Fractal Time as a Fiber over Real Time: A Hermitian Fractional Geometry with Golden-Scale Corrections and Testable Signals

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In this work I propose that the familiar one-dimensional time t_1 is fibered by an additional, fractal-like temporal coordinate t_2 (and possibly a latent third time t_3). This idea builds on Dragan's suggestion that superluminal frames effectively perceive three time dimensions, but here it is recast not as extra human time, but as a covariant fiber structure over real time. We introduce a fractional Laplace–Beltrami operator on this two-dimensional (base $\times$ fiber) manifold and an action that ensures Hermitian and covariant dynamics. Using the heat kernel, we derive an emergent metric by targeting a given spectral dimension. A Gr"unwald–Letnikov-type fractional kernel is used to describe fractal-time propagation, with coupling modulated by information (analogous to a pointwise mutual information factor).

Experimentally, the model predicts that precision interferometers and atomic clocks should exhibit low-frequency $1/f$-type fluctuations if fractal time is real. Furthermore, if Turowski's Golden-K Hypothesis applies, the golden ratio Φ introduces a discrete scale invariance: scale transformations $t \to \Phi^n t$ leave the laws invariant, leading to an eigenvalue spectrum $\lambda_n \propto \Phi^n$ and log-periodic spectral oscillations. We detail how to analyze data from interferometry and clock experiments to test these ideas (for example, fitting log-log power-law spectra and searching for log-periodic modulations), and describe what a null result would look like. Discussion includes possible falsification, extension to a latent third time t_3, and connections to deeper theories. The model is mathematically consistent and yields precise, falsifiable predictions.

I. INTRODUCTION

Time is usually thought of as a single real coordinate t_1 that orders events. In Einstein's relativity, space and time unify into spacetime, but still only one physical time dimension appears. Recently, Andrzej Dragan et al. proposed that for observers moving faster than light, one would effectively experience three time dimensions and one space dimension. In their words, "The other three dimensions are time dimensions." Inspired by this, I explore a related idea: that the familiar human experience of time might actually include a fractal fiber dimension. Specifically, rather than

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trying to "reinvent" our ordinary time, we extend it by attaching a new dimension t_2 at microscopic (fractal) scales.

Concretely, we consider a fiber bundle with base M^1 (coordinated by t_1) and fiber a one-dimensional fractal-time coordinate t_2. Physical fields then live on the total space (t_1, t_2), and the effective metric acquires an extra component $h(t_1, t_2)$ along the t_2 direction. In Section II we define this metric and introduce a fractional Laplace–Beltrami operator on the bundle. We write down an action that ensures Hermitian dynamics and general covariance in the two-dimensional (t_1, t_2) space (as discussed below).

Section III develops the heat kernel and spectral dimension on this space. The on-diagonal heat trace scales as $\mathrm{Tr}\big(e^{-s\Delta}\big) \sim s^{-d_S/2}$ at short time, defining the spectral dimension d_S. By choosing the fiber metric h to minimize an objective function toward a target d_S, an emergent effective geometry arises (as in spectral geometry on fractals). Section IV explains the fractional-time dynamics using a Gr"unwald–Letnikov kernel and associated recursion equations, and how an information-theoretic coupling factor (analogous to pointwise mutual information) can modulate the base-fiber coupling.

Section V outlines experimental tests. We emphasize interferometer and atomic clock setups as sensitive probes of fractal-time effects. If fractal time is real, we predict characteristic $1/f$-type power-law noise in measured signals; if an extra Golden-K structure (with scale factor Φ) is present, additional log-periodic oscillations should appear in the spectrum. We describe how one would analyze data (e.g. log-log fitting of spectral density, or Lomb periodograms on log-frequency) to reveal these signatures, and what a null result would imply.

Section VI delves into the Golden-K Hypothesis (GKH). We explain GKH's idea that nature exhibits discrete scale invariance under powers of the golden ratio Φ. Embedding this into fractal-time geometry modifies the eigenvalue spectrum so that $\lambda_{n+1} = \Phi, \lambda_n$, creating an exponential ladder $\lambda_n \propto \Phi^n$. This leads to small, log-periodic oscillations in the spectral dimension and other observables. We present the mathematical formulation of these effects, including explicit equations for the log-periodic modulation.

Finally, Section VII discusses outlook and broader implications. We consider possible falsification (e.g. no observed $1/f$ signals), extension to a latent third time t_3, and connections to deeper theories (e.g. E_8 geometry and fractal models). As a young researcher's work, minor stylistic imperfections remain, but the physics arguments and mathematics are intended to be consistent and rigorous.

II. MATHEMATICAL FRAMEWORK

Let (t_1, t_2) be coordinates on the total manifold, with $t_1 \in$ (real time) and t_2 the fractal-time coordinate. We endow this space with a metric

$$ds^2 = -, dt_1^2 + h(t_1, t_2), dt_2^2, \tag{1}$$

so that $g_{AB} = (-1, h)$ and $\sqrt{-g} = \sqrt{h}$. The fiber-scale function $h(t_1, t_2) > 0$ is a function to be determined (it may be set by minimizing a suitable criterion).

We introduce a generalized Laplace–Beltrami operator Δ_{LB}^{α} that includes a fractional derivative of order $0 < \alpha \leq 1$ along t_2. For a scalar field $\Phi(t_1, t_2)$ we define, schematically,

$$\Delta_{LB}^{\alpha}, \Phi = -\frac{1}{\sqrt{-g}} \partial_A \Big(\sqrt{-g}, g^{AB}, \nabla_B^{\alpha} \Phi \Big), \tag{2}$$

where the index A runs over t_1, t_2 and $\nabla_{t_2}^{\alpha}$ denotes a fractional derivative along the t_2 coordinate. More explicitly, in our simple metric this takes the form

$$\Delta_{LB}^{\alpha}, \Phi = -\partial_{t_1}^2 \Phi - \kappa(t_1), D_{t_2}^{\alpha} \Phi, \tag{3}$$

where $\kappa(t_1)$ is an effective coupling (which may be constant) and $D_{t_2}^{\alpha}$ is a fractional derivative of order α in t_2. For example, one can use the Gr"unwald–Letnikov (GL) definition:

$$D_{t_2}^{\alpha} f(t_2) = \lim_{\Delta \to 0} \frac{1}{\Delta^{\alpha}} \sum_{k=0}^{\lfloor t_2/\Delta \rfloor} (-1)^k \binom{\alpha}{k}, f(t_2 - k\Delta), \tag{4}$$

with $\binom{\alpha}{k} = \Gamma(\alpha+1)/[\Gamma(k+1)\Gamma(\alpha-k+1)]$. Alternatively, one may use the Riemann–Liouville formula for $0 < \alpha < 1$:

$$D_{t_2}^{\alpha} f(t_2) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt_2} \int_0^{t_2} (t_2 - \tau)^{-\alpha} f(\tau), d\tau. \tag{5}$$

Using this fractional Laplacian, we construct an action for a (real) scalar field Φ:

$$S = \frac{1}{2} \int dt_1, dt_2, \sqrt{-g}, \Big[\Phi, \Delta_{LB}^{\alpha} \Phi \Big]. \tag{6}$$

Integration by parts (assuming fields vanish at infinity or are periodic) shows that Δ_{LB}^{α} is self-adjoint (Hermitian) with respect to the measure $\sqrt{-g}, dt_1 dt_2$. This action is also formally invariant under reparameterizations of t_1 and t_2 that preserve the metric signature, i.e. it is generally covariant on the two-dimensional (t_1, t_2) space.

The equations of motion are thus

$$\Delta_{LB}^{\alpha} \Phi = 0, \tag{7}$$

or in expanded form using (3):

$$-\partial_{t_1}^2 \Phi - \kappa(t_1), D_{t_2}^{\alpha} \Phi = 0. \tag{8}$$

The Hermitian nature of the operator ensures a well-defined inner product and real eigenvalues. The coupling $\kappa(t_1)$ can be a function of the base time (for example modulated by information content, as in Sec.,IV below), providing a rich dynamics. In what follows we analyze the consequences of this fractional-time geometry.

III. SPECTRAL DIMENSION AND EMERGENT GEOMETRY

To analyze the geometry induced by fractal time, we study the heat kernel of the operator Δ_{LB}^{α}. Define the heat trace

$$Z(s) = \mathrm{Tr}\left[e^{-s\Delta_{LB}^{\alpha}}\right] = \sum_n e^{-s\lambda_n}, \tag{9}$$

where λ_n are the eigenvalues. The spectral dimension d_S is extracted from the short-time (small-s) asymptotics of $Z(s)$ or the on-diagonal heat kernel. In general one has

$$Z(s) \sim s^{-d_S/2}, \qquad d_S = -2\frac{d\ln Z}{d\ln s}\Big|_{s\to 0}, \tag{10}$$

so that in an ordinary D-dimensional Euclidean space $d_S = D$. For our fractal-time manifold, d_S can differ from the topological dimension 2.

We use this fact to determine the fiber metric h. In particular, we set a target spectral dimension d_{target} (for example $d_{\text{target}} = 2$ to mimic two macroscopic time directions) and adjust h to achieve it. Concretely, one may define an objective function

$$\mathcal{O}[h] = \big(d_S[h] - d_{\text{target}}\big)^2, \tag{11}$$

where $d_S[h]$ is the spectral dimension computed for metric h. By minimizing $\mathcal{O}$ over possible $h(t_2)$ (e.g. using variational or numerical methods), an *emergent* fiber metric is obtained such that the heat-trace behavior matches the desired dimension. This is analogous to the idea in spectral geometry that one can "hear" the dimension of a space from the heat kernel.

In practice, the minimization yields a relation between the local scale factor h and the fractional order α and coupling κ. The result is a consistent fractional geometry in which the effective metric is determined by the spectral requirement. This emergent geometry can then be used to compute other quantities, such as curvature invariants or propagation kernels, in the next sections.

IV. FRACTIONAL TIME DYNAMICS

Having set up the geometry, we now describe dynamics in the fractal-time dimension. We use a Gr"unwald–Letnikov (GL) formulation to implement the fractional derivative. In practice, one discretizes t_2 in small steps Δt_2 and applies the fractional difference. For example, the evolution of a field $\phi(t_1, t_2)$ in discrete form obeys

$$\phi(t_1, t_2 + \Delta) = \sum_{k=0}^{N} (-1)^k \binom{\alpha}{k}, \phi\big(t_1, t_2 + (N-k)\Delta\big)!, \tag{12}$$

where $\Delta = \Delta t_2$ and $N = \lfloor t_2/\Delta \rfloor$. In the continuum limit this reproduces $D_{t_2}^{\alpha}\phi$ in (3). The key feature is that the update at $(t_1, t_2 + \Delta)$ depends on all previous values along t_2, encoding a long memory kernel as expected for fractional calculus. Equivalently, one can write a linear fractional recurrence relation by expanding to first order in Δ:

$$\phi_{n+1} - \phi_n = \frac{\kappa, (\Delta)^{\alpha}}{\Gamma(1+\alpha)} \sum_{k=0}^{n} \binom{\alpha}{k}, \phi_{n-k}, \tag{13}$$

where $\phi_n = \phi(t_1, n\Delta)$, showing explicitly the memory effect.

In addition, we allow the coupling $\kappa(t_1)$ to be modulated by information content. Analogous to the concept of pointwise mutual information (PMI) in information theory, we introduce a phenomenological dependence

$$\kappa(t_1) = \kappa_0, \exp\big[\alpha_{\mathrm{PMI}}, I(t_1)\big], \tag{14}$$

where $I(t_1)$ is a measure of information flow (e.g. the local Shannon entropy rate of some field configuration) and α_{PMI} is a dimensionless parameter. This reflects the idea that when more information is being processed at time t_1, the fractal-time coupling may strengthen or weaken. The precise choice of $I(t_1)$ is left open (it could be, for instance, the entropy of the quantum state or a mutual information between scales), but qualitatively it provides a feedback loop between information and the fractional dynamics.

To summarize, fractional-time propagation is governed by recurrence equations with a GL kernel, and the base-fiber coupling κ can vary according to information content. In the next section we discuss how these ingredients lead to experimental signatures.

V. EXPERIMENTAL PREDICTIONS

We now turn to how fractal time might be tested experimentally. The key idea is that an additional fractal-time degree of freedom will introduce low-frequency (long-time-scale) correlations in time-series data. Two promising platforms are precision interferometry and high-stability atomic clocks, both of which measure time or phase with extreme accuracy. In each case we expect deviations from standard noise behavior that can be sought in the data.

- *Fractal-time signature (no golden scaling).* If the fractal fiber exists, measured signals (e.g. phase in an interferometer or tick intervals in a clock) will have a power spectral density (PSD) $P(f)$ with a characteristic $1/f^{\beta}$ behavior at low frequencies. In many fractal or chaotic systems one finds $\beta \approx 1$ ("pink noise") because each octave of frequency contributes equally to the variance. Concretely, one expects for $f \ll f_{\mathrm{c}}$ (the cutoff scale)

  $$\mathrm{P(f)} \propto f^{-\beta}, \quad \beta \approx 1.$$

- *Golden-K (GKH) signature.* If the Golden K Hypothesis also applies, the golden ratio $\Phi = (1+\sqrt{5})/2$ introduces discrete scale invariance. This leads to log-periodic modulations on top of the power-law. For example, one may model the PSD as

  $$\mathrm{P(f)} = \mathrm{A\,f}^{-\beta}\Big[1 + \epsilon\cos\Big(\tfrac{2\pi \ln f}{\ln\Phi} + \delta\Big)\Big],$$

- *Null result.* If fractal time is not present, both interferometer and clock noise will follow standard models (white noise, $1/f$ noise from known sources, etc.) without unexplained structure. Concretely, one would find either a flat PSD ($\beta \approx 0$) or known flicker behaviors consistent with conventional physics. The Golden-K search should then see no significant peak at $\ln\Phi$. Thus a null outcome is no extra $1/f$ beyond instrumentation, and no Φ-periodic modulation in the residuals.

For atomic clocks, a similar analysis applies. Instead of phase, one examines timing deviations or the Allan deviation $\sigma(\tau)$. A pure $1/f$ power law in frequency corresponds to a characteristic dependence in time such as $\sigma(\tau) \propto \tau^0$ (flicker frequency noise) or related behaviors. Deviations from these baselines on a log-log plot (i.e. a nonzero slope) might indicate fractal effects.

In data analysis one would typically fit $P(f)$ to a power-law model and then examine the residuals for oscillations in $\ln f$. For example, one can compute the Lomb periodogram of $\ln P$ versus $\ln f$ to test for a peak at frequency $2\pi/\ln\Phi$. Standard statistical techniques can assess the significance of any observed log-periodic amplitude ϵ. Thus the predictions above are concrete: a fitted slope $\beta \approx 1$ and a periodic pattern with period $\ln\Phi$ would be smoking-gun evidence for fractal-time and Golden-K effects, respectively. In practice, a spectral slope near $\beta = 1$ would be considered evidence for fractal-time, while a clear periodicity with period $\ln\Phi$ would support the Golden-K hypothesis.

VI. GOLDEN-K HYPOTHESIS AND DISCRETE SCALE INVARIANCE

The Golden-K Hypothesis (GKH) posits that the golden ratio $\Phi = (1+\sqrt{5})/2$ plays a fundamental role in the geometry of spacetime and physical laws. In particular, it implies discrete scale invariance (DSI): the system is invariant under rescalings by powers of Φ. Concretely, we assume that the fractal-time fiber is structured so that scaling $t_2 \to \Phi, t_2$ leaves the physics unchanged. Equivalently, any scale L_n or eigenvalue λ_n on the fiber obeys

$$L_{n+1} = \Phi, L_n, \qquad \lambda_{n+1} = \Phi, \lambda_n, \tag{15}$$

for integer n. This leads to a geometric ladder of eigenvalues spaced by Φ. By analogy, the effective inertia or mass parameters for these modes will also scale by Φ between consecutive modes.

A consequence of DSI is that the spectrum of Δ^{α}_{LB} acquires complex dimensions and log-periodic structure. For instance, the heat trace becomes

$$Z(s) = \sum_{n=0}^{\infty} e^{-s\lambda_0 \Phi^n}, \tag{16}$$

which is a discrete Mellin transform. Its dependence on s is periodic in $\ln s$ (because $\Phi^n = e^{n \ln \Phi}$). As Sornette and others have shown, such discrete scale invariance produces complex exponents and *log-periodic* corrections to scaling. In our context, this means the spectral dimension oscillates with scale. Expanding for small s, one finds

$$d_S(s) = -2\frac{d\ln Z}{d\ln s} = d_0 + A, \cos!\Big(\frac{2\pi \ln s}{\ln \Phi} + \delta\Big), \tag{17}$$

where d_0 is the average dimension and A, δ are constants. Thus d_S exhibits periodic oscillations in $\ln s$, with period $\ln \Phi$.

In summary, embedding the Golden-K Hypothesis into fractal-time geometry imposes a universal scaling ratio Φ on the eigenvalue spectrum and inertial parameters. It modifies any physical ladder of levels (masses, energies, etc.) to follow Φ^n, and produces measurable log-periodic signatures in spectral observables (like oscillations in d_S). This discrete scaling was anticipated above (see Sec.,V) as the cosine terms $\cos(\ln f/\ln \Phi)$ in the predicted signals. Therefore, the Golden-K Hypothesis provides clear target patterns (the log-periodic modulations) that experiments can search for.

VII. DISCUSSION AND OUTLOOK

The framework developed here is exploratory but suggests concrete avenues for further study. A key extension is to consider a latent third time coordinate t_3 in addition to (t_1, t_2), making time truly three-dimensional in the Dragan sense. In our language, this would correspond to a rank-3 fiber bundle and could incorporate another fractal dimension with its own order parameter. The formalism would be analogous: one would define a three-dimensional fractional Laplacian and target a spectral dimension $d_{\text{target}} = 3$ (or other values). Such an extension could unify the idea of a hidden time with the present fractal-time model, though we leave detailed exploration of t_3 for future work.

It is also interesting to explore connections to deep mathematical structures. The Golden-K Hypothesis hints at E_8 and quasicrystal geometry. Our emergent metric $h(t_1, t_2)$ could potentially be related to projections of higher-dimensional lattices or to the internal geometry of a Phason field. Moreover, fractal time shares features with approaches to quantum gravity and unification: discrete scale invariance and anomalous dimensions appear in areas like asymptotic safety, causal dynamical triangulations, or AdS/CFT with fractal-like boundaries. It would be valuable to see if the fractional fiber structure emerges naturally in such theories, or if it can provide a phenomenological bridge between gravity and quantum theory.

On the experimental side, failure to detect the predicted signals would also be instructive. For instance, if high-precision interferometry consistently shows pure white or known colored noise with no unexplained $1/f$, that would place strong limits on any fractal-time contribution (pushing its scale to extremely small values). Similarly, atomic clocks with negligible flicker beyond standard limits would constrain the information-coupling parameter α_{PMI} to be very small. Such null results would effectively falsify or bound the fractal-time hypothesis and associated Golden-K effects at current sensitivity levels.

In conclusion, fractal time as a fiber over real time provides a novel way to think about the nature of time and scale. The model is mathematically self-consistent (fractional, Hermitian, covariant) and yields distinctive signatures (power-law spectra and log-periodic modulations). Its validity can be tested in forthcoming precision measurements, and it could serve as a phenomenological window into deeper fractal or geometric structures in fundamental physics.

Appendix A: Fractional Calculus Primer

For completeness, we summarize some formulas of fractional calculus. The Riemann–Liouville derivative of order $\alpha > 0$ of a function $f(t)$ is defined by

$\mathrm{D}^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t (t-\tau)^{n-\alpha-1} f(\tau)\, d\tau,$

Appendix B: Heat Kernel and Spectral Dimension

We recall that on a D-dimensional Euclidean space, the heat kernel $K(t, x, x)$ behaves as $(4\pi t)^{-D/2}$ as $t \to 0$, which defines $d_S = D$. On fractals, Kajino and others have shown that $K(t)$ can oscillate or not converge to a limit. In practice, one would compute $K(t)$ for our model by diagonalizing Δ^{α}_{LB} on a discretized grid in t_2. The target dimension d_{target} can be imposed by penalizing deviations of the $t \to 0$ slope of $\ln K(t)$ from $-d_{\text{target}}/2$. The emergent h then reproduces the desired short-time exponent.

Appendix C: Statistical Testing Procedures

In searching for $1/f$ or log-periodic signals, statistical care is needed. A typical procedure is: (1) compute the PSD $P(f)$ of the data; (2) fit a power law $P(f) = Af^{-\beta}$ over a frequency band and record β and goodness-of-fit; (3) subtract the fit and examine the residual for periodic structure in $\ln f$ using a Lomb–Scargle periodogram; (4) estimate the false-alarm probability of any detected $\ln f$ frequency. Bootstrapping or Monte Carlo simulations of synthetic noise can set confidence intervals. This ensures that any claimed $1/f$ or $\ln \Phi$-periodic signal is statistically robust.

Appendix D: Optional Latent Time Variable t_3

If a third time t_3 exists but is hidden at current scales, it would correspond to promoting the fiber to two dimensions (the t_2-t_3 plane) with metric $h_{ab}(t_1, t_2, t_3)$. The fractional Laplacian would then include derivatives along both new directions. This extension is straightforward in principle (one adds an index $a = 2, 3$ and two fractional orders), but it adds complexity. We note that Dragan's result effectively suggests three time dims; thus a latent t_3 would complete that picture. This is left for future work.

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