Prob(e1)Prob(e2) = Prob(e3)Prob(e4) and Quantum Spin Part 2

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In Part I, we argued that one may create a dynamic probability which ensures that an elastic collision of two particles has equal probability for any (ei, ej) pair with the same sum and any (pxi,pxj) pair, also with the same sum (and the same condition for py and pz). This suggests a power law, i.e. number power ei or number power px or exp(e), exp(px). We then argued that probability should be a scalar and should not depend on the direction of the x-axis. This means that px and -px should have the same probability so exp(px) must be changed to a function with px multiplied by a linear measure which compensates for a change in the direction of the x axis. Furthermore, there is no equilibrium of free particles (like in a Maxwell-Boltzmann gas) and so we proposed exp(i px delta x), i.e. a complex probability. There is no real weight to adjust for fre particles . To be more general, one may consider an argument which is invariant to frame changes (Lorentz transforms) and this leads to exp(-iEt+ip dot r). In other words, we created a dynamic (imaginary) probability to allow from a frame independent probability which ensures conservation of energy and momentum with no distinguishability between (ei,ej) or (pxi, pxj) pairs with the same sum.

Noether's theorem also deals with conservation, but argues that there should be a symmetry in the action (Integral Lagrangian dt). In particular, (1) shows that dL/d variable partial = the conserved quantity because the action does not contain x. If this notion is linked to the above ideas, then -Et+px should be linked to Noether's theorem. One may show that -Et+px is in fact the action Lt in both nonrelativistic and relativistic cases if v=x/t. In this case, i d/dt partial and -id/dx partial make use of the symmetry variable which then brings out the conserved quantity, which is a little different from dL/dv=p.

We then focused on operators (matrices) which take the place of basis vectors for the vector p and more generally for the 4-vector (pc,E). These matrices act on vectors, but should play the role of d/dt partial or d/dx partial if in fact there is a conserved quantity linked with spin. In particular, we tried to show that spin should be linked to the exp(ip1x)exp(ip2x) =exp(ip3x)exp(ip4x) statistical property ((1)) if p1+p2=p3+p4. We note that the "spin" operators have the same group theoretic algebra as those of angular momentum and argued that there was a connection with exp(i m phi), where m hbar is the Lz value. We try here to argue why there should be a conserved quantity and thus why ((1)) should also be linked with spin.

Noether's Theorem

According to (1), Noether's theorem is linked with the classical action:

Integral L dt where L is the Lagrangian ((2))

In particular, in the nonrelativistic case:

Action = .5m vv t because L(x,v,t) ((3))

The action has no x present and so there is an invariance in x. This leads to a conserved quantity, namely p = dL/dv partial.

We try to see if this theory may be somehow linked to the ideas of a dynamical probability discussed in Part 1.

Dynamical Probability

In Part 1 we argued that in Newtonian mechanics, one may consider two particles colliding elastically. In such a case,

E1+e2 = e3 + e4 ((4a)) and p1x + p2x = p3x+p4x etc ((4b))

We argued that if all ei, ej pairs with the same sum and all pxi, pxj pairs with the same sum (and the same condition for py,pz) represent the same weight, the the probability should be linked to:

Exp (i ei) or exp(i px) etc ((5))

We use a complex argument "i" because one does not have a distribution of energy like in the Maxwell-Boltzmann case. One only wants to capture the condition of equal product probability for ei,ej or pxi,pxj etc pairs with the same sum.

The problem we have is that the sign of px depends on the direction of the x-axis which should not be the case for a probability. Thus, we suggested introducing delta x to obtain:

exp(i px delta x) ((6))

We then argued that ((6)) should be invariant when seen from different frames (special relativity) and generalized to:

exp(-iEt+ip dot r) ((7))

The question is: How is ((7) linked with Noether's theorem. We first ask: Should it be linked to Noether's theorem in the first place? We argue that it should because one deals with conserved quantities. Here, however, one does not have a Lagrangian as in the Noether theory case.

We note, however, that -Et+px is the classical action both relativistically and nonrelativistically if one uses v=x/t:

Action = .5m vv t = .5m xx/t = .5 px = .5 Et (where E=.5mvv)

Thus: -Et+px = -px.5 or negative the action ((8))

In the relativistic case: Action = -mo sqrt(1-vv/cc) t

Now -Et+px = -mocc t/sqrt(1-vv/cc) + movx/sqart(1-vv/cc) = -mo(t - vx) /sqrt(1-vv) for c=1 t-vx= t(1-vv) so -Et+px = -mot sqrt(1-vv) ((9))

Thus, the dynamical probability created is linked with the action of Noether's theorem, but here the invariant variables t,x are used in operators id/dt partial, -id/dx partial to obtain the conserved quantities E and px.

Noether's action seems to be linked to invariance because one cannot argue for conservation differing with one frame (versus another moving at constant speed). Thus, the action must be linked to invariance as must the dynamical probability. The dynamical probability shows the invariant variable and has the form:

d/d invariant variable exp(i conserved quantity invariant variable) = conserved quantity exp(i conserved quantity invariant variable) ((10))

These ideas, however, apply to E and p, but what about spin?

Spin?

We argued in Part I, that if the px, py, pz values (numbers) are obtained by using -id/dx, -id/dy and -id/dz acting on the dynamical probability, then the basis vectors of the p vector (ex,ey,ex) should be operators acting of a function or vector). Dirac and Pauli showed that these are linked to Pauli matrices, sx, sy,sz which have certain commutator relations shown in Part I.

At this point, there is no notion of any symmetry or conserved quantity. In Part I, we argued that the rxp angular momentum operator (which exists without any notion of spin) has the same commutator relations as the spin operator. Again, this does not point to a symmetry or conserved quantity. Our goal is then to see why a symmetry should appear together with a conserved quantity.

We note that conservation of momentum is a concept which exists in Newtonian mechanics and is directly linked to uniformity/symmetry in space (i.e. no potential). Similarly, angular momentum conservation also exists in Newtonian mechanics and is linked (for a spinning record) to a phi variable with a L pointing in a vertical direction which is conserved.
We suggest that one may map this to a conserved Lz value, i.e. create an eigenstate of Ldot L and Lz because they commute.

Then one has:

Lz |l,m> = m |l,m> ((11))

The problem is that ((11)) is abstract. One needs an operator d/d symmetry variable and an exp(i symmetry variable conserved quantity). From the spinning record case, the symmetry variable is phi and the conserved quantity is hbar m, the eigenvalue of Lz. To see this result formally, one may calculate rxp explicitly in spherical co-ordinates as argued in (2).

Angular momentum operators have the same algebra (commutation relations) as spin operators, and at the same time Lz is linked with exp(i m phi), i.e. the statistical condition of exp(ipx) and exp(-iEt). We thus suggest that a similar feature must exist for spin which may also have an eigenstate of |s,m>, i.e. m being conserved. One does not have the simple 2*3.14 symmetry of phi in the spin case, but we argue that the basic ideas of a conserved quantity which hold for exp(i m phi) should hold for spin because the algebras of spin and angular momentum are the same.

Conclusion

In conclusion, in Part I, we tried to argue that spin has the same exp(ip1x)exp(ip2x)=exp(ip3x)exp(ip4x) feature as momentum (and energy). We did this by showing that commutation relations for the spin operators are the same as those for the angular momentum operator rxp. Here, we try to link these ideas to Noether's theorem and show explicitly how the conserved variable and quantity Lz appear in the angular momentum case. We do not have the same exact feature for spin, but we argue that the eigenvalue of sz should be conserved and be linked with symmetry and should distinguish between different z projections in a product if no further constraints are imposed. (Note: Forcing an eigenvalue s dot s is an extra constraint which can affect weights as in Clebsch-Gordon coefficients), but here we are only interested in the conserved quantity, i.e the sz eigenavlue. We thus argue that spin is linked with the same statistical feature found for E and p and for ei in the Maxwell-Boltzmann distribution, except that here one deals with different probability functions (i.e wavefunctions, spinors).

References

1. https://en.wikipedia.org/wiki/Noether%27s_theorem
2. https://en.wikipedia.org/wiki/Angular_momentum_operator