

Technical Addendum: Mathematical Summary of the Photon Soul Theorem

Foundational Setup

Let X denote the state site of a photonic-crystal waveguide ($\dim \leq 3$), and let F the photon sheaf defined on X_{et} , encoding quantum field amplitudes over the site.

We define a dimensional transition morphism

$$\phi : X \rightarrow X'$$

corresponding to a categorical lift of the system (e.g., local mode reshaping in the presence of a cavity induced transition).

The soul subspace $S(P) \leq H^\theta(\mathcal{Y})^i$ is the conomologically minimal invariant satisfying:

- (Invariance): Extracted via a functor $S : D(P) \rightarrow D^i S$, acting between derived categories.
- (Obstruction Characterization: Nontriviality of the obstruction class $[\omega] \cup [\omega] \in H^2(X, (X_v f^* \circ F^*))$ implies reduced visibility in interferometry.

Core Theorem (Informal Formulation)

Theorem (Soul-Induced Visibility Reduction) Let F' be photon sheaf on a finite-dimensional etale site X , with defect geometry defined by a cavity structure, if $\phi^* L \rightarrow R_{\phi^*} \phi_* \tilde{E}^\theta F$ is not a quasi-isomorphism in degrees $\leq n$, then the interference visibility V

$$V < 1 \quad [\omega] \in Gr^i H_E^{i+p+q}(c, F)$$

with the defect term explicitly computable via quibic truncated spectral sequence $E_{p,q}^\theta \Rightarrow Gr^i H_E^{i+p+q}(F)$ and dominant contribution expected at $(p,q) = (2,2)$.

Interpretation

In standard QED, interference visibility for a coherent photon in a lossless cavity is $V = 1$. But if the photon admits a nontrivial soul component, mismatch between physical and soul cohomology reduces visibility a measurable topological quantum effect.

Complete derivations are included in Appendices C and D.

Appendix: Diagrams

[Diagram A] Topps Transition; $X \rightarrow X'$ via ϕ

[Diagram B] Spectral Sequence Structure: $E_{p,q}^\theta \rightarrow Gr^i H_E^{i+p+q}(X, F)$