Plato's Quantum Cave, Entanglement And Classical Physics

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We demonstrate that quantum entanglement emerges naturally from the fundamental Relator condition $R\omega = c$—linking wavefunction evolution speed, classical conservation laws, and the standard Schrödinger equation. By introducing an internal generator space ($\mathbb{C}$-space) orthogonal to conventional spatial dimensions ($\mathbb{R}^3$-space), the Relator framework offers an explicit and unified explanation of the conceptual gap between classical and quantum physics. Within this formulation, particle interactions generate a shared internal–frequency component and establish resonant coupling in $\mathbb{C}$-space, providing a concrete geometric mechanism for the origin of entanglement.

I. INTRODUCTION

Quantum physics is our most successful description of nature's how, yet it remains silent on nature's why. If quantum theory describes reality correctly, can any particle truly have its own wavefunction, given constant interactions and entanglements in Hilbert space? Perhaps no particle truly possesses an independent wavefunction; rather, a single, universal quantum state envelops the entirety of existence. Alternatively, one might abandon the abstract and somewhat intangible Hilbert space formalism, returning instead to a more concrete geometric ontology—one in which each particle inherently maintains its distinct wavefunction. In such a geometric framework, quantum entanglement naturally emerges from intrinsic interactions between two complementary spaces: the familiar three-dimensional position space $\mathbb{R}^3$ and an internal generator space $\mathbb{C}$, thereby offering a profound philosophical shift from abstraction to tangible geometric reality.

Historically, quantum entanglement was introduced and critically questioned by Einstein, Podolsky, and Rosen (EPR) [1], leading to foundational discussions on the completeness of quantum mechanics. Despite the predictive success of the conventional Hilbert space formalism and the Copenhagen interpretation, significant conceptual ambiguities remain, particularly regarding the physical origins of entanglement and wavefunction collapse [2, 3]. Alternative interpretations, such as Bohmian mechanics and the Many-Worlds interpretation, attempt to address these ambiguities by introducing deterministic hidden variables or parallel universes, respectively [4, 5]. In contrast, the Relator framework presented here offers a distinctly geometric-algebraic perspective, avoiding adhoc assumptions and resolving inherent conceptual puzzles by naturally incorporating classical conservation laws and explicit geometric frequency interactions.

Although quantum theory effectively captures entanglement correlations, it does not elucidate their physical origins or why classical intuition breaks down [6]. In this paper, starting from classical equations and simple conservation laws, we demonstrate how apparently innocuous assumptions lead inevitably to contradictions in classical physics. We then resolve these contradictions using Relator theory [7], defined by the fundamental condition $R\omega = c$ —wavefunction evolves in the speed of light— thus revealing entanglement as an intrinsic feature emerging naturally from fundamental physical constraints.

Consider two interacting particles in free space, each characterized by rest energy, kinetic energy, and mutual potential energy. In classical physics, when calculating the total system energy, the interaction potential term is counted precisely once. However, when analyzing each particle individually, the full interaction potential energy appears explicitly in each particle's equations of motion. While this treatment is standard and rigorously accepted in classical mechanics—exemplified by formulations such as the Virial theorem—in standard quantum mechanics the interaction energy appears naturally as a joint potential in the system's Hamiltonian, rather than being explicitly partitioned between particles. This subtle difference raises a fundamental question: have we implicitly assumed a shared (or "energy-sharing") mechanism between the particles in both classical and quantum treatments? Could such an intrinsic sharing of internal energy underlie the phenomenon of quantum entanglement? If so, under what mathematical and physical conditions can it be rigorously formulated? and where, precisely, does it reside within the fabric of reality? Addressing these profound questions within a non-Hilbertian geometric framework forms the central objective of this article.

In the Relator framework, each particle can be analyzed independently as an individual wavefunction, even when mutual interactions such as Coulomb potentials are present. Traditionally, within standard quantum mechanics, interacting particles cannot be represented by separable wavefunctions:

$$\Psi(\mathbf{r}_a, \mathbf{r}_b, t) \neq \psi_a(\mathbf{r}_a, t)\, \psi_b(\mathbf{r}_b, t).$$

This fundamental inseparability necessitates ambiguous and nonlocal processes such as wavefunction collapse. In contrast, the unified geometric phase-space structure $\mathbb{R}^3 \oplus \mathbb{C}$ intrinsic to the Relator approach naturally allows each particle to retain its own independent geometric wavefunction [7]. Consequently, entangled or interacting particles no longer need to be viewed as inseparable

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components of a single joint state, thus restoring a clear, intuitive, and local description of quantum interactions and eliminating the need for wavefunction collapse entirely.

It is crucial to clarify the conceptual relationship between Einstein's relativistic energy and the Relator framework. Although Einstein's relativistic energy relation naturally emerges within the Relator formalism, the interpretation and formulation differ fundamentally. In the Relator framework, the total energy of a particle is inherently defined within two distinct, orthogonal spaces: the internal complex space $\mathbb{C}$, which encodes the particle's intrinsic rest-energy (mass), and the external three-dimensional space $\mathbb{R}^3$, which accommodates all forms of kinetic and potential energies.

In the external propagation space ($\mathbb{R}^3$), energies manifest as spatial frequencies ($\boldsymbol{\omega}_{\mathbb{R}^3}$) arising either directly from particle momentum or from fundamental potentials, such as gravitational or electromagnetic fields. These spatial frequencies exhibit explicit vectorial addition. The Relator principle, given by the fundamental geometric constraint $R\omega = c$, always ensures the consistency of energy calculations with Planck's relation $E = \hbar\omega$.

Moreover, due to the inherent orthogonality between the internal space $\mathbb{C}$ and the external space $\mathbb{R}^3$, their respective frequency contributions combine through a simple Euclidean summation to yield the particle's total energy magnitude. A notable departure from conventional relativistic and quantum treatments is the introduction of *shared-energy frequencies*, uniquely arising within the Relator framework. These shared-energy terms naturally correspond to joint Hamiltonians typically defined in standard quantum Hilbert spaces. Although Hilbert space facilitates calculation simplicity, it does not explicitly elucidate the physical mechanism behind entanglement and quantum correlations. Within the Relator framework, the explicit construction of Hilbert spaces becomes unnecessary.

Throughout this paper, references to relativistic equations and Hamiltonians are included solely to maintain consistency with established terminology and concepts in modern physics, facilitating clear and intuitive connections to known theoretical structures and experimental results.

In this study, we rely exclusively on three foundational principles:

1. *Classical Physics*: intuitive and familiar laws governing simple interactions.
2. *Schrödinger Equation*: standard quantum-mechanical description of wavefunction evolution.
3. *Fundamental Relator Condition*: $R\omega = c$.

This approach highlights subtle distinctions and overlooked details historically responsible for interpretational confusion, suggesting a fundamentally simple yet profound structure underlying reality.

II. THE RELATOR FRAMEWORK

In Relator theory, a quantum particle is explicitly characterized by two frequencies: the intrinsic internal frequency $\omega_{\mathbb{C}}$, and the spatial frequency $\omega_{\mathbb{R}^3}$, precisely defined as follows:

$$\begin{cases} \omega_{\mathbb{C}}(\mathbf{r},t) = -\dfrac{1}{\hbar}\dfrac{d}{d\tau}S(\mathbf{r},t), \quad (\circlearrowright : \text{CW}\ ,\ \circlearrowleft : \text{CCW}\ ) \\[2ex] \vec{\omega}_{\mathbb{R}^3}(\mathbf{r},t) = \dfrac{1}{mR(\mathbf{r},t)}\nabla S(\mathbf{r},t). \end{cases} \tag{1}$$

Here, m is the particle's rest mass, c is the speed of light, $\hbar$ the reduced Planck constant, $S(\mathbf{r},t)$ the particle's quantum action, and $R(\mathbf{r},t)$ the Relator radius of the particle. The total Relator frequency is explicitly obtained through the Euclidean combination of internal and spatial components:

$$\omega(\mathbf{r},t) = \sqrt{\omega_{\mathbb{C}}(\mathbf{r},t)^2 + \omega_{\mathbb{R}^3}(\mathbf{r},t)^2}. \tag{2}$$

The fundamental Relator principle explicitly imposes the invariant condition:

$$R(\mathbf{r},t)\,\omega(\mathbf{r},t) = c, \tag{3}$$

which governs the evolution speed of the particle's wavefunction, connecting energies across the $\mathbb{R}^3$ and $\mathbb{C}$ spaces [7].

To rigorously incorporate the fundamental Relator condition $R\omega = c$ into the standard Schrödinger equation, we propose the following extended Lagrangian formulation for better clarity:

$$\mathcal{L} = \mathcal{L}_{\text{Schrödinger}} + \lambda(r,t)\left(\sqrt{\frac{R^2}{\hbar^2}\left(\frac{dS}{d\tau}\right)^2 + \frac{|\nabla S|^2}{m^2}} - c\right), \tag{4}$$

where $\lambda(r,t)$ is a Lagrange multiplier enforcing the intrinsic geometric-algebraic constraint between internal and external frequencies. This formulation naturally reproduces the Schrödinger dynamics while explicitly embedding the Relator condition, ensuring the geometric coherence of quantum states within the unified Relator framework.

III. INTERACTION BETWEEN TWO PARTICLES

Consider a system of two interacting particles a and b with arbitrary masses m_a and m_b, charges q_a and q_b, and initial momenta p_a and p_b. Initially, at infinite separation ($r \to \infty$), the total energy and corresponding frequencies of each particle are given explicitly by:

$$E_j^{(\infty)} = \sqrt{m_j^2c^4 + p_j^2c^2}, \quad \omega_j^{(\infty)} = \frac{E_j^{(\infty)}}{\hbar}, \quad R_j^{(\infty)} = \frac{\hbar c}{E_j^{(\infty)}}. \tag{5}$$

A. Two-Particle System Dynamics

Consider two charged particles with rest masses m_j (where $j = a, b$), charges q_j, and equal initial momenta p_0 directed radially toward each other. Total system energy, comprising relativistic kinetic energies and the exact relativistic (Liénard–Wiechert) electromagnetic interaction potential, remains strictly conserved.

Initially, at infinite separation ($r \to \infty$):

$$E_{sys}(r \to \infty) = \sqrt{p_0^2 c^2 + m_a^2 c^4} + \sqrt{p_0^2 c^2 + m_b^2 c^4}. \quad (6)$$

At arbitrary separation r, relativistic energy conservation dictates:

$$E_{sys}(r) = \sqrt{p(r)^2 c^2 + m_a^2 c^4} + \sqrt{p(r)^2 c^2 + m_b^2 c^4} + V_{rel}(r, v), \quad (7)$$

where $V_{rel}(r, v)$ is the exact relativistic interaction potential (Liénard–Wiechert), explicitly velocity-dependent, incorporating retardation and Lorentz effects.

This relativistically exact formulation immediately poses a fundamental question:

$$\underbrace{\frac{p^2}{2m_a} + m_a c^2 + \frac{p^2}{2m_b} + m_b c^2 + \frac{q_a q_b}{4\pi\epsilon_0 r}}_{\text{Classical Formula}} \neq \underbrace{\sqrt{p^2 c^2 + m_a^2 c^4} + \sqrt{p^2 c^2 + m_b^2 c^4} + V_{\text{rel}}(r, v)}_{\text{Relativity+Quantum Formula}} \quad (8)$$

The classical kinetic energy sum combined with the standard Coulomb potential does not equal the relativistically exact total energy. The question: WHY!?

This seemingly straightforward inequality embodies a profound *philosophical* and *physical* puzzle that bridges classical mechanics, quantum physics, and all associated theoretical frameworks. Resolving this puzzle is the central goal of this article.

B. Single-Particle Dynamics with Shared Internal Frequencies

For clarity and consistency, we first explicitly present the starting point, the time-independent Schrödinger equation [8] for a particle with classical momentum in a classical Coulomb potential:

$$\left(-\frac{\hbar^2}{2m_j}\nabla^2 + q_j \Phi(r)\right)\psi_j(r) = E_j\, \psi_j(r), \quad (9)$$

where $\Phi(r) = \frac{q_j Q}{4\pi\epsilon_0 m_j r}$ is the Coulomb potential, and E_j is the total energy of the particle.

Expressing the wavefunction in Madelung form, $\psi_j(r, \theta, \phi) = R_j(r, \theta, \phi) e^{iS_j(r,\theta,\phi)/\hbar}$, the Schrödinger equation straightforwardly yields the extended quantum Hamilton–Jacobi equation that explicitly includes both translational and spin–angular momenta:

$$\frac{(\nabla S_{\text{trans},j})^2}{2m_j} + \frac{(\nabla S_{\text{spin},j})^2}{2m_j} + q_j \Phi(r) - \frac{\hbar^2}{2m_j}\frac{\nabla^2 R_j}{R_j} = E_j, \quad (10)$$

which explicitly connects the translational momentum (associated with the gradient $\nabla S_{\text{trans},j}$) and spin angular momentum (associated with the angular gradient $\nabla S_{\text{spin},j}$) to the particle's energy within the Coulomb field. In the classical limit, neglecting the quantum potential (the last term), this term becomes zero within the Relator framework when no energy is exchanged with the particle (i.e., the Relator radius R_j remains constant).

Defining the frequencies explicitly within the Relator framework, we impose the fundamental condition $R\omega = c$ to obtain intrinsic internal and spatial frequencies as:

$$\star(\textbf{I. } R\omega = c) \quad \begin{cases} \omega_{\mathbb{C},j}(r) = \sqrt{\omega_j(r)^2 - \omega_{\mathbb{R}^3,j}(r)^2}, \\ \boldsymbol{\omega}_{\mathbb{R}^3,j}(r) = \boldsymbol{\omega}_{p,j}(r) + \boldsymbol{\omega}_{\mathcal{J},j}(r) + \boldsymbol{\omega}_{\phi,j}(r), \end{cases} \quad (11)$$

with the explicit definitions for each frequency component:

$$\boldsymbol{\omega}_{p,j}(r,t) = \frac{\nabla S_{\text{trans},j}(r,t)}{m_j R_j} = \frac{p_j(r,t)}{m_j R_j}\,\hat{\mathbf{r}},$$

$$\boldsymbol{\omega}_{\mathcal{J},j}(r,t) = \frac{\nabla S_{\text{spin},j}(r,t)}{m_j R_j} = \frac{\mathcal{J}_j(r,t)}{R_j\sqrt{m_j I_j}}\hat{\mathbf{n}}_{\mathcal{J},j}(r,t), \quad (12)$$

$$\boldsymbol{\omega}_{\phi,j}(r,t) = \frac{\nabla S_{\phi,j}(r,t)}{m_j R_j} = \frac{\sqrt{2\,|q_j \Phi(r)|/m_j}}{R_j}\,\hat{\mathbf{E}}(r),$$

where the unit vector $\hat{\mathbf{E}}(r)$ is defined from the Coulomb field as

$$\hat{\mathbf{E}}(r) \equiv -\frac{\nabla\Phi(r)}{\|\nabla\Phi(r)\|} = \frac{\mathbf{E}(r)}{\|\mathbf{E}(r)\|}, \quad \mathbf{E}(r) = -\nabla\Phi(r) = \frac{q_j Q}{4\pi\epsilon_0\, r^2}\,\hat{\mathbf{r}}, \quad (13)$$

which, for a Coulomb field, points purely in the radial direction $\hat{\mathbf{r}}$.

Gauge–invariance remark: In the electrostatic gauge, the scalar potential is defined up to an additive constant, $\Phi(\mathbf{r}) \to \Phi(\mathbf{r}) + \text{const}$. The physical Coulomb field $\mathbf{E}(\mathbf{r}) = -\nabla\Phi(\mathbf{r})$ is invariant under this transformation, and so is its unit vector $\hat{\mathbf{E}}(\mathbf{r})$. By defining $\boldsymbol{\omega}_{\phi,j}$ through $\hat{\mathbf{E}}(\mathbf{r})$, we ensure that this frequency component depends only on the physical electric field and is manifestly gauge–invariant.

The unit vector $\hat{\mathbf{n}}_{\mathcal{J},j}(r,t)$ is defined as:

$$\hat{\mathbf{n}}_{\mathcal{J},j}(r,t) \equiv \frac{\boldsymbol{\mathcal{J}}_j(r,t) \times \hat{\mathbf{r}}_s}{|\boldsymbol{\mathcal{J}}_j(r,t)|}. \tag{14}$$

Here, the internal radius $R_j = \hbar c/E_j^{(\infty)}$ remains constant for a particle with fixed total energy $E_j^{(\infty)}$. $p_j(r)$ is the radial translational momentum, $\boldsymbol{\mathcal{J}}_j(r,t)$ denotes the internal spin–angular momentum vector, I_j is the effective internal moment of inertia (spin–related), and $\Phi(r)$ is the electrostatic Coulomb potential at separation r:

$$\boldsymbol{\mathcal{J}}_j(r,t) = \mathcal{J}_{j,x}(r,t)\,\hat{\mathbf{x}} + \mathcal{J}_{j,y}(r,t)\,\hat{\mathbf{y}} + \mathcal{J}_{j,z}(r,t)\,\hat{\mathbf{z}}. \tag{15}$$

Important Remark: In the Relator framework, the contributions from translational momentum, spin–angular momentum, and Coulomb potential remain strictly independent. This ensures that each particle individually senses external interactions as separate action gradients. Practically, this allows for a significant simplification: when solving the Hamilton–Jacobi equation, one isolates the desired energy component by setting all others to zero, equating the remaining term directly to its corresponding energy. All fundamental equations of relativity—gravity, electromagnetism, Lorentz transformations, and Liénard–Wiechert potentials—should emerge naturally from classical terms combined with the Relator condition $R\omega = c$.

However, when considering a particle embedded within an interacting two–particle system, it becomes necessary to explicitly include shared–energy contributions to the particle–based frequencies. To accurately account for the quantum mechanical interaction energy (analogous to the joint Hamiltonian commonly employed in standard quantum descriptions of two–particle systems), we introduce additional shared–energy frequency terms:

$\star$(**II.** Particle Energy)

$$\begin{cases} \omega_{\mathbb{C},j}(r,t) = \dfrac{m_j c^2}{\hbar} + \omega_{\mathbb{C}\text{-share}}(r,t), \\ \boldsymbol{\omega}_{\mathbb{R}^3,j}(r,t) = \dfrac{1}{\hbar}\left(\dfrac{p_j(r,t)^2}{2m_j}\hat{\mathbf{r}} + \dfrac{\mathcal{J}_j(r,t)^2}{2I_j}\hat{\mathbf{n}}_{\mathcal{J},j}(r,t) + q_j\Phi(r)\,\hat{\mathbf{E}}(r) + \hbar\,\boldsymbol{\omega}_{\mathbb{R}^3\text{-share}}(r,t)\right), \end{cases} \tag{16}$$

$\omega_{\mathbb{C}\text{-share}}$ is proportional to shared energy in Relator space $\mathbb{C}$, while $\boldsymbol{\omega}_{\mathbb{R}^3\text{-share}}$ corresponds to shared energy in Relator space $\mathbb{R}^3$. To ensure explicit consistency across both I—II, the following equalities must hold:

$$\omega^{(\mathrm{I})}_{\mathbb{C},j}(r,t) = \omega^{(\mathrm{II})}_{\mathbb{C},j}(r,t), \quad \boldsymbol{\omega}^{(\mathrm{I})}_{\mathbb{R}^3,j}(r,t) = \boldsymbol{\omega}^{(\mathrm{II})}_{\mathbb{R}^3,j}(r,t). \tag{17}$$

Enforcing asymptotic boundary conditions at infinite separation explicitly constrains these shared–frequency terms. To maintain strict energy conservation within a two–particle system, the interaction–induced portion of the shared frequency (partner frequency) must carry an explicitly negative sign for one particle and an equal but opposite positive sign for the other particle. This exact balance ensures the precise cancellation of interaction energies, thus preserving total energy conservation. As we shall see, this crucial condition also offers deeper insights into the fundamental nature of particles and their associated momenta. Consequently, the intrinsic geometric sharing of frequencies within the Relator framework naturally provides a rigorous geometric foundation for quantum entanglement.

In the Relator framework, calculating spatial frequencies directly from the vector sum of momenta, spin, and potentials (vector path) inherently captures directional interactions and yields results that differ from the simple scalar sum of corresponding energies. This difference explicitly defines the shared energy (or frequency), reflecting quantum correlations induced by directional interactions in the system.

To simplify forthcoming expressions, we introduce compact scalar variables:

$$\begin{aligned} \beta_{p,j} &\equiv \frac{p_j(r,t)}{m_j c}, & \beta_{\mathcal{J},j} &\equiv \frac{\mathcal{J}_j(r,t)}{c\sqrt{m_j I_j}}, \\ \beta_{\Phi,j} &\equiv \frac{\sqrt{2q_j\Phi(r)}}{c\sqrt{m_j}}, & \gamma_j &\equiv \frac{E_j^{(\infty)}}{m_j c^2}. \end{aligned} \tag{18}$$

These β variables—*Relator Config*—are dimensionless and represent normalized velocity–like scalars.

C. Shared C-Space Frequency

Similarly, to determine the shared internal frequency $\omega_{\mathbb{C}\text{-share}}$, we explicitly impose consistency between the fundamental Relator condition (Eq. (11)) and the particle-energy-based formulation (Eq. (16)). Equating internal frequency components, we have:

$$\omega^{(\mathrm{I})}_{\mathbb{C},j}(r,t) = \omega^{(\mathrm{II})}_{\mathbb{C},j}(r,t), \tag{19}$$

with explicit definitions:

$$\omega^{(\mathrm{I})}_{\mathbb{C},j}(r,t) = \frac{E_j^{(\infty)}}{\hbar}\sqrt{1 - \|\beta_{p,j}\hat{\mathbf{r}} + \beta_{\mathcal{J},j}\hat{\mathbf{n}}_{\mathcal{J},j} + \beta_{\Phi,j}\hat{\mathbf{r}}\|^2}. \tag{20}$$

$$\omega^{(\mathrm{II})}_{\mathbb{C},j}(r,t) = \frac{m_j c^2}{\hbar} + \omega_{\mathbb{C}\text{-share}}(r,t). \tag{21}$$

Thus, solving for the shared internal frequency, we obtain the final consistent form:

$$\boxed{\begin{aligned}&\omega_{\mathbb{C}\text{-share},j}(r,t) = \\ &\frac{E_j^{(\infty)}}{\hbar}\sqrt{1 - \|\beta_{p,j}\hat{\mathbf{r}} + \beta_{\mathcal{J},j}\hat{\mathbf{n}}_{\mathcal{J},j} + \beta_{\Phi,j}\hat{\mathbf{r}}\|^2} - \frac{m_j c^2}{\hbar}\end{aligned}} \tag{22}$$

It is important to explicitly note that, according to Eq. (16), the sum of the shared internal frequency and the intrinsic Compton frequency precisely yields the particle's total internal frequency. Moreover, since the total energy $E_j(t)$ remains strictly conserved within the isolated system, the total frequency $\omega_j = E_j^{(\infty)}/\hbar$ is also constant at all times. Within the Relator framework, the ratio of internal frequency to total frequency $(\omega_{\mathbb{C},j}/\omega_j)$ represents the relativistic time dilation factor, describing the overall wavefunction evolution rate per internal Relator $\mathbb{C}$-space cycle. This formulation generalizes the concept of relativistic time dilation beyond gravitational fields, explicitly demonstrating that Coulomb fields also induce measurable relativistic time dilation effects.

D. Shared R3-Space Frequency

To explicitly determine the shared-frequency terms, we impose consistency between the fundamental Relator condition (Eq. (11)) and the particle-energy-based formulation (Eq. (16)). Equating spatial frequency components, we have:

$$\boldsymbol{\omega}^{(\mathrm{I})}_{\mathbb{R}^3,j}(r,t) = \boldsymbol{\omega}^{(\mathrm{II})}_{\mathbb{R}^3,j}(r,t), \tag{23}$$

with vector definitions for each side:

$$\boldsymbol{\omega}^{(\mathrm{I})}_{\mathbb{R}^3,j}(r,t) = \frac{E_j^{(\infty)}}{\hbar}\left(\beta_{p,j}\hat{\mathbf{r}} + \beta_{\mathcal{J},j}\hat{\mathbf{n}}_{\mathcal{J},j} + \beta_{\Phi,j}\hat{\mathbf{r}}\right) \tag{24}$$

$$\begin{aligned}\boldsymbol{\omega}^{(\mathrm{II})}_{\mathbb{R}^3,j}(r,t) =& \frac{m_j c^2}{\hbar}\left(\frac{\beta_{p,j}^2}{2}\hat{\mathbf{r}} + \frac{\beta_{\mathcal{J},j}^2}{2}\hat{\mathbf{n}}_{\mathcal{J},j} + \frac{\beta_{\Phi,j}^2}{2}\hat{\mathbf{r}}\right) \\ &+ \boldsymbol{\omega}_{\mathbb{R}^3\text{-share}}(r,t).\end{aligned} \tag{25}$$

where we used $R_j = \hbar c/E_j^{(\infty)}$, as the particle energy $E_j^{(\infty)}$ remains constant throughout the isolated system's evolution. So, explicitly solving for the shared spatial frequency yields the clear vector form:

$$\boxed{\begin{aligned}\boldsymbol{\omega}_{\mathbb{R}^3\text{-share}}(r,t) =& \frac{E_j^{(\infty)}}{\hbar}\left(\beta_{p,j} - \frac{\beta_{p,j}^2}{2\,\gamma_j}\right)\hat{\mathbf{r}} \\ &+ \frac{E_j^{(\infty)}}{\hbar}\left(\beta_{\mathcal{J},j} - \frac{\beta_{\mathcal{J},j}^2}{2\,\gamma_j}\right)\hat{\mathbf{n}}_{\mathcal{J},j} \\ &+ \frac{E_j^{(\infty)}}{\hbar}\left(\beta_{\Phi,j} - \frac{\beta_{\Phi,j}^2}{2\,\gamma_j}\right)\hat{\mathbf{r}}\end{aligned}} \tag{26}$$

The resulting spatial shared frequency, $\boldsymbol{\omega}_{\mathbb{R}^3\text{-share}}(r,t)$, precisely captures the discrepancy between action-based and energy-based frequency perspectives, revealing interaction-induced correlations inherent to the Relator framework.

E. Ontological Function of the Relator

From Eqs. (22) and (26), it becomes explicitly clear that all Relator beta-factors ($\beta_{p,j}$, $\beta_{\mathcal{J},j}$, and $\beta_{\Phi,j}$) appear consistently in both the internal $\mathbb{C}$-space frequency $\omega_{\mathbb{C}\text{-share}}$ and the external $\mathbb{R}^3$-space frequency $\omega_{\mathbb{R}^3\text{-share}}$. Consequently, our complete knowledge of these beta-factors, combined with their vector orientations in the internal $\mathbb{C}$-space, allows the explicit and full reconstruction of the external frequency vectors in $\mathbb{R}^3$-space. This fundamental property justifies the designation of the $\mathbb{C}$-space as the generator space.

Indeed, a rigorous geometric depiction of the Relator within the generator space $\mathbb{C}$ can explicitly illustrate the generation and subsequent exposure of frequencies $\omega_{\mathbb{R}^3}$ within the propagation medium $\mathbb{R}^3$.

It is crucial to emphasize that, according to Relator theory, the three-dimensional position space $\mathbb{R}^3$ serves solely as a medium for propagation. The particle's position, energy, and momentum emerge fundamentally from the geometric processes occurring within the generator space $\mathbb{C}$ and subsequently manifest explicitly within the propagation medium $\mathbb{R}^3$.

IV. PARTICLES TANGO

To explore the vectorial structure of shared internal and spatial frequencies, we begin by decomposing the internal angular momentum vector $\boldsymbol{\mathcal{J}}_j(r,t)$ as defined in Eq. (15). For notational and computational simplicity, we align the radial direction with the internal z-axis, i.e., $\hat{\mathbf{r}} \equiv \hat{\mathbf{z}}$.

We now express the shared internal frequency $\omega_{\mathbb{C}\text{-share},j}(r,t)$ explicitly, consistent with Eq. (22) and the component decomposition:

$$\omega_{\mathbb{C}\text{-share},j}(r,t) = \frac{E_j^{(\infty)}}{\hbar}\sqrt{1-\left[\beta^2_{\mathcal{J}_x,j}+\beta^2_{\mathcal{J}_y,j}+(\beta_{p,j}+\beta_{\mathcal{J}_z,j}+\beta_{\Phi,j})^2\right]} - \frac{m_j c^2}{\hbar}. \tag{27}$$

Next, we derive the vectorial expression for the spatial shared frequency $\boldsymbol{\omega}_{\mathbb{R}^3\text{-share},j}(r,t)$, using Eq. (26) and the component decomposition of $\boldsymbol{\mathcal{J}}_j(r,t)$:

$$\begin{aligned}\boldsymbol{\omega}_{\mathbb{R}^3\text{-share},j}(r,t) &= \frac{E_j^{(\infty)}}{\hbar}\left[\beta_{\mathcal{J}_x,j}\left(1-\frac{\beta_{\mathcal{J}_x,j}}{2\,\gamma_j}\right)\right]\hat{\mathbf{x}} \\ &+ \frac{E_j^{(\infty)}}{\hbar}\left[\beta_{\mathcal{J}_y,j}\left(1-\frac{\beta_{\mathcal{J}_y,j}}{2\,\gamma_j}\right)\right]\hat{\mathbf{y}} \\ &+ \frac{E_j^{(\infty)}}{\hbar}\left[\beta_{p,j}\left(1-\frac{\beta_{p,j}}{2\,\gamma_j}\right)+\beta_{\mathcal{J}_z,j}\left(1-\frac{\beta_{\mathcal{J}_z,j}}{2\,\gamma_j}\right)+\beta_{\Phi,j}\left(1-\frac{\beta_{\Phi,j}}{2\,\gamma_j}\right)\right]\hat{\mathbf{z}}\end{aligned} \tag{28}$$

Since no mass-energy conversion occurs in this interaction, the total internal and spatial frequencies, as defined explicitly in Eq. (16), must each remain constant over time:

$$\begin{aligned}&\frac{d}{dt}\sum_{j=a,b}\left(\frac{m_j c^2}{\hbar}+\omega_{\mathbb{C}\text{-share},j}(r,t)\right)=0, \\ &\frac{d}{dt}\sum_{j=a,b}\left(\frac{E_{j,\text{classical}}(r,t)}{\hbar}+\boldsymbol{\omega}_{\mathbb{R}^3\text{-share},j}(r,t)\right)=0.\end{aligned} \tag{29}$$

Note that the Relator parameters β_Φ (Coulomb potential) and β_p (momentum) explicitly depend on position and thus implicitly on time. As shown in the derived equations of motion, these parameters inherently depend on the mass, momentum, and charge of the second particle, reflecting the dynamic interplay of energies between the two interacting particles.

These spatially and temporally dependent terms constitute the dynamic drivers behind energy transformations both within each particle and between the interacting pair. Specifically, for two electrons, the internal shared frequencies ($\omega_{\mathbb{C}\text{-share}}$) possess identical directions, causing their geometric overlap, as illustrated in Fig. 1.

Each particle possessing momentum or situated within a Coulomb field inherently exhibits shared frequency components, denoted as $\boldsymbol{\omega}_{\text{share}}$. Within the Relator framework, we interpret each particle as *the manifestation* of a Relator in $\mathbb{R}^3$-space. An electron initially at rest corresponds to a stationary Relator; upon absorbing a photon, it acquires momentum, signifying an interaction and effective merging of two distinct Relators. While we do not claim strict fundamental independence of particles post-interaction, the absorbed photon's influence remains traceable as the particle's self-contained frequency component, denoted explicitly as $\boldsymbol{\omega}_{\text{self}}$.

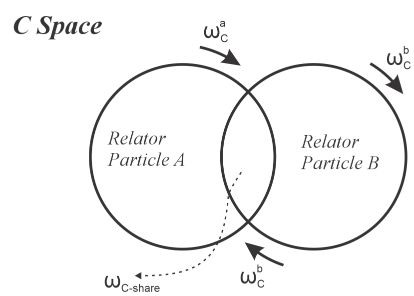

FIG. 1. Geometric interpretation of internal shared frequencies ($\omega_{\mathbb{C}\text{-share}}$) in Relator $\mathbb{C}$ space. For two electrons, the identical directions of internal shared frequencies cause their geometric overlap to subtract and neutralize each other, resulting in stable entanglement.

However, as indicated by Eqs. (27) and (28), these self-contained frequencies are not restricted to individual particles. Instead, frequencies may dynamically exchange

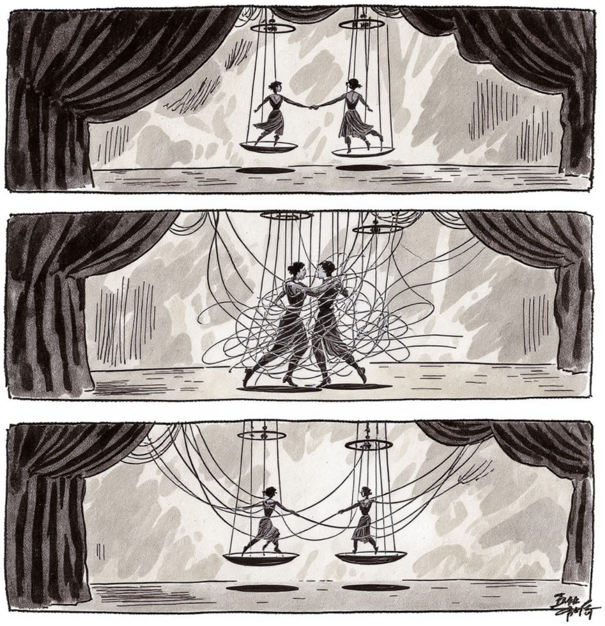

FIG. 2. A metaphorical illustration of the two-particle tango—particles dynamically exchanging frequencies and becoming entangled through their Relators.

and overlap among multiple Relators across both $\mathbb{R}^3$ and $\mathbb{C}$ spaces. This frequency sharing preserves energy conservation and rigorously satisfies the fundamental Relator condition $R\omega = c$ at all times.

We introduce a simple yet insightful decomposition of shared frequencies into two clearly defined components:

$$\boldsymbol{\omega}_{\text{share}} = \boldsymbol{\omega}_{\text{self}} + \boldsymbol{\omega}_{\text{partner}}, \tag{30}$$

where $\boldsymbol{\omega}_{\text{partner}}$ explicitly represents the frequency contribution originating from interactions with other particles—partner Relators.

This intricate yet elegant exchange of frequencies between interacting particles can be metaphorically described as a two-particle tango, as illustrated in Fig. 2, and its precise temporal dynamics can be formulated into explicit time-evolution equations. Such dynamical equations illuminate how, for instance, two electrons approach equilibrium states, spontaneously aligning spins oppositely and sharing their frequency components in a coherent and balanced manner.

From our analysis above, the interaction of two charged particles via the Coulomb field naturally leads to an anti-parallel spin alignment specifically along the z-axis—classical interpretation. This alignment emerges directly from the fundamental Relator condition $R\omega = c$, as clearly indicated by Eq. (28), enforcing the condition:

$$\beta_{\mathcal{J}_z,a} = -\beta_{\mathcal{J}_z,b}. \tag{31}$$

According to the equation (27), any change in $\omega_{\mathbb{C}\text{-share}}$, corresponding to internal energy, must be balanced by an equal and opposite change in the partner particle. Hence, no net frequency—energy is created or destroyed; rather, these internal frequencies are strictly exchanged between the two particles.

$$\begin{array}{ccc} \beta_{\mathcal{J},a} \xrightarrow{\text{changes}} \boldsymbol{\omega}_{\mathbb{R}^3\text{-share},a} & & \\ \downarrow \text{adjust} & & \\ \{\beta_{p,a},\ \beta_{\Phi,a}\} & \xrightarrow{R\omega = c \text{ reflects to}} & \omega_{\mathbb{C}\text{-share},a} \\ \downarrow \text{reflects to} & & \\ & \omega_{\mathbb{C}\text{-share},b} \xrightarrow{\text{affects}} & \boldsymbol{\omega}_{\mathbb{R}^3\text{-share},b} \end{array} \tag{32}$$

This exchange mechanism precisely ensures the conservation of total energy and clearly demonstrates how classical Coulomb interactions inherently establish spin alignment through frequency sharing, without invoking additional fields or energy creation.

V. ENTANGLING AND DISENTANGLING INTERACTIONS

Under Coulomb interactions, internal Relator frequencies—particularly those associated with spin—can geometrically overlap between two or more interacting particles within the internal $\mathbb{C}$-space. Remarkably, this geometric overlap can persist even as particles spatially separate in $\mathbb{R}^3$-space, provided no subsequent interactions disrupt their connection. I interpret this persistent geometric overlap of internal Relator frequencies as quantum entanglement.

Formally, the entanglement condition is represented by the persistence of the partner frequency components in the asymptotic limit:

$$\lim_{r\to\infty} \boldsymbol{\omega}_{\text{partner}}(r,t) \neq 0. \tag{33}$$

Physically, particles—understood as manifestations of Relators in $\mathbb{R}^3$—may become spatially separated after interactions, such as collisions or interactions with detectors, yet maintain a frequency-sharing connection in the internal $\mathbb{C}$-space. This sustained frequency-sharing, referred to here as a two-particle tango, represents the core mechanism behind quantum entanglement within the Relator framework.

Measurement, from this viewpoint, constitutes an interaction between a particle and a measurement apparatus, explicitly designed to probe and resolve specific shared frequencies, such as spin components. Consequently, *measurement interactions disrupt frequency sharing*, causing the collapse of entanglement correlations, see Eq. (32) backward flow. The intricate relationship between measurement and quantum superposition states will be analyzed comprehensively in a separate, dedicated article.

In the Relator framework, two distinct limiting behaviors thus emerge:

1. Disentangling Interaction Fork:
Within the Relator framework, the *shared* components of the internal and spatial partner frequencies represent the physical carrier of entanglement energy. These shared energies are explicitly located in the internal generator space ($\mathbb{C}$-space), where interaction between particles establishes a resonance condition linking their internal frequencies. This resonance is the geometric origin of the two-particle joint Hamiltonian in conventional quantum theory, now understood in explicit physical terms.

At infinite separation, the shared components vanish:

$$\lim_{r\to\infty} \omega_{\mathbb{C}\text{-partner}}(r) = 0, \quad \lim_{r\to\infty} \boldsymbol{\omega}_{\mathbb{R}^3\text{-partner}}(r) = 0, \quad (34)$$

indicating complete disentanglement and the restoration of independent particle states without persistent quantum correlations.

Measurement processes (e.g., spin measurements) are described here as physical interactions between the particle and external systems—often mediated by the Coulomb field—which induce a redistribution of the particle's internal spin momentum. This redistribution directly breaks the $\mathbb{C}$-space resonance by eliminating the shared-frequency components $\boldsymbol{\omega}_{\text{partner}}$ between previously entangled particles. As discussed earlier in the ontological analysis, spin momentum information is encoded in the internal shared frequency $\omega_{\mathbb{C}\text{-partner}}$, remaining independent of the particle separation in $\mathbb{R}^3$-space.

In a subsequent article, *Relator Bifurcation Theory*, we will show that this resonance-breaking mechanism—applicable equally to entanglement creation and to its destruction—naturally leads to violations of Bell inequalities [9], even for particles separated by intergalactic distances in $\mathbb{R}^3$-space. The present work establishes the location and nature of the shared energies; the forthcoming work will present the explicit geometric–algebraic derivation of the Bell-test correlations and their bifurcation-induced breakdown.

2. Entangling Interaction Fork: Nonzero shared partner frequencies persist even at infinite separation:

$$\lim_{r\to\infty} \omega_{\mathbb{C}\text{-partner}}(r) \neq 0, \quad \lim_{r\to\infty} \boldsymbol{\omega}_{\mathbb{R}^3\text{-partner}}(r) \neq 0, \quad (35)$$

representing stable quantum entanglement and sustained nonlocal correlations.

Consequently, single-particle dynamics within the Relator framework span continuously from disentangled interactions—where no lasting correlations remain—to fully entangled quantum states characterized by persistent shared internal frequencies.

Notably, the head-on (180°) elastic collision of two electrons, originally analyzed by Mott [10], vividly exemplifies a profound philosophical puzzle in physics: deterministic quantum equations—such as Schrödinger's equation and mode analysis—consistently yield inherently stochastic and nonlocal correlations. This suggests that the conventional quantum Hilbert-space formulation, despite its mathematical consistency, does not fully capture the deeper, underlying physical reality. To address precisely this conceptual shortcoming, the present work introduces the Relator framework, explicitly distinguishing external spatial ($\mathbb{R}^3$-space) and internal generator ($\mathbb{C}$-space) structures. Within this ontological paradigm, background reality is viewed as arising from individual—Relators, each as fundamental computational processing units. These units collectively generate and sustain the universe through deterministic frequency-sharing dynamics, resulting naturally in the appearance of quantum entanglement and observed nonlocal correlations.

VI. RELATOR MODES AND RESONANCE STABILITY

This section clarifies the physical and mathematical connection between Hilbert space and Relator space in the context of quantum entanglement. While a complete mapping between the two will be presented in future publications, our goal here is to formalize how entangled states in the Relator framework arise from well-defined *resonance conditions* in the coupled $\mathbb{C}$-space (internal) and $\mathbb{R}^3$-space (external) frequency domains.

Importantly, the analysis below does *not* use a Hilbert-space inner product in the conventional sense. Instead, we construct a *physical correlation measure* between the two particles in $\mathbb{R}^3$-space to identify the specific configurations where a stable exchange of energy between them occurs. Such a configuration is defined here as a *resonance* in $\mathbb{R}^3$-space, which—due to the structure of the Relator equations—necessarily has a direct and simultaneous counterpart in $\mathbb{C}$-space. This follows from the earlier results showing that all β-parameters in $\mathbb{R}^3$ are reflected into $\mathbb{C}$ through the fundamental Relator condition.

In the Relator picture, each particle is represented as a superposition of discrete *Relator modes*. Each mode corresponds either to the intrinsic rest-mass energy (internal $\mathbb{C}$-space mode, $\omega^{(m)}_{\mathbb{C},j}$) or to contributions from absorbed photons and external potentials (external $\mathbb{R}^3$-space mode, $\boldsymbol{\omega}^{(m)}_{\mathbb{R}^3,j}$). These modes collectively determine the particle's total momentum and internal energy. For particle a:

$$\mathcal{R}_a(\mathbf{x},t) = \sum_m A_m \, e^{i(\omega^{(m)}_{\mathbb{C},a}\, t + \boldsymbol{\omega}^{(m)}_{\mathbb{R}^3,a}\cdot\mathbf{x})}, \quad (36)$$

and similarly for particle b:

$$\mathcal{R}_b(\mathbf{x},t) = \sum_n B_n \, e^{i(\omega^{(n)}_{\mathbb{C},b}\, t + \boldsymbol{\omega}^{(n)}_{\mathbb{R}^3,b}\cdot\mathbf{x})}. \quad (37)$$

We then define a *resonance correlation function*:

$$\mathcal{M}_{ab}(t) = \int_{-\infty}^{+\infty} \mathcal{R}_a(\mathbf{x},t)\, \mathcal{R}_b(\mathbf{x},t)\, d^3x, \quad (38)$$

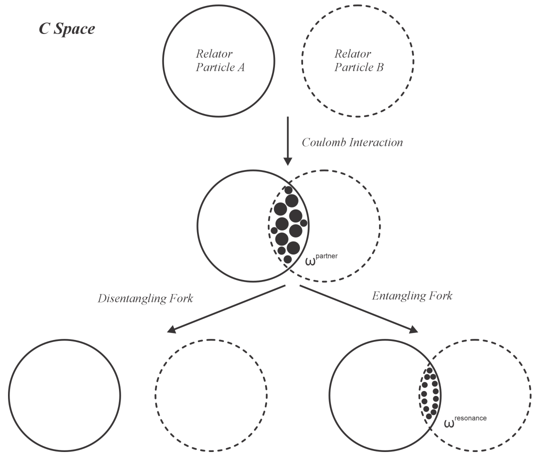

FIG. 3. Illustration of resonance dynamics in Relator $\mathbb{C}$-space. Two particles (Relators A and B) interact via a Coulomb potential, generating a shared region of partner frequencies (ω_{partner}). Depending on the system's evolution, the resonance is either lost (*disentangling fork*) or maintained as a stable resonance mode ($\omega_{\text{resonance}}$), corresponding to persistent quantum entanglement.

which quantifies the degree of spatial-frequency matching between the two particles in $\mathbb{R}^3$. Substituting the explicit mode expansions yields:

$$\mathcal{M}_{ab}(t) = \sum_{m,n} A_m B_n \, e^{i(\omega^{(m)}_{\mathbb{C},a} + \omega^{(n)}_{\mathbb{C},b})t} \int_{-\infty}^{+\infty} e^{i(\boldsymbol{\omega}^{(m)}_{\mathbb{R}^3,a} + \boldsymbol{\omega}^{(n)}_{\mathbb{R}^3,b})\cdot \mathbf{x}} \, d^3x. \tag{39}$$

The spatial integration imposes the vector condition:

$$\mathcal{M}_{ab}(t) = (2\pi)^3 \sum_{m,n} A_m B_n \, e^{i(\omega^{(m)}_{\mathbb{C},a} + \omega^{(n)}_{\mathbb{C},b})t} \delta^{(3)}(\boldsymbol{\omega}^{(m)}_{\mathbb{R}^3,a} + \boldsymbol{\omega}^{(n)}_{\mathbb{R}^3,b}), \tag{40}$$

so that resonance occurs when:

$$\boldsymbol{\omega}^{(m)}_{\mathbb{R}^3,a} = -\,\boldsymbol{\omega}^{(n)}_{\mathbb{R}^3,b}. \tag{41}$$

Physically, this expresses exact momentum–frequency cancellation between the two modes in $\mathbb{R}^3$-space. By the Relator principle, such matching in $\mathbb{R}^3$ directly maps to phase-locked matching in $\mathbb{C}$-space, creating a stable resonance state.

Under this condition, certain mode pairs (m, n) form *stable resonance modes* with a shared energy:

$$E^{(mn)}_{\text{resonance}} = \hbar \sqrt{\left(\omega^{(m,n)}_{\mathbb{C}}\right)^2 + \left(\omega^{(m,n)}_{\mathbb{R}^3}\right)^2}, \tag{42}$$

where $\omega^{(m,n)}_{\mathbb{C}} = 0$ for $m \neq 0$. Only the fundamental internal mode ($m = 0$) contributes long-term internal $\mathbb{C}$-space energy; higher-order modes are transient.

A crucial point is that the stability of $E^{(mn)}_{\text{resonance}}$ does not require a continuous strong interaction in $\mathbb{R}^3$; once phase-locking in $\mathbb{C}$-space is established, it persists over arbitrary separations, explaining observed long-distance entanglement. In conventional Hilbert-space language, these stable resonance modes correspond to entangled eigenstates of the joint Hamiltonian. Here, however, the mechanism is expressed explicitly in geometric–frequency terms, revealing how $\mathbb{R}^3$-space correlations necessarily reflect into $\mathbb{C}$-space dynamics.

Connection to Experimental Observations. While the present work is primarily theoretical, the Relator framework offers a clear interpretational link to key experimental tests of quantum entanglement. For instance, landmark photon–polarization experiments by Aspect *et al.* (1982) [11] and the long-distance Bell tests of Hensen *et al.* (2015) [12] both reveal correlations that violate classical bounds while preserving no-signaling constraints. In our formulation, these correlations correspond directly to persistent $\mathbb{C}$-space partner frequencies, which remain unaffected by separation in $\mathbb{R}^3$-space. The observed high-visibility interference fringes map naturally to the stable resonance modes described in Sec. VI, while decoherence effects in experimental data correspond to gradual disruption of these resonance conditions, as described in Sec. V. This correspondence suggests that future high-precision Bell tests could serve as direct probes of Relator-space resonance stability.

VII. CONCLUSION

The Relator framework offers a clear geometric-algebraic ontology that explicitly resolves conceptual ambiguities inherent to standard Hilbert-space formulations, directly addressing why classical equations fail to reproduce quantum phenomena without invoking nonlocality or instantaneous collapse.

We have shown that quantum entanglement naturally emerges from the fundamental Relator condition $R\omega = c$, combined explicitly with classical conservation laws. Within the Relator framework, entanglement is revealed as a clear geometric phenomenon arising from coherent frequency-sharing in coupled internal $\mathbb{C}$ and external $\mathbb{R}^3$ spaces. Crucially, this geometric-algebraic structure provides a rigorous physical interpretation of entanglement, directly addressing longstanding ambiguities related to energy distribution and correlation between entangled particles.

In particular, the intrinsic shared-frequency mechanism derived from particle interactions eliminates the need for ad-hoc assumptions or instantaneous wavefunction collapse. Our formalism yields explicit dynamical equations describing how spatial ($\mathbb{R}^3$-space) and internal ($\mathbb{C}$-space) frequencies mutually influence each other, clearly demonstrating the dynamic origin of spin orientations and quantum correlations. This approach directly clarifies the physical mechanisms underpinning quantum entanglement.

The detailed examination of Bell-type inequalities and quantum measurement processes is deferred to future work, the present formulation strongly suggests a unified geometric ontology underlying these phenomena. Future work will explicitly examine experimental tests and further implications of the Relator formalism, potentially yielding novel predictions testable in precision quantum experiments.

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