Review
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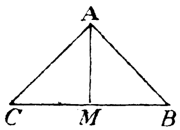
Source: *The Mathematical Gazette*, Vol. 2, No. 39 (May, 1903), pp. 290-292
Published by: Mathematical Association
Stable URL: http://www.jstor.org/stable/3603560
Accessed: 02-03-2016 11:16 UTC

Ex. 2. $a=45{\cdot}2$, $b=32{\cdot}9$, $c=15{\cdot}4$.



	No.	Log.
b	32·9	
c	15·4	
$b+c$	48·3	1·6839
$b-c$	17·5	1·2430
b^2-c^2		2·9269
$CM+BM=a$	45·2	1·6551
$CM-BM$	18·70	1·2718
$2CM$	63·90	
$2BM$	26·50	

	No.	Log.
CM	31·95	1·5045
$CA(=b)$	32·9	1·5172
$\cos C$		$\bar{1}$·9873

$\therefore\ C=13^\circ\ 47'$

	No.	Log.
BM	13·25	1·1222
$BA(=c)$	15·4	1·1875
$\cos B$		$\bar{1}$·9347

$B=30^\circ\ 38'$

Finally, $A=180^\circ-13^\circ\,47'-30^\circ\,38'=135^\circ\,35'$.

The solutions of these two examples by conventional methods would be as follows:

Ex. 1.

	No.	Log.
a	684	
b	504	
$a-b$	180	2·2553
$a+b$	1188	3·0748
		$\bar{1}$·1805
$\cot\frac{1}{2}C$	$\cot 47^\circ\,8'$	9·9676
$\tan\frac{1}{2}(A-B)$		9·1481
$\frac{1}{2}(A-B)$	$=\ 8^\circ\ 0'$	
$\frac{1}{2}(A+B)$	$=\ 42^\circ\,52'$	
A	$=\ 50^\circ\,52'$	
B	$=\ 34^\circ\,52'$	
$a+b$		3·0748
$\sin\frac{1}{2}C$		9·8651
		12·9399
$\cos\frac{1}{2}(A-B)$		9·9958
c	879·2	2·9441

Ex. 2.

	No.	Log.
a	45·2	
b	32·9	
c	15·4	
	2) 93·5	
s	46·75	1·6693
$s-a$	1·55	0·1903
$s-b$	13·85	1·1415
$s-c$	31·35	1·4962
$(s-a)\div s$		$\bar{2}$·5305
$(s-c)\div(s-b)$		·3557
$\tan^2\frac{1}{2}B$		$\bar{2}$·8862
$\tan^2\frac{1}{2}C$		$\bar{2}$·1748
$\tan\frac{1}{2}B$	$\tan 15^\circ\,30'$	9·4437
$\tan\frac{1}{2}C$	$\tan\ 6^\circ\,58'$	9·0874
B	$31^\circ\ 0'$	
C	$13^\circ\,56'$	

Finally, $A=180^\circ-31^\circ\,0'-13^\circ\,56'=135^\circ\,4'$.

A comparison of the alternative methods shows that there is no appreciable saving of labour in the use of the elaborate formulae commonly taught, and that by the use of the simpler methods here suggested the subject may be taught to beginners at an earlier stage than has been hitherto customary.

G. H. Bryan.

REVIEWS.

A Course of Modern Analysis. By E. T. Whittaker, M.A. Pp. xvi 378. (Cambridge University Press. 1902.)

This work is sure of a favourable reception because it gives in a moderate compass an attractive account of some of the most valuable and interesting results of recent analysis. The first part deals mainly with infinite series, especially power-series and Fourier expansions: it also includes the elements of complex integration and of the theory of residues. The second part comprises chapters on the gamma-function, Legendre functions, the hypergeometric series, Bessel functions, the equations of mathematical physics, and elliptic functions.

The author has secured brevity by adopting, on the whole, a deductive method of exposition. The advantages of this are shown very clearly in the chapter on the gamma-function. By taking Weierstrass's definition of $\Gamma(z)$ as an infinite product, the properties of the function are deduced with extreme simplicity, and although the historical order of development is almost reversed, we feel that the new treatment is really the true and natural one. The gamma-function is, in fact, the simplest case of a transcendent function with an infinite number of simple poles, and this property is made obvious by the new definition. (It is curious, by the bye, to see how very nearly Weierstrass was anticipated in this matter by Gauss.) Again, the new method has suggested interesting extensions to Alexeiewsky, Barnes, and others, so that the change of view has amply justified itself.

On the other hand, Mr. Whittaker's chapter on the hypergeometric series seems to me to illustrate the risk of becoming artificial, which is incurred by an author who sacrifices induction overmuch. Here, for instance, we have Papperitz's form of the differential equation satisfied by the most general form of Riemann's P-function set down without any explanation of the method by which it is obtained. For all the reader can tell, it might have dropped from the skies, or have been written on a wall by some spirit from another world. It has not entered into the plan of the treatise to give any general account of linear differential equations: but in this context it would have been possible to bring out the essential fact that the equation defines a function with three critical points, each associated with two indices. The suppression of this fundamental property obscures the real meaning of much that follows, and gives an air of hocus-pocus to the demonstrations. The principal vice of Cambridge methods and Cambridge text-books has always been the encouragement of students to take their mathematics ready-made, to assimilate facts without inquiring into their sources. By some of us, at least, it was hoped that the institution of Part II. of the Mathematical Tripos would help to counteract this tendency. Alas! the conscientious ingenuity of tutors and lecturers, aiming at immediate results, is likely to defeat the main purpose of the innovation, and produce a state of things worse than that under the old régime.

But to return to Mr. Whittaker's book. One very good feature is that the relations of the functions of Legendre, Bessel, etc., to the general hypergeometric function have been brought out. If Chaps. x., xi. had been transposed this might perhaps have been done even more effectively. Such generalisations as this are the direct result of recent function-theory, and help to lighten the burden with which the progress of mathematics appears to threaten us. Apart from special subtleties, which appeal to the adept, the general tendency of function-theory is towards simplicity and clearness. There is no reason, for instance, why a school-boy should not learn the proper expression for $\sin x$ as an infinite product, or the elementary theory of power-series for a complex variable. New ideas are not difficult simply because they are new: if there is something which I never heard of until I was fifty, it does not follow that no one else is to hear of it before reaching the same age. And why persist in putting before boys and girls ideas which are positively erroneous, and methods entirely out of date?

This same gain of simplicity is illustrated by Mr. Whittaker's chapters on elliptic functions. It is not sufficiently shown why the ratio of the periods should be non-real; and, personally, I should prefer the omission of proofs of the addition-theorem which are not straightforward applications of Abel's method. But with these exceptions the discussion, as far as it goes, is very clear; and apart from the theories of transformation and complex multiplication (the omission of which is natural enough), the reader will find all the facts he is likely to require. One reflection suggested by reading this and other parts of the book is that the reputation of Cauchy and Liouville are likely to be enhanced as time goes on. It seems to me that even yet their contributions to science are insufficiently recognised. In this connection it may be remarked that Mr. Whittaker gives, besides Dirichlet's proof of the Fourier expansions, Cauchy's second demonstration (published in 1827) with some necessary amendments.

In some respects, I think, this treatise improves as it goes on. The first two or three chapters are neither so agreeably nor so accurately written as those which follow, and there is an occasional want of proportion. For instance (p. 12) "Series whose convergence *is due to* the convergence of the series formed by the moduli of their terms . . . are called absolutely convergent series" (the italics are mine): again, it is assumed (p. 13) that u^{n+1} tends to zero for $n=\infty$ when $|u|<1$, although facts quite as obvious, or more so, are proved in detail; on p. 19 the statement

"As n increases, v_{n+1}/v_n will therefore tend to the limit
$1-(1+\frac{1}{2}c)/n$"

is not satisfactory; while the second part of Art. 6 seems to me to obscure a very simple matter. Finally, I confess that I cannot follow all the argument of Arts. 84, 85: this is probably my own fault.

Of casual slips and misprints there are apparently few. It is, of course, wrong to say that $(z-1)!$ is a polynomial (p. 209); on p. 24 $(u+p)$ should be $(n+p)$: p. 46 for "excluded" read "included"; p. 201 for "Alexerewsky" read "Alexeiewsky."

G. B. MATHEWS.

Practical Exercises in Geometry. By W. D. EGGAR, M.B. Macmillan & Co.

A New Geometry for Beginners. By R. ROBERTS, B.Sc. Blackie & Son.

Geometry. By S. O. ANDREW, M.A. John Murray.

Elements of Geometry. By R. LACHLAN, Sc.D. and W. C. FLETCHER, M.A. Edward Arnold.

Elementary Geometry. By W. M. BAKER, M.A. and A. A. BOURNE, M.A. G. Bell & Sons.

Plane Geometry. By T. PETCH, B.A. Edward Arnold.

Theoretical Geometry for Beginners. By C. H. ALLCOCK. Macmillan & Co.

Works on Geometry, whose appearance is due to alterations in the regulations of various examining bodies, continue to issue from the press. They may be divided roughly into two classes—(1) those which devote themselves to a course of Experimental Geometry, intended to precede and prepare for the future course of Deductive Geometry; (2) those intended to supply that future course, and replace the editions of Euclid's Elements, modernised or otherwise, now current in the schools.

There seems more immediate need of the former, the experimental courses throwing a great strain on teachers who have to deal with many other subjects besides Geometry, and have neither the time nor the inclination to develop a systematic course of experimental work. Excellent as are the little works by Mault, Bert, and Spencer, something more is wanted, and we welcome heartily Mr. Eggar's book as one that should supply a widely-felt want. It can be used both by those who are preparing their pupils for, and by those who are taking their pupils through, a deductive course. We hope it will be used largely by teachers of both categories, for we hold that experimental methods should *accompany*, as well as *precede*, deduction. How much vividness a theorem in loci gains, for instance, by such experimental treatment as that given on pp. 170, 186! The book is well got up; the figures are effectively drawn on an ample scale. We see everywhere signs of a teacher whose heart is in his work, and whose efforts, we feel assured, will help to kindle enthusiasm, not only among those who are under his direct personal influence, but among those also who are trained after the model he has set.

Mr. Roberts' work is of an intermediate character. Though without special experience of their needs, we imagine it excellently suited for technical schools. It is a rapid course through the essentials of Geometry. Theorems and problems seem judiciously chosen, and calculated to interest the student and lead to further studies and applications. We can best explain how condensed the scheme of treatment is by the statement that out of the 87 pages of which it consists, 5 are devoted to graphs and curve tracing, and that in the other 82 the author treats the subject matter of the ordinary school Euclid both theoretically and practically. The diagrams are numerous and well drawn; the five pages on curve tracing are