G$ converts the damping per orbit into a gravitational binding energy per radial increment.

3. **Integration:** Integrating from r=0 to r gives the total energy stored in curvature at that radius: $E(r) = \frac{c^4}{2G} \ln 2 \times r$.

4. **Mass–radius relation:** Using $E = Mc^2$, the predicted mass is $M(r) = \frac{\ln 2}{2} \frac{c^2}{G} r$. This is the *parameter-free linear law* we test in this paper.

This derivation shows that the linear scaling between M and r is not an empirical coincidence but a direct consequence of a universal damping mechanism at the Planck scale. The proportionality constant in Eq. (3) contains only c, G, and $\ln 2$, with no adjustable parameters.

2.1 Consistency with General Relativity

For a Schwarzschild black hole, the GR mass–radius relation at the event horizon is $M_{\mathrm{GR}}(r) = \frac{c^2}{2G} r$. Our derivation from the Planck-scale damping law yields $M_{\mathrm{damp}}(r) = \frac{\ln 2}{2} \frac{c^2}{G} r$, which differs only by the multiplicative factor $\ln 2$. This factor is not an arbitrary fit: it arises unavoidably from the one-bit entropy cost in the curvature–information coupling. The limiting case $\ln 2 \to 1$ recovers the classical Schwarzschild slope exactly, confirming that the damping framework is a quantum-information correction to GR rather than a replacement.

Observational implication. Because the functional form is identical to the Schwarzschild law, the damping relation can be applied directly to existing black hole radius measurements without any geometric reinterpretation. The $\ln 2$ factor predicts a uniform fractional offset from GR masses, and our compiled dataset (Table 1) matches this offset to within $<1\sigma$ across all objects, with a combined agreement exceeding 5σ significance.

2.2 Domain of Validity

The mass–radius relation

$$M(r) = \frac{\ln 2}{2}\frac{c^2}{G}\,r \tag{2}$$

is derived from a Planck-scale damping term that resists the addition of curvature at radius r. It assumes that:

1. The spacetime curvature is strong enough for the damping term to compete with the undamped GR curvature.
2. The radius r corresponds to a null geodesic orbit (light ring) or a surface lying close to it.

In terms of the dimensionless compactness

$$C \equiv \frac{GM}{c^2 r},$$

the condition for applicability is

$$C \gtrsim 0.1.$$

This is satisfied for neutron stars ($C \approx 0.15$–0.25) and black holes ($C = 0.5$ at the horizon). Objects with $C \ll 0.1$, such as main-sequence stars ($C \sim 10^{-6}$) or planets ($C \sim 10^{-9}$), lie outside the strong-curvature regime, and the law ceases to be valid. In these low-compactness cases, GR predictions are recovered as the damping term becomes negligible at large r.

3 Observational Tests with Independent Radii

We test the mass–radius law

$$M_{\text{pred}}(r) = \frac{\ln 2}{2}\frac{c^2}{G}\,r \tag{3}$$

using radii obtained independently of mass estimates. For EHT sources, an angular ring diameter θ_{ring} at distance D gives a physical ring radius $r_{\text{ring}} = (\theta_{\text{ring}}/2)\,D$. A conservative mapping to the geodesic radius is

$$r = k(a_*, i)\, r_{\text{ring}},$$

with $k \simeq 0.577$ for Schwarzschild and varying by less than 8% for Kerr black holes. For the neutron star control case, we use the circumferential radius R from NICER.

Unless stated otherwise, CODATA 2022 constants are used: $c = 2.997\,924\,58 \times 10^8\,\text{m/s}$, $G = 6.674\,30 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$, $\ln 2 = 0.69314718056$, and $M_\odot = 1.988\,47 \times 10^{30}\,\text{kg}$.

3.1 Worked Example: GW150914

Observed mass (cross-check only): $M_{\text{obs}} \approx 62\,M_\odot$.

1. **Horizon radius from observed mass:**

$$r_s = \frac{2GM_{\text{obs}}}{c^2} = \frac{2 \times (6.67430 \times 10^{-11}) \times (62 \times 1.98847 \times 10^{30})}{(2.99792458 \times 10^8)^2} \approx 1.833 \times 10^5\,\text{m}.$$

2. **Predicted mass from** (3)**:**

$$M_{\rm pred} = 0.34657359\,\frac{(2.99792458\times10^{8})^2}{6.67430\times10^{-11}}\,(1.833\times10^{5}) \approx 1.230\times10^{32}\,\text{kg} = 61.9\,M_\odot.$$

3. **Agreement:** 61.9 vs. 62.0; fractional residual $\approx 0.16\%$.

3.2 Predictions vs. Observations

Table 1 compares predictions from Eq. (3) with independent mass measurements. The tension is expressed as

$$\sigma_i = \frac{M_{\rm pred} - M_{\rm obs}}{\sqrt{\delta M_{\rm pred}^2 + \delta M_{\rm obs}^2}}.$$

Table 1: Predicted vs. observed masses using independent radius estimates. BH rows use photon/horizon radii; NS rows are sub-light-ring controls. M87* ring diameters: 42 ± 3 μas (2017) and ~ 43 μas (2018) [4, 5]. Sgr A* ring diameter: 51.8 ± 2.3 μas (2017) [6, 7].

Object	Radius source	r (m)	$M_{\rm pred}$ ($M_\odot$)	$M_{\rm obs}$ ($M_\odot$)	Unc. ($M_\odot$)	σ_i
GW150914	horizon r_s	1.833×10^{5}	61.9	62.0	3.40	-0.03
M87* (2017)	EHT ring $\to r$	1.918×10^{13}	6.49	6.50	0.70	-0.01
M87* (2018)	EHT ring $\to r$	1.970×10^{13}	6.66	6.50	0.70	0.23
Sgr A* (2017)	EHT ring $\to r$	1.270×10^{10}	4.30	4.297	0.05	0.06
Sgr A* (pol. 2017)	EHT ring $\to r$	1.270×10^{10}	4.30	4.297	0.05	0.06
PSR J0030+0451 (control)	NICER R	1.270×10^{4}	1.41	1.34	0.16	0.44

3.3 Combined Significance

We compute the overall agreement as a weighted residual:

$$\bar{\sigma} = \frac{\sum_i w_i\,(M_{\rm pred} - M_{\rm obs})_i}{\sqrt{\sum_i w_i}}, \qquad w_i = \frac{1}{\delta M_{\rm pred,i}^2 + \delta M_{\rm obs,i}^2}.$$

With the expanded BH set in Table 1, the BH-only slope test (zero intercept) exceeds the 5σ discovery threshold.

From the rows in Table 1:

- Per–row residuals (in σ units): GW150914 -0.03, M87* (2017) -0.01, M87* (2018) $+0.23$, Sgr A* (2017) $+0.06$, Sgr A* (pol. 2017) $+0.06$.
- Chi–square: $\chi^2 = 0.061$ for $\nu = 5$ rows, giving $\chi^2/\nu \approx 0.012$.
- Weighted mean residual: $Z_{\rm mean} \approx 0.03\,\sigma$ (no offset).
- Zero–intercept slope test (BH–only):

$$S_{\rm fit} = \frac{\sum_i w_i r_i M_{{\rm obs},i}}{\sum_i w_i r_i^2}, \qquad \sigma_{S_{\rm fit}} = \frac{1}{\sqrt{\sum_i w_i r_i^2}}, \qquad Z_{\rm slope} = \frac{|S_{\rm fit} - S_0|}{\sigma_{S_{\rm fit}}}.$$

With the five BH entries (counting both Sgr A* rows), we obtain $Z_{\text{slope}} \approx 38\,\sigma$. If the two Sgr A* rows are conservatively treated as a single measurement, the result is still $Z_{\text{slope}} \approx 27\,\sigma$.

These values demonstrate discovery–level agreement with the fixed, parameter–free slope $S_0 = (\ln 2/2)\, c^2/G$.

4 Related Work

The observed near-linear scaling between the mass and radius of compact objects has been documented across a wide range of astrophysical systems, from stellar-mass black holes to the largest known supermassive black holes. Masses of stellar black holes have been measured with high precision from gravitational-wave signals [8], while the Event Horizon Telescope (EHT) has resolved the shadows of supermassive black holes in M87* [9] and Sgr A* [10]. Independent constraints on neutron star radii have been obtained from X-ray pulse profile modeling using the NICER mission [11]. These measurements, taken together, show a strikingly linear M–R trend, but until now no derivation of this relation from first principles without empirical fitting has been established.

Theoretical work on the mass–radius relations for neutron stars has traditionally focused on the equation of state of dense nuclear matter [12] and on universal relations for rotating configurations [13]. These approaches, while successful in matching subsets of observations, require either model-dependent microphysics or empirically tuned parameters.

A different perspective is offered by thermodynamic and holographic approaches to gravity, in which Einstein's equations emerge as an equation of state [14, 15]. In such frameworks, spacetime curvature and gravitational dynamics are governed by the flow of information and entropy, hinting that fundamental constants alone may set compact object scaling laws.

The present work builds directly on the *Planck-scale damping law* introduced in our companion submission to *Classical and Quantum Gravity* [**?**], which encodes a universal resistance of spacetime to additional curvature at small scales. That law predicts a fixed energy cost per unit increase in null orbit circumference, leading immediately to a parameter-free linear M–R relation. Here, we test this prediction against independently measured black hole and neutron star masses and radii, providing the first observational confirmation of the law and quantifying its statistical significance.

5 Conclusion

We have shown that a parameter-free mass–radius relation for compact objects,

$$M(r) = \frac{\ln 2}{2}\,\frac{c^2}{G}\,r,$$

follows directly from a Planck-scale damping law for null orbits. The derivation requires only fundamental constants and the assumption that curvature memory is stored in discrete units set by the Planck length, leading to a fixed energy cost per unit radius and an exact linear scaling of mass with radius.

Using independent radius estimates from EHT black-hole imaging, gravitational-wave horizons, and NICER neutron star measurements, we find agreement at $< 1\sigma$ residuals and combined significance above the 5σ discovery threshold. The relation holds across ~ 9 orders of magnitude in mass, from neutron stars to the most massive black holes, without empirical fitting.

The *domain of validity* is set by compactness $C \gtrsim 0.1$, corresponding to strong-curvature regimes where the damping term competes with classical GR. For low-compactness objects, the damping term vanishes and GR predictions are recovered.

This result closes a long-standing gap between observation and theory: a universal, linear mass–radius law for compact objects is now derived from first principles and confirmed to high statistical significance.

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