The Scrutineers reported that the following candidates had been duly elected as

Associate Members.

THOMAS EDWARD BEACHAM, B.Sc. (Engineering) (*Lond.*).
WILLIAM JAMES PURKIS BEWLEY, Stud. Inst. C.E.
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ROBERT CECIL BLACKWOOD, M.C., B.Sc. (*Edin.*).
CECIL GUSTAVE BRADLEY, D.S.O.
FRANCIS EDGAR BURNETT.
STANLEY CYRIL CARTER.
ERIC CARUS-WILSON, B.A. (*Cantab.*), B.Sc. (*McGill*).
HERBERT WALLIS COALES, M.C.
GILBERT THORP COTTERELL, Stud. Inst. C.E.
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THOMAS ALEXANDER FINCH, B.A., B.A.I. (*Dubl.*).
LAURENCE EDWIN GREENING, Stud. Inst. C.E.
ARTHUR LAFONE FRANK HILLS.
HENRY JAMES NEWSON, M.Sc. (*Victoria*).
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HENRY SNOWDEN ROWELL, B.Sc. (Engineering) (*Lond.*).
JOSEPH ERIC SWINDLEHURST, B.A. (*Cantab.*).
DENIS JOSEPH TIERNEY, B.E. (*National*).
SYDNEY TONKIN, B.Eng. (*Liverpool*).

(*Paper No. 4166.*)

"The Commercial Metering of Air, Gas, and Steam."

By JOHN LAWRENCE HODGSON, B.Sc., Assoc. M. Inst. C.E.

IN the autumn of 1906 Messrs. George Kent, Limited, were asked by Messrs. Harper Brothers to prepare designs for the meters which would be required in connection with the Victoria Falls and Transvaal Power Scheme,[1] which was then under consideration.

This work was undertaken by the Author, and the preliminary designs having been prepared and provisionally accepted, the necessary experimental work was carried out during the summer and autumn of 1909. The first meters were constructed during 1910, and erected on the Rand under the Author's supervision during 1911. Since that date a number of additional meters have been sent out to Johannesburg, so that at the present time the aggregate capacity of the air-meters installed is over 300 million horse-power-hours per annum.

In this Paper the Author proposes to give some account of his work in connection with these meters and of the subsequent work carried out by himself and by the firm with which he is associated in developing the commercial metering of air, gas, and steam.

[1] A full account of this scheme will be found in Mr. A. E. Hadley's Paper on "The Power Supply on the Rand" (Journal Inst. Elec. Engineers, vol. 51 (1913), p. 2). See also Mr. G. M. Clark's Paper, Journal South African Inst. Engineers, 1914 (abstract in Minutes of Proceedings Inst. C.E. vol. ccxix. p. 523).

The Air Unit.

Under the terms of the agreement which was drawn up between the Power Company and the consumers, it was stipulated that air-meters should be obtained which would register accurately to within $\pm$ 3 per cent.

It was provided that the Power Company and the consumers should each install their own meters, which should preferably operate upon different principles, and that the charges should be based upon the mean of the readings of the two meters thus installed at each point of supply.

It was further provided that the air unit, in terms of which the meters were to register, should be the quantity of air which would be compressed from mean atmospheric pressure and temperature on the Rand [1] to the pressure of delivery, by the expenditure of one kilowatt-hour of energy in an isothermal compression process of the same overall efficiency as that which obtained between the indicated power in the steam-cylinders and the air delivered in the case of certain specified compressors then in use upon the Rand; it being understood that these compressors should be put in first-class order for the purpose of the test and run under normal conditions as regards speed and pressure.

In choosing a suitable energy unit for the measurement of compressed air, it has to be considered whether the basis of the measurement should be the energy required to compress each pound of air to the temperature and pressure of delivery or use, or the energy available in each pound of air at that temperature and pressure when compressed. Either method would form a satisfactory basis of sale; but it is obvious that the former, which is more closely related to the cost of production, is for that reason the more satisfactory.

The formulas which give the number of kilowatt-hours required, on the basis adopted by the Victoria Falls and Transvaal Power Company, to compress isothermally the weight of air delivered per second to the pressure at the point of supply are established in Appendix V.

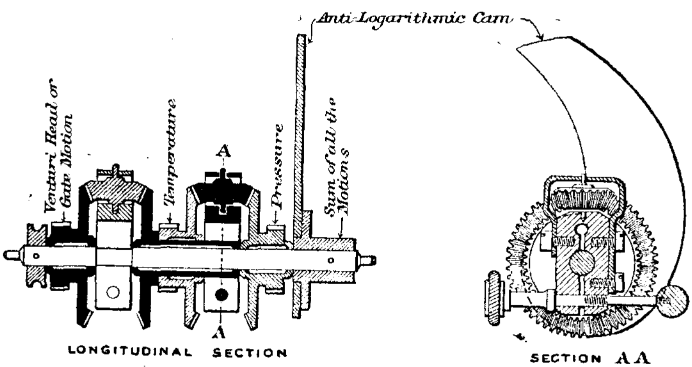
The "Venturi" and "Gate" Air-meters.

In the type of meter which the Author designed for the Power Company, the flow of air was measured in terms of the differential pressure obtained from a Venturi tube of known up-stream area and throat-ratio; while in the type of meter which he designed for the consumers, the flow was measured in terms of the angular displacement of a weighted gate hung in the air-way.

[1] 12·086 lbs. per square inch (absolute) and 60° F.

In order to simplify the construction of the meters, it was agreed that, except in those cases where it was found necessary to measure the flow in close proximity to the compressors, the correction for temperature should not be automatic; but that a given mean temperature of delivery should be assumed for each mine, and the recorders should be set so as to register in accordance with these various mean temperatures. Assuming, then, that the temperature in any particular case is constant, it will be seen from the equations obtained in Appendix V that in the case of the Venturi meter the air-units measured can be taken, without sensible error, as equal to the product of a constant multiplied by functions of the pressure in the main and the differential pressure obtained from the Venturi tube; and in the case of the gate meter, to the product of a constant multiplied by functions of the angle moved through by the gate and the pressure in the main.

Figs. 1.



By the employment of suitable devices, movements are obtained which depend upon the pressure and the flow, and which are proportional respectively to the logarithms of the functions to be multiplied. These movements are then added together on a differential gear, *Figs. 1*, which communicates to the central spindle, on which it is carried, the algebraic sum of the motions transmitted to it. The motion of this spindle is therefore proportional to the logarithm of the product of these functions, and, by means of an anti-logarithmic cam and feeler, it is transformed into one proportional to the product itself. In this way an angular displacement proportional to the rate of flow in air-units per hour is obtained.

Amounts proportional to this angular displacement are added to the counter-reading at equal intervals of time by means of a clutch, one member of which is rotated continuously at a uniform rate by a small air-turbine controlled by a centrifugal escapement.

The other member, which is connected through a train of wheels to the counter, is thrown into gear at a fixed point in the revolution of the first member; and out of gear at a variable point, the position of which is determined by the angular displacement proportional to the rate of flow referred to above. The adjustment for mean temperature is effected very simply by altering the setting of the anti-logarithmic cam on the differential-gear spindle[1] in accordance with a scale of temperatures engraved thereon. The counter thus registers the total number of air-units passed.

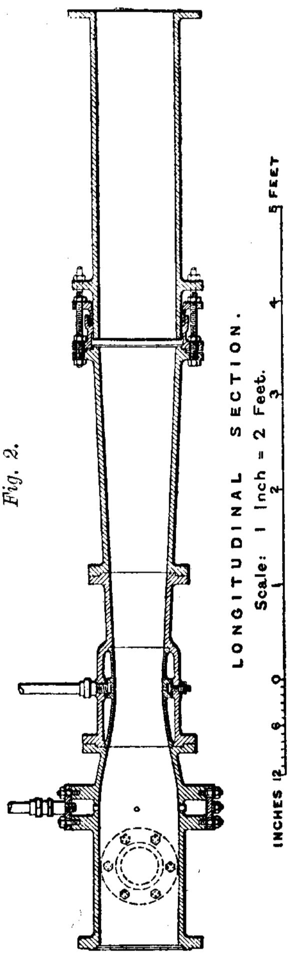


Fig. 2.

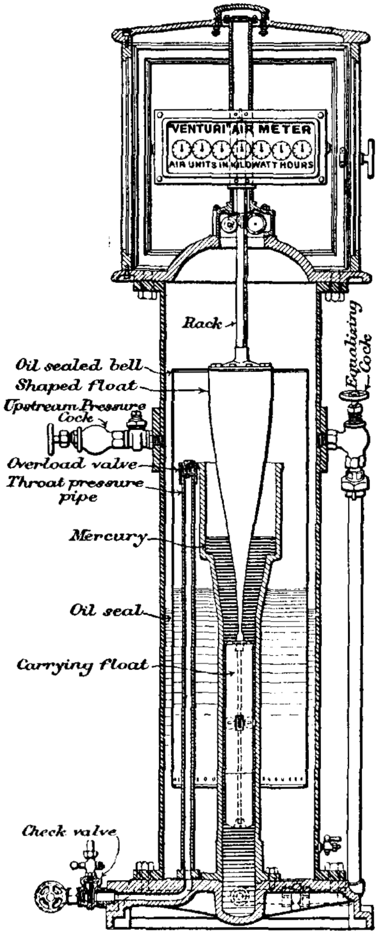
The size of the throat of the Venturi tube installed at each point of supply is chosen so that in every case the same differential pressure is obtained at the maximum flow called for by the consumer. This enables all the recorders, whose action depends upon the differential pressure, to be identical and interchangeable. When it becomes desirable to increase the capacity of any meter, it is only necessary to replace the existing throat-section of the Venturi tube by one of larger diameter, and to alter the gear-wheels in the counter-train of the recorder. A special expansion-joint is provided in connection with the Venturi tube to enable this exchange to be made easily and quickly.

The gate meter is arranged so that its capacity can be increased even more simply. This is effected by increasing the loading of the gate in such a way that the ratio of the initial and final moments tending to close it remains unchanged, i.e., if the change of loading is always made in such a way that the centre of gravity of the moving parts remains on the line passing through the original centre of gravity and the

[1] Each unit of angle through which the anti-logarithmic cam is advanced multiplies the rate of registration by a specific amount.

axis of the bearings, then the original calibration still holds good, except that the quantity passed by the meter is increased in the ratio of the square root of the new loading to that of the old.

Fig. 3.



LONGITUDINAL SECTION
(FRONT VIEW)
Scale: ¾ Inch = 1 Foot.

Fig. 2 (p. 111) shows a section through the Venturi tube and the expansion-joint. The "throat-ratios" of the tubes supplied range from about 2·0 to 5·0.

Fig. 3 is a section through the Venturi recorder, which shows the arrangement whereby the differential pressure obtained from the Venturi tube is measured.

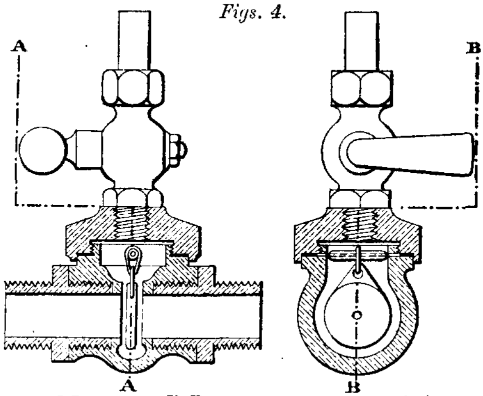
It consists of a light inverted bell immersed in an oil seal, the throat pressure acting on the inside of the bell, and the up-stream pressure on the outside. An increase of the flow thus causes the bell to descend. The weight of the bell is taken by a carrying float which is always totally immersed in mercury. The displacement of the bell is made proportional to the logarithm of the number of air-units passing by means of a shaped float, the bell descending until the increase in the difference in pressure is balanced by the increase in buoyancy.[1] The carrying float is built up of slate and vulcanite in the ratio of 1 to 2·46 in order to compensate for changes of temperature, and so prevent error in the zero flotation-level. The arrangement is sensitive to a Venturi head of less than $\frac{1}{10,000}$ lb. per square

[1] The formulas for calculating this float are derived in Appendix VI.

inch, and will measure Venturi heads up to 0·85 lb. per square inch, an accurate range of flow-measurement down to one-twentieth of the maximum flow being obtained. A valve is provided whereby the pressure on both sides of the bell may be equalized and its zero checked. Various protective devices are installed in order to prevent the oil seal from being blown over by careless manipulation of the valves. Among these is the small check-valve shown in *Figs. 4*, which is fitted between the up-stream and down-stream cocks and the bell-chamber. This consists of a circular plate suspended between two machined faces, against one or other of which it is blown if there is any sudden flow into or out of the bell-chamber. A small hole in the plate allows the pressure to equalize gradually.

The pressure of delivery, p_1, is measured by means of a battery of

Figs. 4.

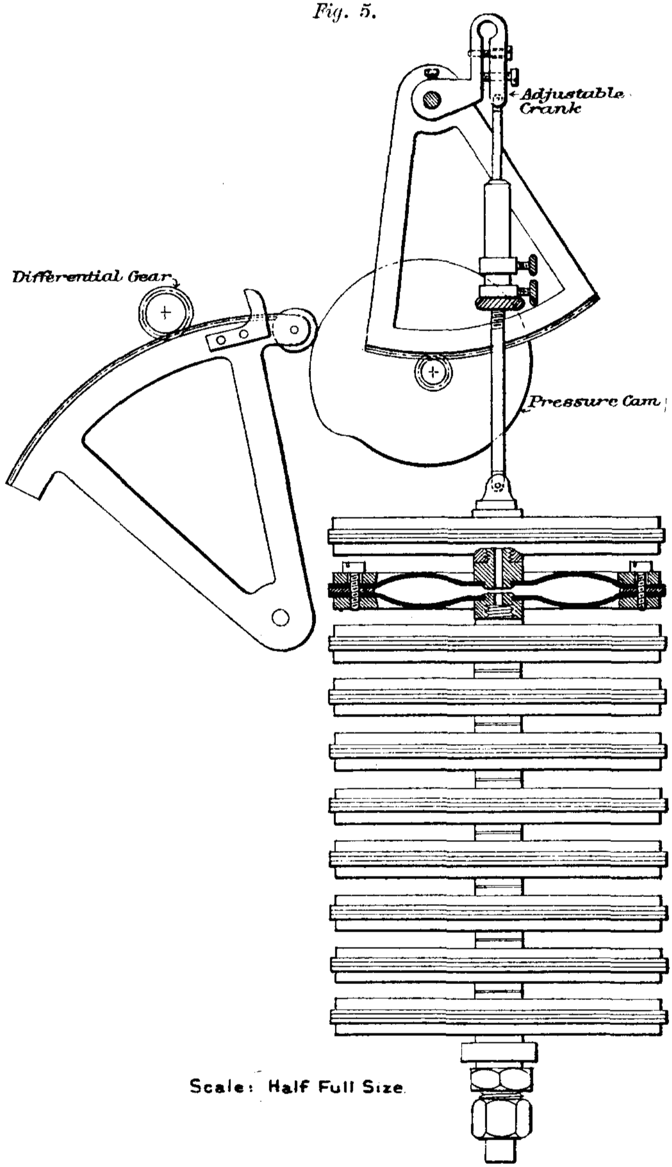


SECTION B B. SECTION A A.

steel diaphragms, which are shown in *Fig. 5.* The required angular motion proportional to log $[p_1^{\frac{1}{2}} \log p_1/p_a]$ is obtained by means of a cam. These diaphragms are made of untempered steel of special quality. They give a small movement of considerable power, and the resulting motion of the cam has rather less lag due to the elastic inertia of the metal than that observed on an ordinary test gauge of good quality.

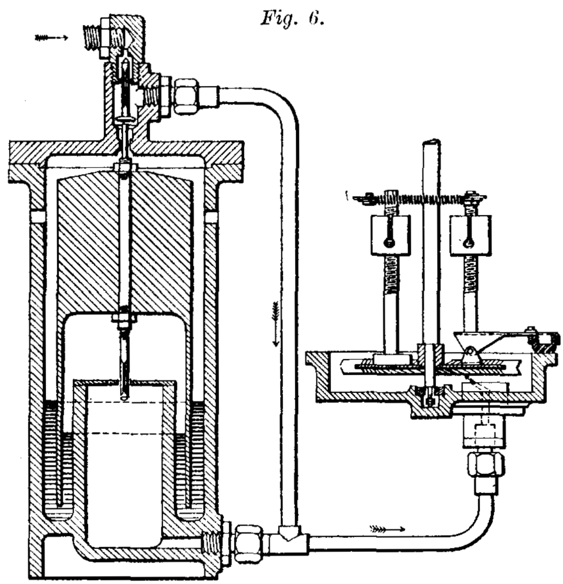
The centrifugal escapement, with its air-turbine and regulating valve, is shown in *Fig. 6.* It will be seen that with an increase of speed there will be an increase in the friction between the brake and the surface upon which it rubs, due to the increase in the centrifugal force of the movable weight. It can be shown from theoretical considerations, and is borne out by actual tests, that after such a speed has been reached that the brake is continuously in contact with the braking surface the revolutions

Fig. 5.



per minute are increased by approximately the same amount for each pound rise in the pressure at the turbine-jet. As this increase in the revolutions per pound increase of pressure may be made small, it is possible to obtain very close regulation of the speed if an accurate reducing-valve is installed between the variable high pressure in the main and the desired constant low pressure at the turbine-jet. In their latest form these centrifugal escapements run at about 600 revolutions per minute, and there is an increase of about 3 revolutions per minute for each pound change in the air-pressure at the turbine-nozzle. The standard pressure at the turbine-nozzle is 1 lb. per square inch, and the regulating-valve is capable of maintaining this pressure constant

Fig. 6.

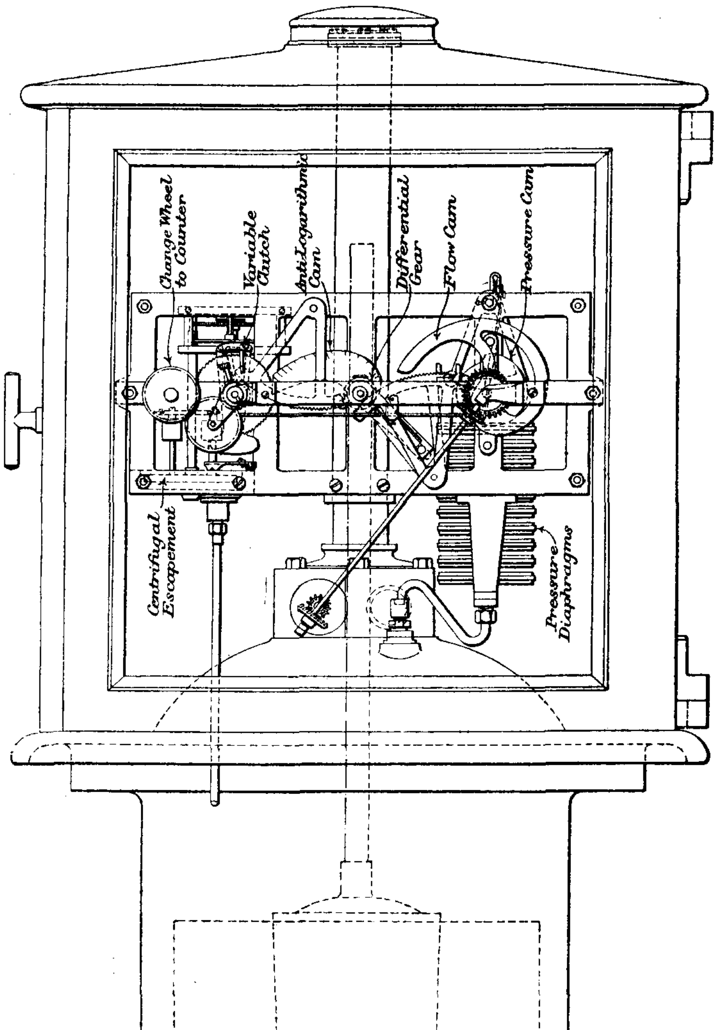


SECTIONAL ELEVATION
Scale $\frac{1}{3}$ Full Size

with a variation of less than $\frac{1}{10}$ lb. for any pressure in the main between 60 lbs. and 120 lbs. per square inch. The maximum error in the speed of the turbine, therefore, does not exceed $\frac{1}{10} \times \frac{3}{600} \times 100 = \frac{1}{20}$ per cent., and the actual average error is less than this. When a regulating-valve has to operate between large differences of pressure it is difficult to prevent it from hunting. This difficulty is overcome in the valve shown in the Figure by constricting, at one point, the area of the passage which communicates the pressure at the turbine-jet to the underside of the bell.

The centrifugal escapement described above is replaced by a

Fig. 7.



BACK ELEVATION OF RECORDER.

Scale: 2 Inches = 1 Foot.

INCHES 6 5 4 3 2 1 0 6 INCHES

clock or an electric motor in cases in which the small passages in the regulating-valve are liable to be choked by dirt or oil, or by frozen moisture deposited from the air.

Fig. 7 shows the general arrangement of the mechanism of the Venturi recorder, which shows the positions of the pressure diaphragms and the various cams and the centrifugal escapement. The motion of the bell is transmitted to this recorder through a gland, the spindle of which is rotated by means of a rack attached to the bell itself.

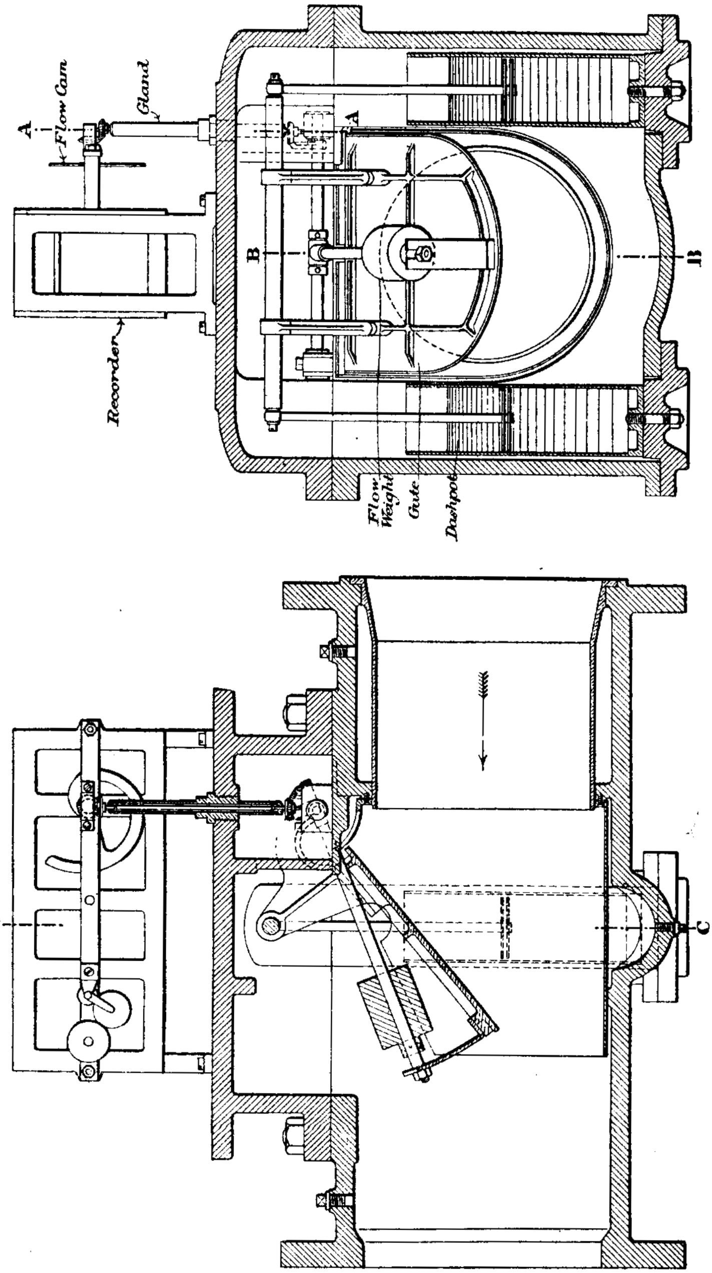
Figs. 8 are sections through one of the gate meters supplied to Rand Mines, Limited. The drawing shows the hanging gate and the dashpots by which any tendency of the gate to oscillate is checked. The bar which carries the weight by means of which the capacity of the meter is altered is also shown. This bar passes through the centre of gravity of the gate, and is radial to the axis about which the gate moves. The motion of the gate is transmitted, by bevel wheels which are housed in a special pocket out of the way of the air-flow, through a gland to the recorder which is placed above.

The gate meters are so designed that, even when working at their maximum capacity, the pressure-drop across the gate is always small. Under these conditions the stream-lines are practically identical for water and air, and the weight of fluid passing for any particular gate-opening and gate-loading is inversely proportional to the square root of the relative densities of water and air. It is therefore possible to calibrate these meters accurately by means of water flows, if the buoyancy of the gate during the water calibration is known and compensated for.

The Cut-out Valve.

As neither the Venturi meters nor the gate meters are arranged to register at a higher load than the maximum for which they happen to be adjusted, the Author designed a simple type of "cut-out" valve which comes into operation when the maximum capacity of the meter is exceeded. This valve, which is shown in *Figs. 9*, consists of a hanging gate so weighted that it always tends to remain open. By means of a screw passing through the casing, the gate can be set at various distances from the orifice which it closes. The air passing tends to close the gate; and it will be obvious that the nearer to the orifice the gate is set, the smaller will be the flow at which it closes. The exact flow at which the valve closes is indicated by a scale against which the position of the screw is read, the graduations being determined by actual

Figs. 8.

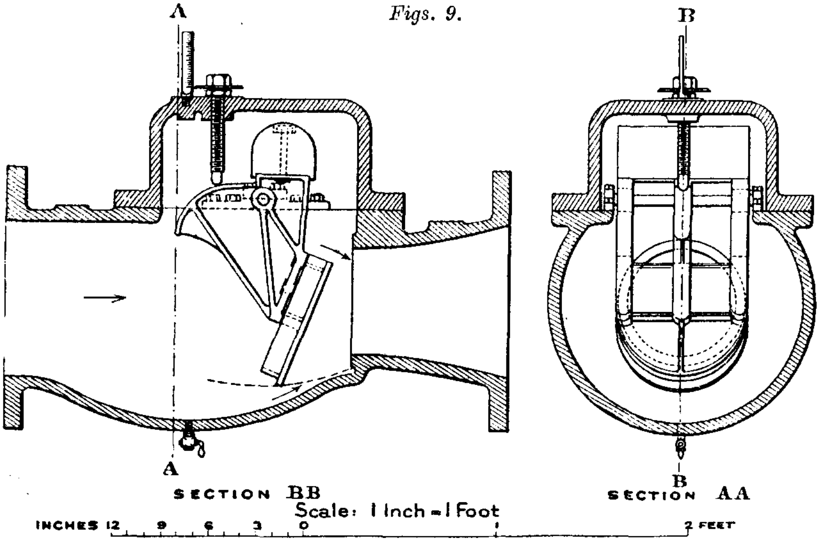


SECTION THROUGH A A & B B. Scale: 1½ Inch = 1 Foot. PART SECTION THROUGH C C.

INCHES 6 5 4 3 2 1 0 1 2 FEET

calibration.[1] In addition to preventing overloading of the meters, these valves have proved very useful in restricting the rate of supply of air at any particular point to that contracted for by the

Figs. 9.



consumer, and in this way they have reduced very considerably the difficulties of the power-station in maintaining the necessary air-supply at all points of the system.

The Limit Valve.

Another type of valve which the Author designed in 1910 is so arranged that only a fixed maximum quantity of air can be taken. This valve, which may be termed a limit valve, does not cut off the consumer when the flow for which he has contracted has been reached, but simply limits the air he can take to that maximum. It consists of a balanced throttle-valve moved vertically by a

---

[1] *Theory shows that the rise of pressure $p - p_1$ due to the sudden closing of* this valve cannot exceed that given by the relation

$$p - p_1 = \frac{V}{12}\left(\frac{\gamma p_1 W_1}{g}\right)^{\frac{1}{2}} \text{ lbs. per square inch.}$$

Thus, when $V = 40$ feet per second.

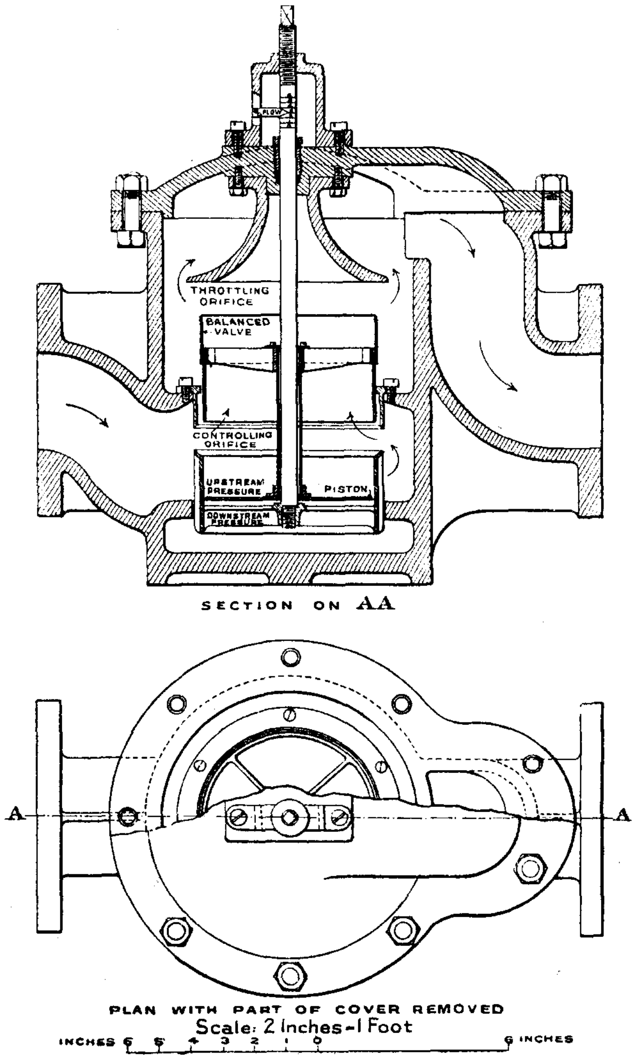
$p_1 = 120$ lbs. per square inch abs.

$f = 60°$ F.

$p - p_1$ (for air) $= 6 \cdot 03$ lbs. per square inch.

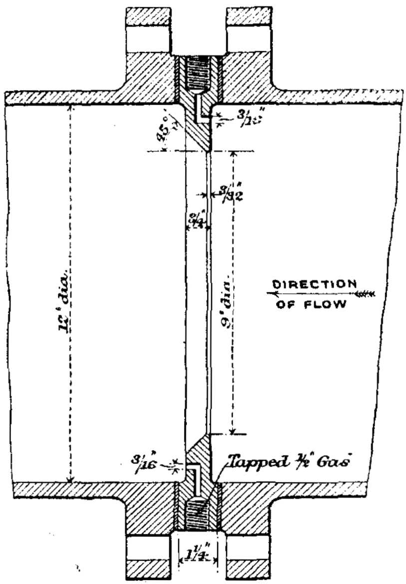
piston on the same spindle, the piston being actuated by the difference of pressure across an orifice through which the air passes.

Figs. 1C.



As soon as the flow for which the valve is set is slightly exceeded, and the lifting force on the piston, due to the difference of pressure across the orifice, becomes greater than the weight of the moving part, the piston rises and partially closes the valve; thus throttling the flow until the load on the piston is again balanced by the drop of pressure. *Figs.* 10 (p. 120) shows the arrangement adopted in order to obtain a perfectly-balanced throttle-valve. It will be seen that the valve consists of two concentric cylinders, and that the pressure on the two ends of the inner cylinder is always that of the air on the upstream side of the valve, while the pressure on the two ends of the outer cylinder is always that of the

Fig. 11.



SECTIONAL ELEVATION.
Scale: 2 Inches = 1 Foot
INCHES 0 1 2 3 4 5 6 INCHES

air on the downstream side. The upper edge of the outer cylinder is bevelled as shown, in order to prevent an unbalanced force from being introduced by the suction which would otherwise act on this edge when the valve is nearly closed down.

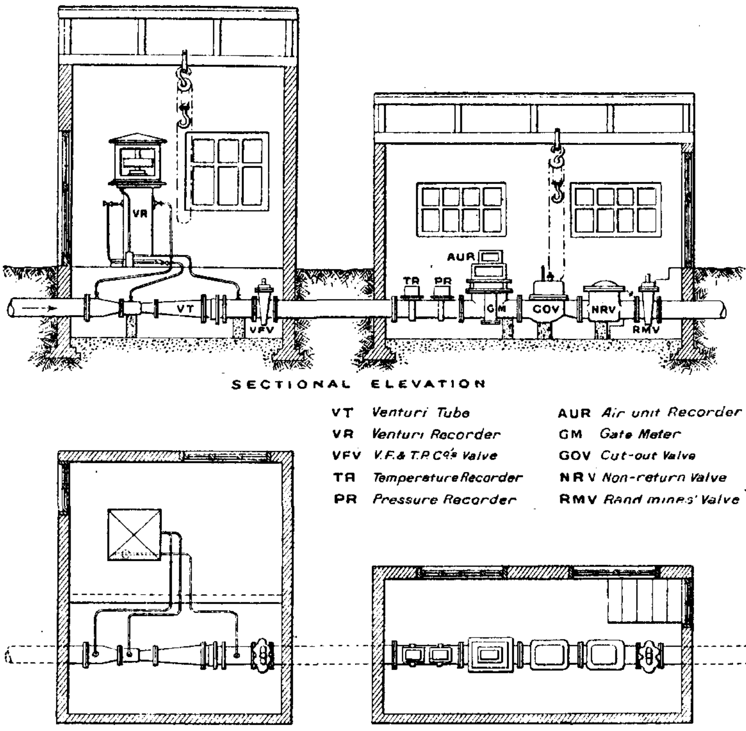
The valve can be set to regulate at various flows by altering the width of the orifice formed between the top edge of the cylinder in which the piston works and the bottom edge of the casting through which the throttle-valve passes. It is arranged so that this adjustment can be made from outside while the valve is under pressure, a scale being provided which shows at what flow the valve will come into operation.

It may be of interest to note in passing that a number of these limit valves are in use in various parts of the world as filter-bed modules controlling the rate of filtration of potable water.

The Square-Edged Orifice.

The orifices shown in *Fig. 11* (p. 121), which the Author designed and calibrated in 1910, were installed in connection with manometers

Figs. 12.



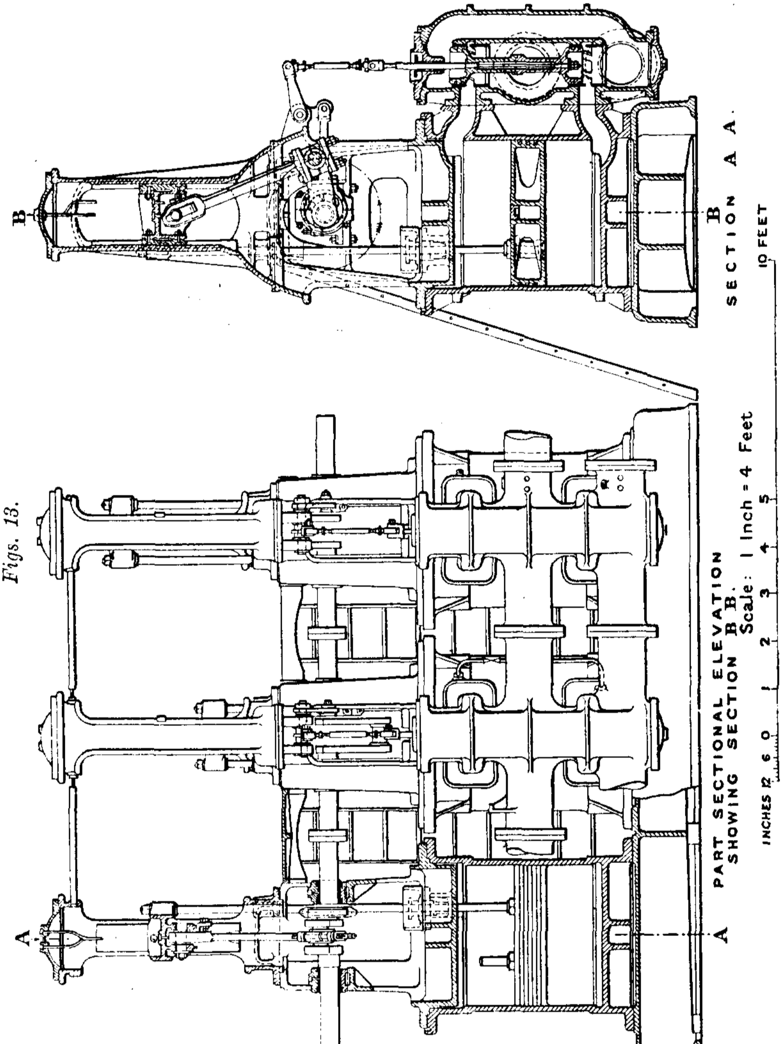
at the Rosherville and Robinson Central Deep stations in order to indicate the compressor-discharges.[1] This arrangement proved to be a very accurate and convenient means of measuring the rate of flow, and a number of similar installations are now in use upon the Rand.

A typical air-meter installation on the Rand is shown in *Figs. 12.*

[1] J. L. Hodgson: "The Metering of Compressed Air," Proc. South African Inst. of Engineers, vol. x, 1911.

THE RAND AIR-CALIBRATION PLANT.

The meters required in connection with the Victoria Falls and Transvaal Power Scheme, together with various sizes of Venturi



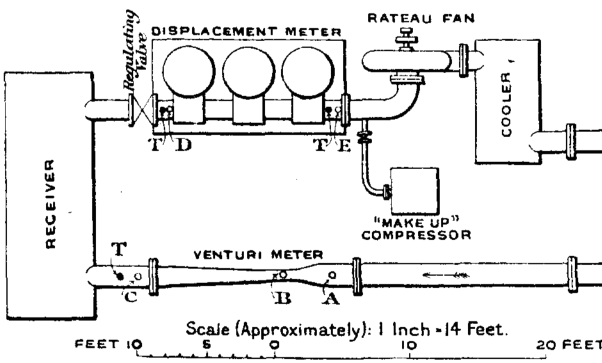
Figs. 13.

tubes, nozzles, and orifices, were calibrated on a special plant, which was designed and built at Erith by Messrs. Fraser and

Chalmers, in consultation with Mr. A. M. Robeson, Mr. A. Smart, and the Author. The plant was erected in the first instance to the Author's specification in 1909, and, after being placed at the disposal of Messrs. Kent for about 6 months by Messrs. H. Eckstein and Company at Erith, was taken down and re-erected at Ferreira Deep on the Witwatersrand. It is now the standard air-calibration plant for South Africa.

The air to be measured is passed through a large displacement meter (*Figs. 13*, p. 123) which is built somewhat on the lines of a steam-engine. It has three cylinders 36 inches in diameter, and the stroke is 27 inches. The admission and discharge of the air to the cylinders are controlled by means of piston-valves which are set to cut off exactly at the top and bottom of the stroke. The stroke-volume is 95·156 cubic feet, and the meter therefore passes 50 lbs.

Fig. 14.



weight of air per revolution at 100 lbs. per square inch (abs.) and 513° F. (abs.); or, roughly, 1 ton per minute at the maximum speed at which it is designed to run. This displacement meter forms part of a closed air-circuit (*Fig. 14*) wherein the air is circulated by a single-stage Rateau fan, which is driven by a 50-HP. motor at 2,500 revolutions per minute, and produces a difference of pressure of about 2·5 lbs. per square inch when the air is at a pressure of 100 lbs. per square inch. The pressure in the circuit is maintained by a small "make-up" compressor, requiring about 30 HP., which delivers into the circuit between the displacement meter and the Rateau fan. After leaving the fan the air passes through a cooler, so that the temperature in the remainder of the circuit may be kept constant. It is then passed successively through the meter to be tested and the displacement meter.

Figs. 15.

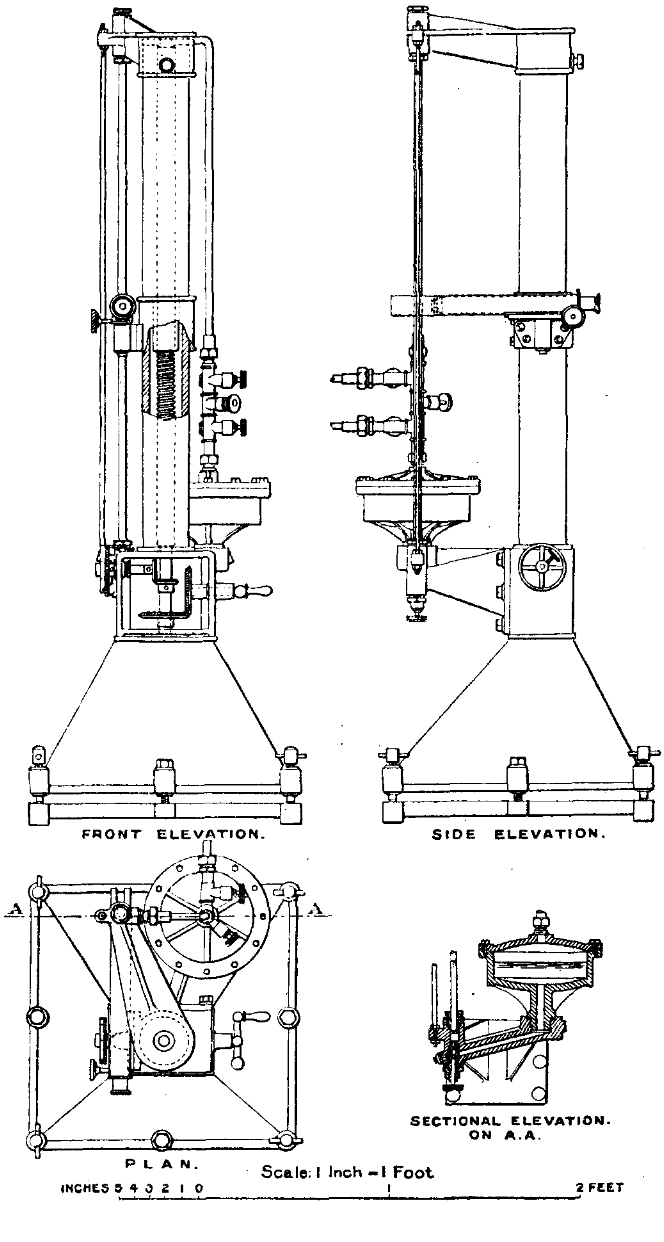
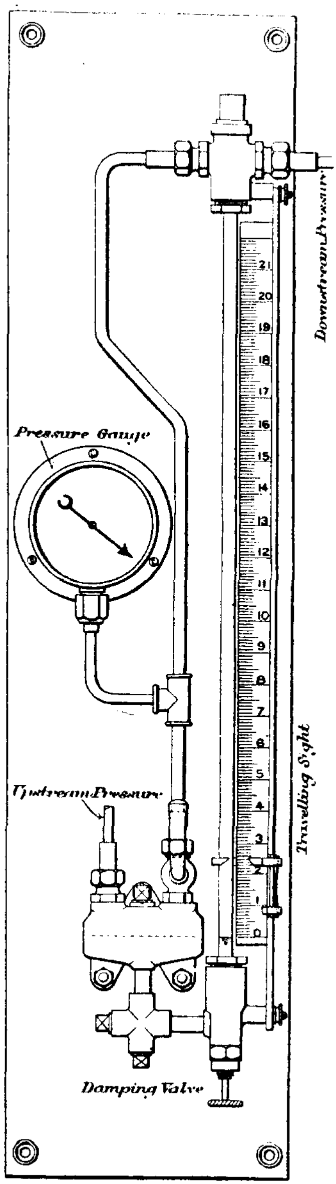

A
A
FRONT ELEVATION.
SIDE ELEVATION.
SECTIONAL ELEVATION. ON A.A.
PLAN.
Scale: 1 Inch = 1 Foot
INCHES 5 4 3 2 1 0
1
2 FEET

Fig. 16.



FRONT ELEVATION.
Scale 2 Inches = 1 Foot.
INCHES 0 1 2 3 4 5 6 INCHES

THE PRECISION MANOMETER.

In order to obtain great accuracy in the differential pressure measurements, the Author designed the precision manometer shown in *Figs. 15* (p. 125). In this instrument practically the whole change of level takes place in one limb, so that it is only necessary to read the change of level in this limb and make a correction which depends upon the ratio of its cross-sectional area to that of the reservoir into which the upstream pressure is introduced. The diameter of this reservoir is large, so that any variation in the internal diameter of the gauge-glasses fitted involves no appreciable change in the constant by which the readings of the instrument have to be multiplied. The change of level of the liquid in the gauge-glass is measured by means of a travelling microscope which is moved by a guide-screw having ten threads per inch. A graduated dial is fitted in connection with the handle by which this screw is rotated, so that readings accurate to less than $\frac{1}{1000}$ inch can be taken. One of the difficulties met with in designing an instrument that would enable a change of liquid level to be read to anything like this degree of accuracy was the difficulty of obtaining a definite and consistent reading on the meniscus. It was overcome by using a microscope of thirty diameters magnifying power and a gauge-glass of $\frac{5}{16}$ inch internal diameter, and illuminating the meniscus always in the same direction. It will be seen (*Figs. 15*) that the gauge-glass passes through a tube which slides on the end of the microscope, and into which light can enter only at the end. The inside of this tube is blackened to prevent cross-reflection. High accuracy can be obtained only by using oil or alcohol as the manometer liquid, since, even with the cleanest possible gauge-glass, a water meniscus is apt to be sluggish. The accuracy of the guide-screw, by means of which the microscope is traversed, need not be of very high order, as great precision in the measurement is only essential at the low heads.

A steady reading on this instrument was obtained, in spite of the pulsations due to the displacement meter, by damping the manometer by means of a needle-valve placed between the reservoir and the gauge-glass. In order to determine that no appreciable error was caused by this procedure, the investigation which is summarized in Table II, p. 135, was made.[1]

Various other instruments were designed by the Author for use in conjunction with this calibration plant, among them the single tube manometer illustrated in *Fig. 16:* a large number of these are now in use for measuring differential pressures.

The pressure was measured at one point only in the circuit, and differential pressures were taken by means of manometers between this and other points.

It was found that, owing to the large mass of metal in the various parts of the circuit, and more especially in the displacement meter, the most accurate results could be obtained only when the tests were taken at the temperature of the atmosphere, whatever that happened to be.

Comparison of Air and Water Calibrations.

At the same time that these tests were carried out on the Erith calibration plant, the Author carried out corresponding tests on a water calibration plant which had been installed at Messrs. Kent's works at Luton during 1908. The object of these tests was to determine to what extent the air- and water-discharges through the same nozzle were comparable, and whether allowance had to be made for any more complicated relation than that, for a given difference of pressure across the nozzle, the weight discharged per second varied

[1] See also Appendix VIII.

as the square root of the density of the fluid passing. This comparison was all the more important owing to the cost of calibrating large air-meters, which measured up to, say, 4,000 HP., with air. An investigation, summarized in Table I, had shown that the power required for calibrating any given meter in a closed circuit, such as was employed in the Luton and Erith plants, varied inversely as the square root of the density of the fluid circulating. On reference to the Table, it will be seen that the cost for power when using water as the calibrating fluid is only 1/28·6th part of the cost when air at atmospheric pressure is used, and $\frac{1}{925}$ part of the cost when superheated steam is used. The economy effected in the cost of calibration is actually much more marked than is indicated by the figures in the Table, as the capital outlay and the staff required are much less in the case of the more efficient plants.

TABLE I.—COMPARISON OF POWER REQUIRED FOR VARIOUS TYPES OF CALIBRATION PLANTS.

The following Table gives, according to the type of calibration plant used, the approximate power in kilowatts required for testing a 6-inch meter, which has a capacity of 6 lbs. of air per second at 80 lbs. per square inch (abs.) and 60° F., and which causes a friction drop of 1 lb. per square inch in the main.

The Table has been calculated on the following assumptions :—

(*a*) That where the meter is calibrated in a closed circuit, there is an additional fall of pressure of 1 lb. per square inch due to friction in the circuit ;

(*b*) That the efficiency of the compressing or circulating plant is 0·64 ;

(*c*) That 20 lbs. of steam, if used for the purpose of producing power instead of for the purpose of calibrating meters, would produce 1 kilowatt-hour.

		Kilowatts.	Relative Figures.
I	Discharging steam at 80 lbs. per square inch (abs.) and 100° superheat through the meter and a valve into a condenser	666	925
II	Discharging air at 80 lbs. per square inch (abs.) through the meter and a valve to atmosphere	598	830
III	Discharging steam at 20 lbs. per square inch (abs.) and 100° superheat through the meter and a valve into a condenser	348	483
IV	Circulating air at 14·7 lbs. per square inch (abs.) in a closed circuit . . .	20·63	28·6
V	Circulating air at 80 lbs. per square inch (abs.) in a closed circuit.	8·82	12·2
VI	Circulating water in a closed circuit . .	0·721	1

These comparative tests between air and water have been continued at intervals during the last 6 years. The following are the Author's general conclusions as to the relation between liquid and gaseous flows through nozzles and orifices.

In considering these conclusions, it will be necessary to refer to the discharge-formulas (Nos. 24 to 29) for nozzles and orifices which are derived in Appendix II. (The symbols and constants used are tabulated in Appendix I.)

For high values of the ratio P_2/P_1 (say when P_2/P_1 is less than 1·0 and greater than 0·98, or when $(P_1-P_2)/P_1$ does not exceed one-fiftieth) the coefficient of discharge for a nozzle or orifice is sensibly the same for water and air flows, provided that the discharge in the case of the water flow is proportional to the square root of the pressure-difference across the nozzle. If the water discharge does not follow a square-root law—and it usually does not do so unless special precautions have been taken in designing the nozzle—there is, except where P_2/P_1 is very nearly unity, no simple and exact relation between water and air flows.

For lower values of P_2/P_1, say between 0·98 and 0·527 in the case of air, the discharge-coefficient,[1] which may be as low as 0·85 in the case of a shaped nozzle, approaches unity as P_2/P_1 diminishes.

It finally reaches unity, or some limiting value between 0·97 and unity, about the same time that the ratio P_2/P_1 approaches the critical value 0·527, at which value the velocity of the particles in the throat of the nozzle is theoretically equal to the velocity of sound at the temperature and pressure which prevail there. Above this value (i.e., when P_2/P_1 lies between 0·527 and zero) the discharge-coefficient becomes constant at some value between 0·97 and unity.[2]

The value of the discharge-coefficient for air flows may therefore differ very considerably from the discharge-coefficient[3] for water flows when the value of P_2/P_1 deviates appreciably from unity. It is possible, however, to construct a shaped nozzle which has a constant discharge-coefficient of about 0·985 over a range of 1 to 30 when tested with water, and for such a nozzle the air coefficient will be sensibly constant for all values of P_2/P_1.

In this connection it is interesting to note that a shaped nozzle for which the water discharge does not follow a square-root law, has

[1] In equation (8).

[2] In the case of steam flows the discharge-coefficient may exceed unity, owing to heat being lost by radiation from the nozzle.

[3] In equation (11).

for air a discharge-coefficient that is between 0·97 and unity when the value of P_2/P_1 lies between 0·527 [1] and zero.

The most usual relation which is found to exist between the drop of pressure across a shaped nozzle and the water discharge through it above the critical velocity, is that the discharge varies as the *mth* root of the drop of pressure, where *m* has some value between 1·94 and 2·2. This deviation from the square-root law—which can be obviated if the walls of the nozzle immediately upstream of the section at which the throat pressure is taken approximate to the shape that the degree of convergence of the walls still farther upstream would impose on a jet discharging from them, and if the throat pressure is not measured downstream of the section at which the walls of the nozzle again become parallel to the axis [2]—is comparatively unimportant unless the nozzle is required to be used for measurements over a large range of flow. In that case, if the discharge be calculated on the assumption of a constant coefficient of discharge and a square-root law, the actual discharge for badly-designed nozzles may differ from the calculated by as much as ± 5 per cent. at the extreme flows.

Figs. 17 show how the discharge coefficient varies with P_2/P_1 for various typical nozzles and orifices. It was found experimentally that the value of the air-discharge coefficient in the limiting case, when P_2/P_1 equals unity, agrees approximately for each of these nozzles with the value of its water-discharge coefficient.

The values of the discharge-coefficient given in Cases I, II, and III are for small nozzles. For larger nozzles these values would be more nearly unity.

Case I shows a badly-designed nozzle. The discharge-coefficient would be much more constant, and more nearly equal to unity, if the parallel part were removed.

Case II shows a better form.

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[1] On reference to *Fig. 17* it will be seen that these changes do not take place exactly at the theoretical value 0·527. The existence of a change in the nature of the flow at or about the theoretical value can best be shown by plotting actual values of V_1 against P_2/P_1 for a shaped nozzle which has a constant discharge-coefficient at the smaller values of P_2/P_1; such a nozzle is shown at II, *Fig. 17*. It will be found that at or about the theoretical value, and for all values of p_2/p_1 which are less than this, the velocity is constant. In the case of a square-edged orifice such as that shown at IV, *Fig. 17*, and for which Ω varies with P_2/P_1, this effect cannot be observed, as in this case there is no particular value of P_2/P_1, at which the velocity becomes constant.

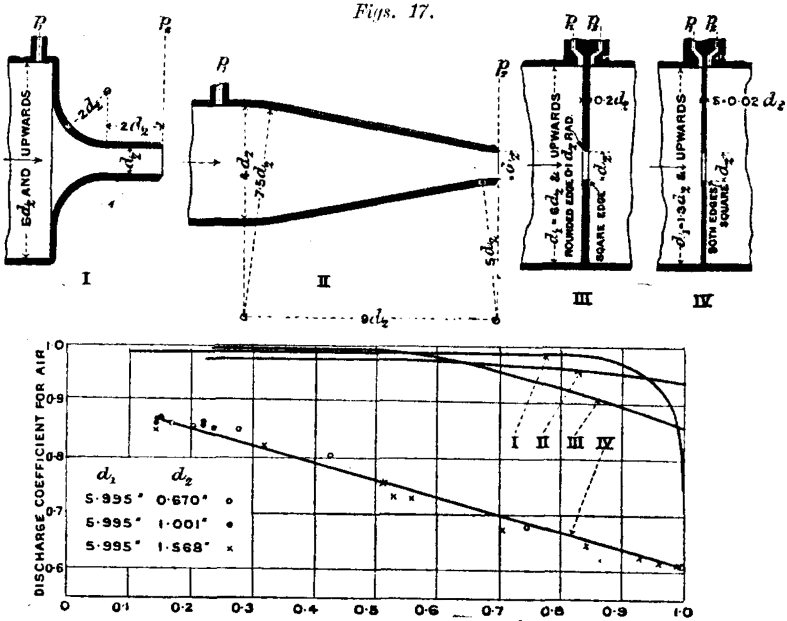
[2] It should be pointed out that the form of nozzle which Professor Gibson used in his investigation on "Abnormal Coefficients of the Venturi Meter" (Minutes of Proceedings Inst. C.E., vol. ccxix, p. 391) does not conform to either of these conditions.

Tests on Square-Edged Orifices.

In 1910 the Author carried out a number of tests on the discharge through square-edged orifices, in order to establish an easily reproducible standard whereby the flow of air, steam, and water might be measured.

The coefficient of discharge for water for such an orifice, when it is fitted in a length of bored pipe, and the upstream and downstream pressures are measured in the plane of the orifice, as shown at Case IV, *Figs. 17*, does not differ materially from 0·608 for values

Figs. 17.



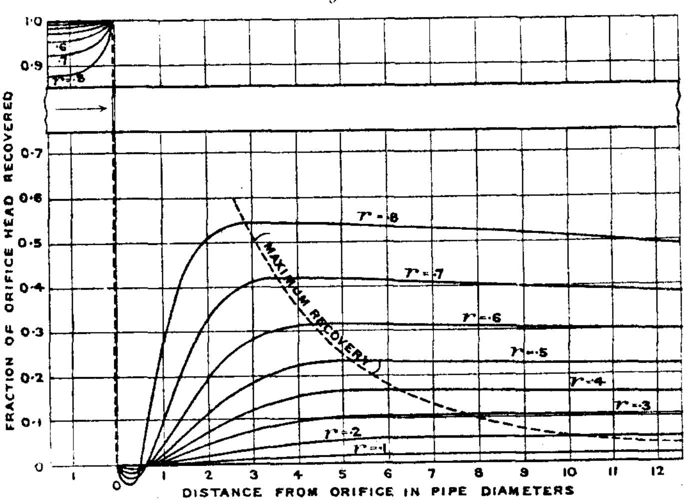
VALUES OF P_2/P_1.

of the ratio of the diameter of the orifice to the diameter of the pipe, d_2/d_1, which lie between zero and 0·7. For values of d_2/d_1 which are greater than this, the discharge-coefficient gradually diminishes, and is so sensitive to minute variations in the conditions under which the orifice is installed, owing to the rapid variation in the pressure immediately upstream and downstream of the orifice, that it is preferable to calibrate each individual orifice in place.

It is important that the pressure holes should be in the plane of the orifice, especially when d_2/d_1 is large, if the results obtained from experiments with orifices in one size of main are to be used in calculations for mains of other diameters; because when d_2/d_1 is large there is a rapid increase of pressure immediately upstream of the orifice and a further fall of pressure, and then a rapid recovery, on the downstream side. *This necessitates that the position of the pressure holes in different sizes of pipes should be geometrically similar if the discharge-coefficients for orifices having the same values of* d_2/d_1 *are to be identical.*[1]

The curves plotted in *Fig. 18* show, for various values of d_2/d_1, and for one particular velocity of flow, how the pressure varies in the vicinity of the orifice.[2] They also show that the recovery of pressure is greatest for large values of d_2/d_1, and that the point of

Fig. 18.



maximum recovery of pressure travels farther downstream as the ratio d_2/d_1 is reduced. It is worthy of note that in cases where d_2/d_1 is large, more than half the differential pressure is recovered without any special means being taken, such as the fitting of a diverging cone, to effect such recovery.

For air flows the discharge-coefficients for these square-edged orifices are identical with the water values only in the limit when P_2/P_1 is unity. As P_2/P_1 diminishes, the discharge-coefficient

[1] See also Appendix III.

[2] Compare *Fig. 2* of Mr. Holbrook Gaskell's Paper, "The Diaphragm Method of Measuring the Velocity of Fluid-Flow in Pipes" (Minutes of Proceedings Inst. C.E., vol. cxcvii, p. 250).

increases according to what is apparently a straight-line law, which, if assumed to hold beyond the limits of the experiments which the Author has been able to make, would seem to indicate a value of 0·914 when the ratio P_2/P_1 is zero.[1] The values plotted[2] were calculated from the experimental results by means of the β curve for $n = \infty$ (*Fig. 25*, p. 154) and equation (26). The Author has investigated the value of this discharge-coefficient for square-edged orifices for which n is large only for values of P_2/P_1 which lie between 0·15 and unity, and he is by no means certain of the value of the coefficient for values of P_2/P_1 which are less than 0·3, as the tests made with the smaller values of P_2/P_1 were not sufficiently numerous. This will be easily understood when it is stated that for the sizes of orifice with which the Author was working, at the time these tests were taken, more than 1,000 HP. was required when working at low values of the ratio P_2/P_1. These tests and others were taken at Ragged Chutes in Northern Ontario by the courtesy of the Northern Ontario Light and Power Company, who put their test plant at the Author's disposal during the autumn of 1913. The conditions for testing at Ragged Chutes were ideal, as a supply of dry air at a uniform temperature and pressure and free from pulsations was available from a Taylor hydraulic compressor installed under the bed of the Montreal river close to the test-house.

On reference to *Figs. 17*, it will be seen that the value of the discharge-coefficient for various values of P_2/P_1 is given with fair accuracy by the relation

$$\Omega = 0\cdot914 - 0\cdot306 \frac{P_2}{P_1}.$$

As far as the Author's investigations show, this relation also holds approximately for steam flows. A number of tests were carried out on square-edged orifices placed in a 6-inch main at Messrs. Fraser and Chalmers's works at Erith during October, 1910, using steam at 50 to 70 lbs. per square inch absolute and about 50° F. superheat. The results of these tests, which were confined to the higher values of P_2/P_1, agreed with the above formula within $\pm$ 1 per cent.

The investigation of the value of the discharge-coefficient for square-edged orifices is interesting from a laboratory point of view only, as the sharpness of the square edge is very apt to be damaged

[1] Compare also the following tests on *sharp-edged* orifices. Rateau, "Experimental Researches on the Flow of Steam through Nozzles and Orifices"; Dr. Fisher, "The Discharge of Steam through Nozzles and Orifices" (Proc, Inst, M.E. 1914, p. 927).

[2] See also the equation on this page.

by handling or by erosion due to the flow. The value of the discharge-coefficient then increases, and the discharge through the orifice can no longer be inferred with certainty from previous tests. For this reason the Author, although retaining the use of square-edged orifices for standard work[1] on account of the ease with which they may be reproduced accurately, and the exactness with which the coefficient is known, uses in commercial work orifices which have slightly rounded edges. The value of the discharge-coefficient for such orifices, though it must be determined in each case by actual calibration, changes far less with erosion. A note on the coefficient of discharge of square-edged orifices is given in Appendix III.

THE MEASUREMENT OF PULSATING FLOWS IN TERMS OF THE MEAN DIFFERENTIAL PRESSURE ACROSS A CONSTRICTION.

The equations given in Appendix II are derived on the assumption that the flow has reached its steady value and is non-pulsating. In almost every case which occurs in practice the flow to be measured pulsates to a greater or less degree. It therefore becomes a matter of considerable importance to determine to what extent the readings of instruments whose design is based on these equations are liable to be in error when the flow pulsates.

In order to simplify the investigation, it has been assumed that the flow at any instant is proportional to the square root of the differential pressure which exists at that instant between the upstream and the throat of the nozzle, and that the reading is taken on a simple manometer which is damped by means of a needle valve placed between the two limbs. Under these conditions the quantity calculated from the reading of the manometer will be in error, owing to the fact that, when the flow is pulsating, the square root of the head, which is indicated by the damped manometer, differs from the mean of the square roots of instantaneous heads over any given period of time. The actual amount of error depends upon the ratio of the amplitude of the pulsation to the maximum differential pressure produced by the flow, *and upon the wave-form of the differential pressure*. If this wave-form be unsymmetrical about the mean value, as is usually the case in practice, then the quantity calculated from the reading of the damped manometer will in general be greater or less than the true value, according to whether

[1] The actual discharge for a square-edged orifice, for which d_2/d_1 is less than 0·7, can be *predicted* from the results of previous tests with much greater accuracy than that for any form of shaped nozzle.

TABLE II.[1]—VALUES OF η.

To be used when the rate of flow of liquid through the damping orifice in the manometer is *proportional to the square root* of the differential pressure.

α.	Case I. $b = 1 \cdot 0$.  $\mu = \dfrac{1 + \sqrt{1 - \alpha}}{2\sqrt{1 - 0 \cdot 5\,\alpha}}$	Case II. $b = 2 \cdot 0$.  $\mu = \dfrac{1 + 2\sqrt{1 - \alpha}}{3\sqrt{1 - 0 \cdot 8\,\alpha}}$	Case III. $b = 0 \cdot 5$.  $\mu = \dfrac{2 + \sqrt{1 - \alpha}}{3\sqrt{1 - 0 \cdot 2\,\alpha}}$
0	1·000	1·000	1·000
0·2	0·998	1·014	0·984
0·4	0·992	1·031	0·962
0·6	0·976	1·046	0·935
0·8	0·934	1·052	0·891
0·9	0·887	1·028	0·853
0·95	0·844	0·984	0·824
1·00	0·707	0·745	0·745

TABLE II A.[1]—VALUES OF η.

To be used when the rate of flow of liquid through the damping orifice in the manometer is *directly proportional* to the differential pressure.

α.	Case I. $b = 1 \cdot 0$.  $\mu = \dfrac{1 + \sqrt{1 - \alpha}}{2\sqrt{1 - \alpha/2}}$	Case II. $b = 2 \cdot 0$.  $\mu = \dfrac{1 + 2\sqrt{1 - \alpha}}{3\sqrt{1 - \alpha\,2/3}}$	Case III. $b = 0 \cdot 5$.  $\mu = \dfrac{2 + \sqrt{1 - \alpha}}{3\sqrt{1 - \alpha/3}}$
0	1·000	1·000	1·000
0·2	0·998	0·999	0·998
0·4	0·992	0·992	0·994
0·6	0·976	0·975	0·980
0·8	0·934	0·926	0·952
0·9	0·887	0·862	0·923
0·95	0·844	0·797	0·897
1·00	0·707	0·578	0·816

[1] The derivation of the equations from which these Tables are calculated is given in Appendix VIII.

the peaks of the wave-form are relatively broader than the hollows or not.

Values of the coefficient, η, by which the discharge, as calculated from the reading of a damped manometer, must be multiplied in order to determine the true flow are given in Tables II and II A for three wave-forms which may be considered as approximately typical of the mean and of the extreme cases which occur in practice. It will be seen from the Tables that this coefficient varies very considerably with the wave-form, as well as with the amplitude of the pulsation, but that its variations with wave-form are, for such amplitudes as usually occur in practice, not great when the manometer is damped with a *viscous* liquid. This fact is of considerable importance when designing apparatus for measuring pulsating flows.

Another source of error is introduced by the pulsation of the pressure in the main. It is quite possible, for instance, to obtain a definite indication on a manometer which is connected to adjacent holes in the wall of a parallel pipe in which there is no flow, if the pressure in the main is pulsating, and one of the limbs of the manometer is of distinctly larger capacity than the other; provided that the pressure-pipe connected to this larger capacity has a different coefficient of discharge when the capacity is filling up from the main from that which it has when the capacity is again discharging into the main. In order to avoid this error, constrictions which are of very much less area than the pressure-pipes or the pressure-holes in the nozzle or orifice, and which are so shaped that their coefficient of discharge in both directions is the same, should be inserted in the pressure-pipes between the manometer and the main.

If the capacities of the pressure-pipes between these constrictions and the manometer are not equal, no additional error will be introduced into the manometer reading, but if the pressure is suddenly turned on when the manometer is not equalized, the liquid will be blown over owing to the pressure rising more quickly on that side in which there is least capacity. Also, if the capacities are equal and the constrictions are different in area the same thing will occur.

A rather different type of error must be guarded against when determining the rate of air-consumption of high-speed petrol-engines by means of a nozzle or an orifice and a manometer, or by means of a gate meter, such as is shown in *Figs. 8*, p. 118.

In the case of a four-cylinder engine there may be as many as 5,000 pulsations per minute, whose wave-form approximates to that dealt with in Case III, Table II, and for which α may be as high as 0·8 or 0·9.

For pulsations of this frequency the inertias of the water column or the gate are relatively so great, that perfectly steady, and apparently quite accurate readings, may be obtained, which, however, may be in error by as much as 15 per cent.

In such cases as these it is necessary to employ an antipulsator in order to reduce the value of a as far as possible before applying the correction indicated in Table II.

The antipulsator consists essentially of a chamber fitted with a flexible wall placed on the downstream side of the meter. The flexible wall must be so devised that only a small change of pressure is required to produce a large displacement. Further, it must be combined with some device which throttles the flow entering the flexible walled chamber to approximately its mean value.

In cases where the pulsating gas to be measured is under pressure, an equivalent arrangement is obtained by surrounding the flexible wall by a second chamber which is connected by a passage of large area to the upstream side of the meter.

The amplitude of the pulsation a may be measured by means of an apparatus such as has been described by Dr. Watson and Mr. Herbert Schofield.[1] There is a slight discrepancy between the values of Ω for square-edged orifices given by Dr. Watson and the value which is now put forward by the Author. This discrepancy is almost completely explained by the facts that the orifices used by Dr. Watson were not installed under the same conditions as those calibrated by the Author, and Dr. Watson's orifices were not similar to one another, nor were their pressure-holes similarly situated.

The Pulsating-Flow Meter.

The foregoing remarks, which are by no means exhaustive, indicate a few of the considerations which have to be taken into account when pulsating flows are measured by means of the differential pressure across a constriction.

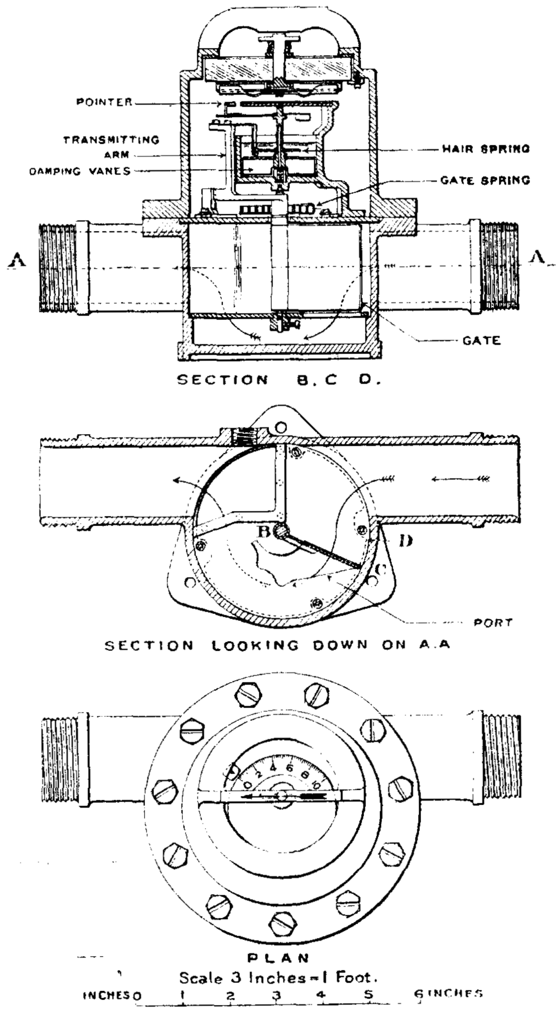
It will therefore be interesting to consider a special type of meter which is able to measure accurately pulsating flows of any wave-form and amplitude. This meter, which was designed by Mr. F. Gray and the Author, is illustrated in *Figs. 19*, p. 138.

The air on passing through the meter displaces a gate whose movement is controlled by a flat spiral spring. The gate is pivoted in the centre of a cylindrical cavity, and in moving uncovers a port

[1] "On the Measurement of the Air-Supply to Internal-Combustion Engines by means of a Throttle-Plate" (Proc. Inst. Mech. E., 1912, pt. 2, p. 517).

in the bottom of this cavity through which the air discharges to the downstream side of the meter. There is a small clearance between this gate and the walls of the cavity through which the air also

Figs. 19.



passes. The port is so shaped as to make the movement of the gate proportional to the rate of flow through the meter. The curious shape of the port necessary to obtain this result will be observed on reference to the Figure. The gate has a small moment of inertia, so that it follows each pulsation of the flow. The motion of the gate is transmitted by means of a hair spring to a pointer which is fixed to another spindle pivoted in the same axial line. The yielding transmission provided by the hair spring enables the pointer, which is damped by means of vanes immersed in oil, to remain steady while the gate itself oscillates. It will be seen, on reference to Appendix IX, that since the turning moment acting on the pointer due to the hair spring is proportional to the relative displacement between the gate and the pointer, and the rate of motion permitted by the oil is also proportional to this turning moment, the pointer will indicate the true mean flow, whatever be the wave-form according to which the gate is displaced. The particular meter illustrated was designed, in its various sizes, to indicate the rate of air-consumption of coal-cutters, rock-drills and small pneumatic tools. In its earlier forms it has been used very largely on the Rand.

The Fan Proportional Meter.

The importance of securing the highest possible economy in the use of compressed air in mining will be realized when it is stated that in the case of those mines on the Rand which use machine stoping, approximately 3 tons weight of air has to be delivered underground for each ton of ore milled, and that the power used in compressing the air which is required by the rock-drills is at least equal to the mill-engine power. This economy can be obtained by maintaining a uniform pressure of supply, by employing only the most efficient tools and maintaining these in a high state of efficiency, by laying the underground mains of sufficient capacity, and by preventing leakage. It is considered by those in charge of the compressed-air distribution on the Rand, that by careful attention to detail at least 20 per cent. of the air now required could be saved.

The meter last described provides means whereby the air-consumption of each type of machine may be determined, both on the test-bench and under working conditions. If each machine in use is brought to the surface periodically and overhauled, and its air-consumption is checked by such a meter before it is again sent underground, a very marked economy in the total air-consumption of any group of machines may be obtained: but, while considerable economies can be effected in this way, the most efficient check on the air-consumption can be obtained only by means of permanent counter-meters installed on the main distribution lines and on the principal branches. Such meters should be of simple construction and not easily put out of order. They should correct automatically for variations of pressure, though not necessarily for variation of temperature. They should preferably register in energy-units rather than by weight or volume, and they should be so arranged that a diagrammatic record of the flow can be obtained if desired.

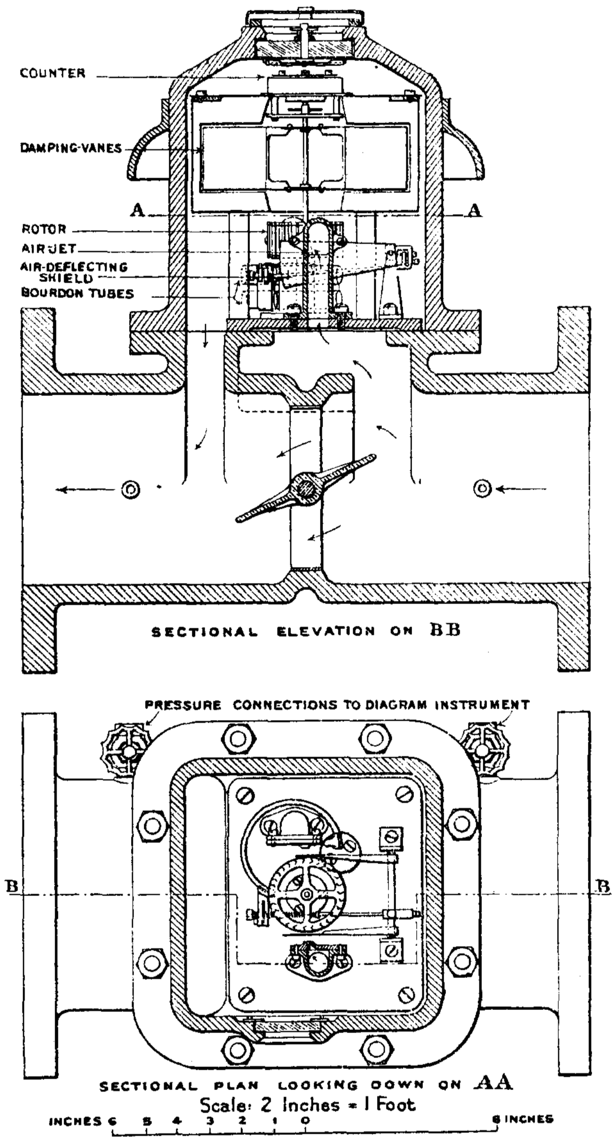
The fan proportional meter, which is shown in *Figs. 20*, fulfils these requirements. In this meter an obstruction is placed in the air-way to divert a portion of the flow through a shunt circuit, where it drives a small impulse-wheel connected by gearing to the counter of the meter. The motion of this impulse-wheel is retarded by a fan attached to it, and its speed of revolution is made to vary with the pressure, by deflecting, in such a way as not to alter the resistance of the shunt circuit, a portion of the air which drives it. The driving force on the impulse-wheel is proportional to the pressure-difference across the jets which drive it, and this, being equal to the pressure-difference across the obstruction in the air-way, varies, for any given density of air, as the square of the weight passing. Since the motion of the impulse-wheel is retarded by a fan whose resistance to rotation is proportional to the density of the air in the main and to the square of the speed of revolution,[1] the area of the undeflected portions of the jets must vary approximately as the cube of the pressure, if the speed of rotation is to be proportional to the energy passing. This is effected by making the nozzles of the necessary cross section, and deflecting a portion of the air issuing from them by plates whose motion is determined by the pressure in the main. These plates are moved by sealed Bourdon tubes which are subjected externally to the pressure of the air in the main. If these tubes have been previously evacuated, their motion will be independent of the temperature of the air and proportional to the external pressure (abs.) to which they are subjected.

In order that the meter may register with sufficient accuracy when the flow is pulsating, the parts are so proportioned that the maximum resistance to rotation is obtained for the minimum moment of inertia of the impulse-wheel and the fan. The use of heavy damping and extremely light rotating parts is also advantageous in that it minimizes the error due to counter-friction and reduces the wear on the bottom pivot by keeping down both the speed

[1] The law of resistance of rotating fans of various proportions is dealt with in a Paper by the Author on "The Fan Dynamometer" (Proc. Inst. of Automobile Engineers, 1916).

of rotation and the load to be carried. The accuracy of the meter at

Figs. 20.



low flows is greatly reduced if there is undue friction in the counter-train; though the accuracy at high flows is scarcely at all affected by this cause.

Referring to the Figure, it will be seen that two jets are used to drive the impulse-wheel in order to avoid side pressure on the pivot. There are also two Bourdon tubes to operate the plates which deflect the air, and so cause the impulse-wheel to rotate at a speed proportional to the energy passing. It will be noticed that the obstruction which causes the pressure-difference across the impulse-wheel takes the form of a butterfly valve, which is so arranged that it can be locked in various definite positions. If, now, the relation between the flow and the drop of pressure across the impulse-wheel jets is determined by calibration for these positions, each meter can be immediately set so as to deal most efficiently with any flow within its range.

The maintenance of these meters in any mine or group of mines in which they may be installed is simplified by the fact that the recording mechanism in all of them is interchangeable, the body and the butterfly-valve being the only parts which vary with the size of the main.

The whole of the moving parts can be clamped for transit by means of a single screw, and the meters can be put into use without skilled supervision.

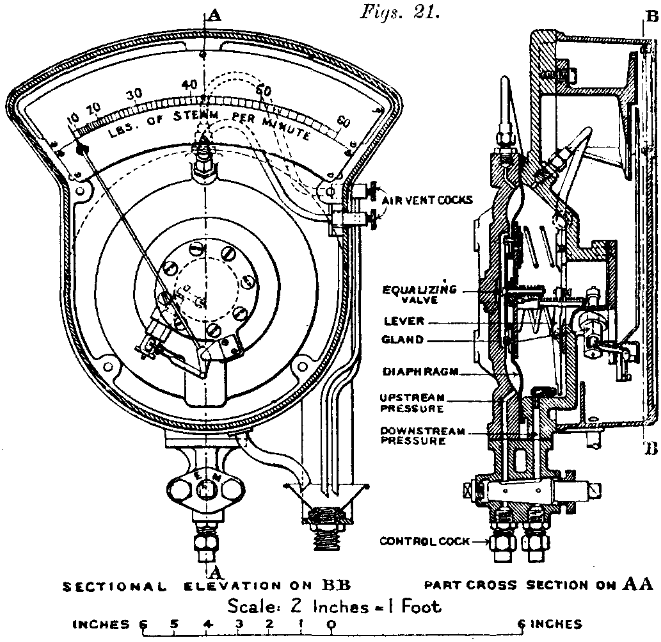
A diagram record of the flow may be obtained by connecting a differential pressure recording device, such as is described below, across the butterfly-valve. If the diagrams are divided in tenths of maximum flow, they can be used for meters of any capacity, provided that the readings are multiplied by the figure which corresponds with the size of the main and the angle at which the butterfly-valve is set.

The Steam Rate-of-Flow Indicator.

In the steam-metering instruments designed by the Author a flexible diaphragm controlled by springs is used to measure the differential pressure. It is impossible to obtain any large range of measurement with an instrument of this type unless the springs which control its movement are arranged to come against stops as the diaphragm moves, so that their "rate" increases with the head, or unless two or more diaphragms, which come into operation successively as the head increases, are used. In any instrument of this type difficulties are caused by the "lag" in the springs and in the material of which the diaphragm is made, and by the liability of the diaphragm to be damaged by careless manipulation of the pressure-cocks.

One of the simplest of these flexible-diaphragm instruments, which are equally applicable to the measurement of steam, water, or compressed air, is illustrated in *Figs. 21.* The main point of interest to be noted in this instrument is the way in which the diaphragm is protected from damage by careless manipulation of the pressure-cocks or by overload. It will be seen that if the pressure on the downstream side of the diaphragm becomes greater than that on the upstream side, the diaphragm will turn inside out and protect itself from damage by bedding on

Figs. 21.



the curved surface behind it. At the same time the upstream pressure-hole will be sealed. If, on the other hand, the pressure on the upstream side becomes much greater than that on the downstream side, then the resulting movement of the diaphragm will cause the downstream pressure-hole to be sealed and a valve, which equalizes the pressure on both sides of the diaphragm, to be opened. In the case of the steam-measuring instruments, it is very necessary that the protective devices should be arranged so that under no condition of overload or manipulation of the pressure-cocks can live steam come into contact with the diaphragm. The diaphragm-case is filled with water, and in order to ensure that the line of demarcation between live steam in the main and the water that condenses in the pressure-pipes is at the same level on both sides of the instrument, so that there is no "false head" on the diaphragm due to this difference of level, it is necessary to lay the pressure-pipes horizontally and on the same plane for some distance after they leave the steam main. These pressure-pipes may be either laid straight or bent up into the form of a flat spiral. It is necessary to adopt the latter form when the instruments are installed on shipboard, owing to the rolling and pitching of the vessel. The motion of the diaphragm is transmitted to the outside of the diaphragm-case through a gland.

The diaphragm is arranged to work in a vertical plane, in order to facilitate the expulsion of air from the diaphragm-chamber. The instrument shown in the Figure is a rate-of-flow indicator. The diagram-recording instrument is practically the same, except that a pen is attached to the indicating-pointer. The pressure-corrected counter instrument has in addition a suitable pressure-correcting device and a counter-recording gear.

The principal uses of steam-meters are in indicating or recording the rate of flow in various chemical processes, the rate of steaming of boilers, and the rate of steam-consumption of turbines. In most of these cases an indication of the rate of flow, or at most a diagram record which shows that the operator has kept to his instructions, is sufficient.

The Venturi Gas-Meter.

The metering of large gas-mains is an obvious application of the Venturi tube and its recorder. Here it offers substantial advantages over the drum type of station gas-meter, on account of its smaller initial cost and cheaper installation, and from the fact that the recorder, which is connected with the Venturi tube by pressure-pipes, need not be placed in the pipe-line.

The first Venturi gas-meter was installed at the Bromley Gas-works in 1909, and on this meter, which is still working satisfactorily, most of the practical difficulties which lay in the way of the successful measurement of gas by this method were met with. The chief of these were the deposition of naphthalene and tarry matters in the throat of the Venturi tube, and the variation of the density of the gas with the quality of the "make."

In measuring town gas, difficulty is seldom experienced from the deposition of tarry matter or naphthalene at the throat, because in the older gas-plants the bulk of the naphthalene is usually removed by "scrubbing" the gas with anthracene oil or other solvent, and in the newer plants the naphthalene is present in small quantities only, as the more modern retorts are so designed that the volatilized products are subjected to high temperatures for as short a time as possible. An inspection of the Venturi throat once every 3 months, and the provision of some simple form of heating, such as a steam radiator, for use during the colder months of the year, is all that is necessary to ensure satisfactory working.

In order to facilitate the inspection of the throat, the gas Venturi tubes are installed with a by-pass, and a special and easily removable type of throat-section is provided.

In the more modern Venturi gas-meters of the Author's design, shown diagrammatically in *Fig. 30* (p. 165), the Venturi head is measured by a water-sealed bell whose motion is transmitted by suitable means to a cam. This is so shaped that the feeler which comes into contact with it adds on to the counter-reading an amount proportional to the square root of the Venturi head at each revolution of the heart-shaped cam which actuates it. If this heart-shaped cam were rotated at a uniform speed, the rate of registration for a given flow in the main would be proportional to the square root of the density of the gas passing, and the meter would therefore only read correctly for gas of a particular density. The correction for variation in the density of the gas is obtained by making the speed of rotation of the heart-shaped cam depend upon the density by driving it by means of a small wet gas-meter which is rotated continuously by gas escaping from the main through a small orifice to the atmosphere, the pressure across this orifice being maintained constant by a specially sensitive regulating-valve. Since the rate of flow through this orifice, across which the difference of pressure is maintained constant, is inversely proportional to the square root of the density of the gas, the variation in speed of the wet meter gives the exact compensation required, and the counter registers the actual volume passing. It will be observed, from *Fig. 30* (p. 165), that the regulating-valve is compensated for changes of level of the liquid seal by means of the displacer D, and for variations in the inclination of the balance-arm by the weight C. The ratio of the area of the bell to the area of the controlled orifice is made large enough to prevent variations in the pressure from affecting the accuracy of working. In practice the valve maintains the head across the orifice O correct to within $\pm$ 0·002 inch of water. A more complete statement of the theory of this meter is given in Appendix XI. The first Venturi gas-meter to be constructed on this principle was installed in series with an existing drum-type station gas-meter at the Luton Gas-works in 1913. It has now been under observation for nearly 2 years, and over this period its readings have been found to agree with those of the station meter to within ± 1 per cent.

In concluding this Paper, the Author would like to thank all those who have placed facilities at the disposal of Messrs. George Kent, Limited, and himself to enable him to carry out the work described, and he would especially desire to thank Mr. A. M. Robeson, late Consulting Engineer to Messrs. H. Eckstein and Company, Mr. Bernard Price, Mr. G. M. Clark, of Johannesburg, Mr. M. A. Vielé, of New York, Messrs. Fraser and Chalmers and Mr. A. Smart, of Erith, Messrs. C. A. Parsons and Company, of Newcastle, Mr. Woodward, the late Manager of the Bromley Gas Company, and Mr. Phillips, the Manager of the Luton Gas Company. He would also wish to thank those who have assisted him in the actual calibration work, and in the design of the apparatus described, more especially Mr. F. Gray, Mr. E. W. Hovenden, B.Sc., and Mr. N. H. Hunt.

The Paper is accompanied by tracings, from which the Figures in the text have been prepared.

[APPENDIXES.

APPENDIXES.

APPENDIX I.

Table of Symbols and Constants.

$A_1, A_2 =$ areas in square feet.

$a_1, a_2 =$,, ,, ,, inches.

$d_1, d_2 =$ diameters in inches.

$r = d_2/d_1$.

* $n = \dfrac{\text{area of up-stream}}{\text{area of orifice or of nozzle throat}} = A_1/A_2 = a_1/a_2 = \dfrac{1}{r^2}$.

† $N = \dfrac{a_1}{[n^2 - 1]^{\frac{1}{2}}} = \dfrac{na_2}{[n^2 - 1]^{\frac{1}{2}}}$.

$s =$ thickness of orifice plate in inches.

$\psi =$ coefficient of contraction for square-edged orifices.

$\chi =$,, ,, friction ,, ,, ,, ,,

$= 0{\cdot}97$.

$V_1, V_2 =$ velocities in feet per second.

‡ $P_1, P_2 =$ pressures in pounds per square foot (abs.).

‡ $p, p_1, p_2 =$,, ,, ,, ,, ,, inch (abs.).

$(p_1 - p_2) =$ differential pressure in pounds per square inch.

$R = P_2/P_1 = p_2/p_1$.

$R_1 =$ the value of P_2/P_1 which satisfies equation (23).

$f =$ the temperature in ° F.

$T =$ temperature in ° F. (abs.)

$= (459{\cdot}5 + f)$.

$Q =$ discharge in pounds per second.

$C =$,, ,, cubic feet per second.

D, D_1 etc. $=$,, ,, ,, ,, ,, minute.

$\Omega =$ "discharge coefficient,"

§ $= \dfrac{\text{actual discharge}}{\text{theoretical discharge}}$.

$\Delta =$ specific gravity of gas relative to air.

$\gamma =$ ratio of the specific heats,

$= 1{\cdot}408$ for air.

$\eta =$ pulsation coefficient as defined in equation (42).

$W_1, W_2 =$ weight of fluid in pounds per cubic foot.

---

* In the case of a Venturi tube this quantity is usually termed the "throat ratio."

† This factor completely specifies the size and proportions of the nozzle or orifice.

‡ The symbols P_2 and p_2 are used to represent the pressure at the narrowest section of the discharging jet, except when the ratio P_2/P_1 is less than R_1, as defined above. P_2 and p_2 are then used to represent the external pressure into which the jet discharges.

§ By "theoretical discharge" is meant the discharge calculated on the assumption that there is no loss of energy due to friction, that the stream lines are parallel at the cross sections between which the differential pressure is measured, and that the velocity of flow is uniform across both of these sections.

L 2

According to Regnault 1 cubic metre of pure dry air at 0° C. and 760 millimetres of mercury weighs 1·29319 kilogram.

Taking 1 cubic metre as 35·31659 cubic feet,

1 kilogram as 2·20462 lbs.,

T as $(459\cdot5 + f^{\circ}$ F.),

and 760 millimetres of mercury at 0° C. as equivalent to a pressure of 14·69628 lbs. per square inch, $W = 2\cdot69981\ p/T$ is the weight of 1 cubic foot of air at pressure p and temperature T.

The weight of a cubic foot of distilled water freed from air at 62° F., as given by Mr. H. Chaney, of the Standards Department of the Board of Trade in the Philosophical Transactions of the Royal Society in 1892, is 62·2786 lbs.

g = acceleration due to gravity,
= 32·2 feet per second per second at Greenwich.

ϕ δ β and ξ are terms which are defined in equations (15), (17), (13) and (22) respectively, and whose values for air can be found on reference to *Figs. 21, 22* and *23.*

w y z a and b are defined in Appendixes VIII and IX.

K K_1 etc., are numerical constants.

M = revolutions per minute.

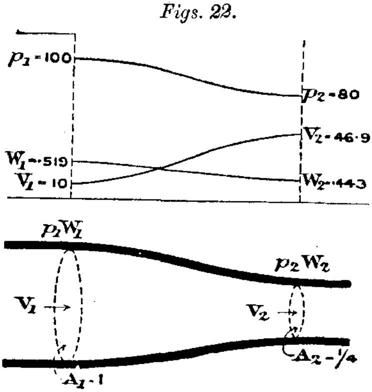
Other symbols are defined in Appendixes V and VI.

APPENDIX II.

Theory of Discharge through Nozzles and Orifices.

Let A_1 and A_2 in *Figs. 22* be two cross-sectional areas in a converging nozzle at which the stream lines are parallel and therefore normal * to the sections considered; and at which the velocities, pressures and densities are respectively V_1, P_1, W_1 and V_2, P_2, W_2.

Figs. 22.



If a steady flow be maintained, the same weight of fluid will pass each cross section in unit time.

Assume that no energy is supplied to or abstracted from the fluid between the two sections, so that the energy passing remains the same.

In the case of an incompressible fluid, the velocity at A_2 will be greater than that at A_1 in the ratio of A_1/A_2. This corresponds to an increase in the kinetic energy; and since the total energy which passes each of the cross sections is the same, the potential energy at A_2 is less than at A_1.

* The volume passing any cross section in unit time is equal to the velocity of the fluid multiplied by the area of the section *only* when the stream lines are perpendicular to the section.

If the fluid considered be a gas, there will be a further increase of velocity and consequent reduction of pressure, owing to the increase of volume which takes place when the pressure-energy, and consequently the pressure, falls.*

When $(P_1 - P_2)$ is large in relation to P_1,† the change in volume which is consequent upon the fall in pressure between A_1 and A_2 causes the discharge-formula for gases to differ materially from that which holds for liquids.

If, however, the difference between the physical properties of liquids and gases be adequately taken into account, the gaseous discharge can under certain conditions be calculated with accuracy when the liquid discharge is known.

The general discharge formulas for nozzles and orifices are derived as follows :—

If the energy per pound of fluid is assumed to be constant, the potential energy liberated between A_1 and A_2 will be equal to the kinetic energy gained.

Also, since there is no net gain or loss of energy, the expansion will be adiabatic.

Hence—

$$\left.\begin{array}{l}\text{The potential energy}\\ \text{liberated per lb. of fluid}\\ \text{between } A_1 \text{ and } A_2\end{array}\right\} = \frac{P_1}{W_1}\frac{\gamma}{\gamma - 1}\left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} \quad \ldots \quad (1)$$

$$\left.\begin{array}{l}\text{The kinetic energy}\\ \text{gained per lb. of fluid}\\ \text{between } A_1 \text{ and } A_2\end{array}\right\} = \frac{V_2^2 - V_1^2}{2g} \quad \ldots \quad (2)$$

Equating $$V_2^2 - V_1^2 = 2g\frac{P_1}{W_1}\frac{\gamma}{\gamma - 1}\left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} \quad \ldots \quad (3)$$

For adiabatic expansion $$\frac{W_2}{W_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} \quad \ldots \quad (4)$$

Now $$Q = A_1V_1W_1 \quad \ldots \quad (4a)$$

$$= A_2V_2W_2 \quad \ldots \quad (5)$$

$\therefore$ $$V_1 = V_2 \cdot \frac{A_2}{A_1} \cdot \frac{W_2}{W_1} = V_2 \frac{1}{n}\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} \quad \ldots \quad (6)$$

(6) in (3) gives

$$V_2^2\left\{1 - \frac{1}{n^2}\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}\right\} = 2g\frac{P_1}{W_1}\frac{\gamma}{\gamma-1}\left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} \quad \ldots \quad (7)$$

(4) and (7) in (5) give

$$Q = A_2V_2W_2 = A_2V_2W_1\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

$$= \Omega A_2\left[\frac{2g\,P_1W_1\frac{\gamma}{\gamma-1}\left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}\left\{\frac{P_2}{P_1}\right\}^{\frac{2}{\gamma}}}{1 - \frac{1}{n^2}\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}}\right]^{\frac{1}{2}} \quad \ldots \quad (8)$$

This is the general discharge formula required, where Ω is the experimental discharge-coefficient.

* Corresponding changes in the velocity, pressure and density are shown in *Figs. 22.* They have been calculated on the basis that $p_1 = 100$, $W_1 = 0{\cdot}519$, $V_1 = 10$ and $A_1/A_2 = 4$.

† Say, when $\frac{P_1 - P_2}{P_1} > \frac{1}{50}$.

The actual volume passing (measured at P and T, or W) may be obtained from the relation $C = \frac{Q}{W}$ . . . . (9)

Equation (8) being cumbersome to use, its evaluation may be simplified by combining it with the less complex formula obtained on the assumption that W is constant.

To derive this formula, the loss of potential energy is equated to the gain of kinetic energy per pound of fluid between A_1 and A_2, thus :—

$$\frac{P_1 - P_2}{W_1} = \frac{V_2^2 - V_1^2}{2g} = \frac{V_2^2 - \left(\frac{V_2}{n}\right)^2}{2g} = \frac{V_2^2}{2g}\left(1 - \frac{1}{n^2}\right)$$

$$\therefore \qquad V_2 = \left\{(P_1 - P_2) \cdot \frac{2g}{W_1} \cdot \frac{n^2}{n^2 - 1}\right\}^{\frac{1}{2}} \qquad (10)$$

Inserting (10) in (5) and multiplying (5) by Ω, we have

$$Q = \Omega A_2 \left[(P_1 - P_2)\, 2g\, W_1 \frac{n^2}{n^2 - 1}\right]^{\frac{1}{2}} \qquad (11)$$

This is the approximate equation corresponding to (8). Combining (8) and (11) after reduction,

$$Q = \Omega A_2 \left[\frac{2g\, n^2}{n^2 - 1} P_1 W_1\right]^{\frac{1}{2}} \left[\beta\right] \qquad (12)$$

where

$$\beta = \left[\frac{\frac{\gamma}{\gamma - 1}\left\{1 - R^{\frac{\gamma - 1}{\gamma}}\right\}\left\{R\right\}^{\frac{2}{\gamma}}}{\frac{n^2 - R^{\frac{2}{\gamma}}}{n^2 - 1}}\right]^{\frac{1}{2}} \qquad (13)$$

Equation (13) enables Q to be calculated in terms of P_1 and P_2/P_1.

In most cases that occur in actual metering the value of P_2/P_1 lies between 0·98 and 1.

Under these circumstances the pressure-difference $(P_1 - P_2)$ can be measured much more accurately than the pressure-ratio P_2/P_1.

It is therefore necessary to express the discharge equation (12) in terms of the pressure-difference.

This can be done in two ways, the first of which involves what is practically P_1 and $(P_1 - P_2)$ and the second what is practically P_2 and $(P_1 - P_2)$.

These equations, which are easily derived from (12), are :—

$$Q = \Omega A_2 \left[\frac{2g\, n^2}{n^2 - 1} (P_1 - P_2)\, W_1\right]^{\frac{1}{2}} \left[\phi\right] \qquad (14)$$

where

$$\phi = \left[\frac{\left(\frac{1}{1 - R}\right)\left(\frac{\gamma}{\gamma - 1}\right)\left(1 - R^{\frac{\gamma - 1}{\gamma}}\right)\left(R\right)^{\frac{2}{\gamma}}}{\frac{n^2 - R^{\frac{2}{\gamma}}}{n^2 - 1}}\right]^{\frac{1}{2}} \qquad (15)$$

and

$$Q = \Omega A_2 \left[\frac{2g\, n^2}{n^2 - 1} (P_1 - P_2)\, P_2 \frac{W_1}{P_1}\right]^{\frac{1}{2}} \left[\delta\right] \qquad (16)$$

where

$$\delta = \frac{\phi}{R^{\frac{1}{2}}} = \left[\frac{\frac{1}{R} \cdot \frac{1}{1 - R}\, \frac{\gamma}{\gamma - 1}\left(1 - R^{\frac{\gamma - 1}{\gamma}}\right) R^{\frac{2}{\gamma}}}{\frac{n^2 - R^{\frac{2}{\gamma}}}{n^2 - 1}}\right]^{\frac{1}{2}} \qquad (17)$$

By (13) and (15),

$$\beta = \phi\,(1-R)^{\frac{1}{2}} \quad \ldots \quad (18)$$

Hence, by (17),

$$\beta = \delta\,[R\,(1-R)]^{\frac{1}{2}} \quad \ldots \quad (19)$$

and

$$\phi = \beta\left(\frac{P_1}{P_1-P_2}\right)^{\frac{1}{2}} \quad \ldots \quad (20)$$

$$= \delta\left(\frac{P_2}{P_1}\right)^{\frac{1}{2}} \quad \ldots \quad (21)$$

The formulas derived up to this point are general for air, gas or steam.

The relation between P_2/P_1 and δ and P_2/P_1 and ϕ for air is plotted for various values of n in *Figs. 23* and *24* respectively.

It will be noticed, on reference to these Figures, that when $n > 3$, δ can be taken as unity for values of P_2/P_1 which lie between 0·98 and 1·0; while the corresponding values of ϕ differ considerably from unity.

Equation (16), which involves δ, is therefore in many cases a more convenient equation to use than equation (14), which involves ϕ.

The values of ξ plotted in *Fig. 27* are calculated from the relation

$$\phi = \left(1 - \xi\,\frac{p_1 - p_2}{p_1}\right)$$

i.e.,

$$\xi = \frac{1-\phi}{1-R} \quad \ldots \quad (22)$$

The relation between P_2/P_1 and β for air discharge is plotted in *Fig. 25*.

Upon reference to equation (13), it will be seen that β is a maximum when

$$\frac{\left(1 - R^{\frac{\gamma-1}{\gamma}}\right)\left(R\right)^{\frac{2}{\gamma}}}{n^2 - R^{\frac{2}{\gamma}}}$$ is a maximum,

i.e., when

$$\frac{\gamma+1}{R^{\frac{2}{\gamma}}} - \frac{2}{R^{\frac{\gamma+1}{\gamma}}} - \frac{\gamma-1}{n^2} = 0\,^{*} \quad \ldots \quad (23)$$

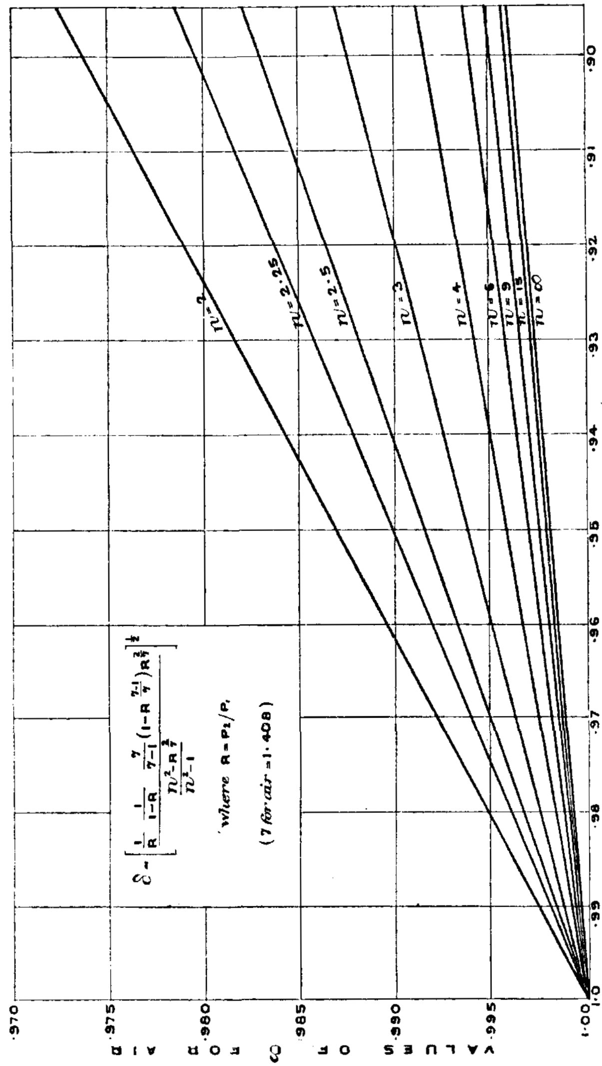
and that the discharge, which is proportional to β, apparently diminishes for values of P_2/P_1 which are less than the value R_1 which satisfies equation (23). Actually, however, the pressure at the narrowest section of the issuing jet cannot fall below $P_1 R_1$; for when this velocity is reached the velocity of discharge becomes equal, as is well known, to the velocity of wave propagation at the temperature and pressure which exist at this cross section, and any external pressure less than $P_1 R_1$ is unable to transmit itself backwards along the issuing jet and affect the discharge.

---

* When $n = \infty$, this reduces to the well known relation $R = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$.

For values of the external pressure less than $P_1 R_1$ the weight discharged therefore becomes directly proportional to P_1, β remaining constant at the value which obtains when $P_2/P_1 = R_1$.

Fig. 23.



The curve connecting β and P_2/P_1 (see *Fig. 25*) is therefore drawn parallel to the P_2/P_1 axis for values of P_2/P_1 which are less than that which satisfies equation (23).

Substituting the following values in equations (16), (12) and (14),

$$A_2 = \frac{A_1}{n} = \frac{a_2}{144}$$

$$N = \frac{a_1}{[n^2 - 1]^{\frac{1}{2}}} = \frac{n a_2}{[n^2 - 1]^{\frac{1}{2}}} = a_2 \text{ when } n = \infty.$$

$$P_1 = 144\, p_1$$

$$P_1 - P_2 = 144\,(p_1 - p_2).$$

$$g = 32 \cdot 2$$

and putting

$$W = 2 \cdot 6998^* \, p_1/T_1$$

there is obtained, as the actual discharge equation for pure dry air,

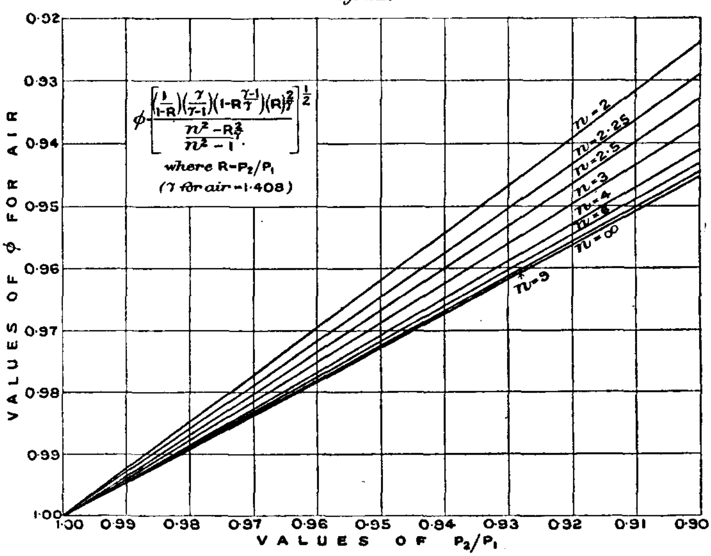
$$Q = 1 \cdot 099\; \Omega\, N\, \phi \left[(p_1 - p_2)\frac{p_1}{T_1}\right]^{\frac{1}{2}} \text{ lbs. of air per second} \quad . \quad . \quad (24)$$

$$= 1 \cdot 099\; \Omega\, N\, \delta \left[(p_1 - p_2)\frac{p_2}{T_1}\right]^{\frac{1}{2}} \quad ,, \quad ,, \quad ,, \quad . \quad . \quad . \quad . \quad (25)$$

$$= 1 \cdot 099\; \Omega\, N\, \beta \quad p_1 \left[\frac{1}{T_1}\right]^{\frac{1}{2}} \quad ,, \quad ,, \quad ,, \quad . \quad . \quad . \quad . \quad (26)$$

where Ω = the experimental discharge-coefficient.

Fig. 24.



When $n = \infty$ and p_2/p_1 is less than $0 \cdot 5269$ [the value of R_1 which satisfies (23) for air discharges] we have

$$Q = 0 \cdot 533\; \Omega\, a_2\, p_1 \left[\frac{1}{T_1}\right]^{\frac{1}{2}} \text{ lbs. of air per second} \quad . \quad . \quad . \quad . \quad . \quad (27)$$

* This value applies only to pure dry air. In any actual case a correction which depends upon the moisture and CO_2 content of the air must be applied.

The values of ϕ, δ and β for air may be obtained from the curves plotted in *Figs. 24, 23* and *25* respectively. Various values of Ω are given in Table III.

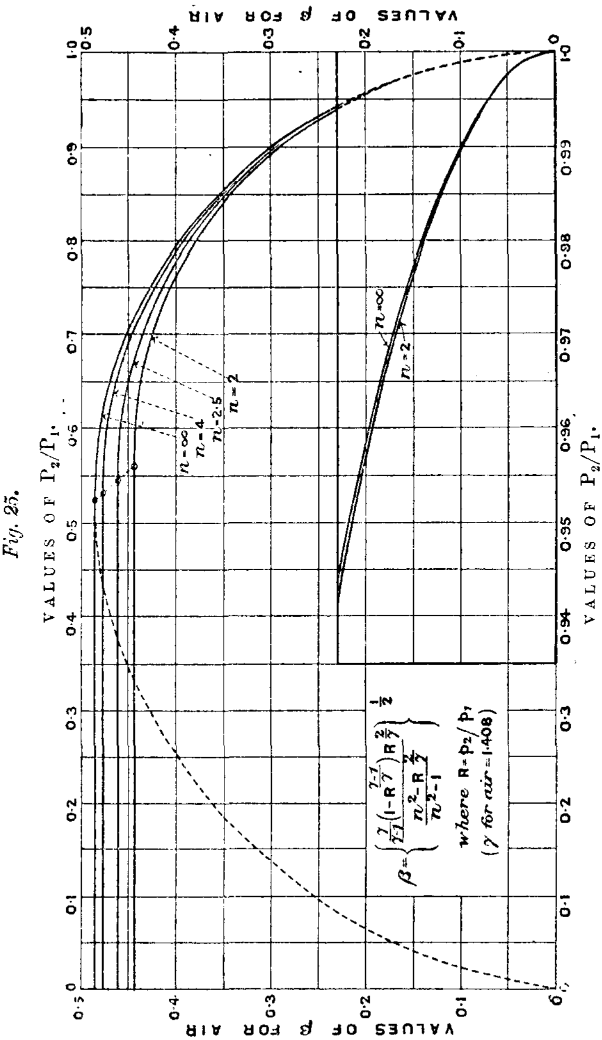


Fig. 25.

When calculating the discharge through square-edged orifices, the ratio of the area of the up-stream bore of the pipe to the area of the *vena contracta* of the issuing jet, n/ψ, must be used instead of n when obtaining β, δ, ϕ or ξ from the plotted curves.

When Ω and χ are known the ratio n/ψ may be calculated as follows :—

By equation (12),

$$Q = \Omega \frac{nA_2}{[n^2 - 1]^{\frac{1}{2}}} [2\,gP_1W_1]^{\frac{1}{2}}\,\beta$$

$$= \chi \frac{nA_2}{\left[\left(\frac{n}{\psi}\right)^2 - 1\right]^{\frac{1}{2}}} [2\,gP_1W_1]^{\frac{1}{2}}\,\beta$$

whence $\Omega^2 \left[\left(\frac{n}{\psi}\right)^2 - 1\right] = \chi^2\,[n^2 - 1].$

If $\Omega = 0{\cdot}608,$*

and $\chi = 0{\cdot}97.$

this reduces to

$$\frac{n^*}{\psi} = \left[\frac{n^2}{0{\cdot}393} - 1{\cdot}55\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

TABLE III.—VALUES OF Ω.

Type of Nozzle or Orifice.	Value of Ω for Water Flows ; or for Air, Steam or Gaseous Flows when p_2/p_1 lies between 0·98 and Unity.	Value of Ω for Air, Steam or Gaseous Flows when p_2/p_1 is less than 0·98.
Badly-designed nozzle, as in Case I, *Fig. 17.*	Ω varies with different nozzles and also with the flow through any given nozzle. In general it may be taken that its value will lie somewhere between 0·75 and 0·95	Ω approximates to unity when p_2/p_1 is less than $\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$
Well-designed nozzle, as in Case II, *Fig. 17*	Ω varies from 0·94 for small nozzles to 0·99 for large nozzles, and is constant for any one nozzle	
Square-edged orifice in thin plate of thickness $0{\cdot}02\,d_2$, with pressure holes arranged as in Case IV, *Fig. 17* . .	$\Omega = 0{\cdot}608$ for ratios of d_2/d_1 less than 0·7, and for all values of d_2	$\Omega = 0{\cdot}914 - 0{\cdot}306\,p_2/p_1$
Orifice in $\frac{1}{16}$-inch plate with upstream edges rounded to $\frac{1}{32}$-inch radius, and with pressure-holes arranged as in Case III, *Fig. 17* .	$\Omega = 0{\cdot}82$ when $d_2 = 0{\cdot}25$ inch and d_1 is not less than 1 inch	Ω approximates to unity when p_2/p_1 is less than $\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$.
	$\Omega = 0{\cdot}62$ when $d_2 = 7$ inches and d_1 is not less than 9 inches	$\Omega = 0{\cdot}92 - 0{\cdot}30\,p_2/p_1$ (approximate)
Butterfly-valve orifice, as shown in Fig. 20 .	0° open : $\Omega N = 0{\cdot}051\,a_1$ 40° „ : $\Omega N = 0{\cdot}296\,a_1$ 70° „ : $\Omega N = 0{\cdot}770\,a_1$	

* It should be noted that the value $\Omega = 0{\cdot}608$, and the values of $\frac{n}{\psi}$ given by (28) apply only to orifices installed in a parallel pipe, and to orifices for which

The water discharge equation

$$C = 0 \cdot 08474\ \Omega\ N\ (p_1 - p_2)^{\frac{1}{2}} \text{ cubic feet per second (at 62° F.)} . \quad (29$$

is obtained by substituting $W = 62 \cdot 2786$ together with the above values of A_2, N, P_1 and g in equation (11).

APPENDIX III.

Note on the Coefficient of Discharge for Square-edged Orifices.

There is considerable divergence in the experimental results published in the different textbooks on hydraulics as to the nature of the variation of this discharge-coefficient with the diameter of the orifice and with the head producing discharge.

In the Author's opinion, this divergence in the results is mainly due to the orifices tested not being geometrically similar, and to the pressure-holes not being in the plane of the orifice.

The Author's tests cover a range of diameters which vary from $\frac{1}{16}$ inch up to 9 inches, the orifices being in each case proportioned and installed as shown at IV, *Fig. 17.*

His results show that, for all values of d_2 within these limits, and for all values of d_2/d_1 less than $0 \cdot 7$, after the critical head has been reached the coefficient of discharge for any orifice will not differ by more than $\pm$ 1 per cent. from the value $0 \cdot 608$.

Below the critical head the water discharge varies as $(p_1 - p_2)^m$, where m is about $0 \cdot 488$.

The experimental values of the discharge-coefficient, and approximate* values for the critical head, are given for the smaller orifices in the following Table :—

Diameter of Upstream. d_1	Diameter of Orifice. d_2	s/d_2 (approximate).	Ω.	Critical Head in Inches of Water.
	Inch.			
1·5	0·0660	0·04	0·612	340
1·5	0·1256	0·02	0·605	250
1·5	0·2412	0·02	0·607	154
1·5	0·3539	0·02	0·606	87

$d_2/d_1 < 0 \cdot 7$, and to cases where the density of the fluid on the up-stream and down-stream sides of the orifice is sensibly the same.

These values do not apply to gaseous discharges when there is an appreciable drop of pressure across the orifice (see *Fig. 17*, Curve IV); or to those cases where a liquid jet discharges into air through an orifice for which $d_2/d_1 > 0 \cdot 2$, or when the pipes up-stream and down-stream of the orifice are of widely different diameters.

* The critical head does not occur at any very well marked point.

Great care was taken in the preparation of these smaller orifices. They were made in hardened steel to ensure that their edges were exactly square and without the least trace of any "wire edge."

Their diameters were measured in four directions at 45° to one another by means of a travelling microscope, and these measurements were checked to the nearest $\frac{1}{10,000}$ inch by means of specially made plug gauges.

The orifices were calibrated with water, which was first filtered in order to avoid the least trace of deposit.

These tests on very small orifices are of importance, as they help to show that similar orifices similarly installed may be expected to have the same coefficient of discharge, whatever be their diameter.

APPENDIX IV.

The Victoria Falls and Transvaal Power Company Air Unit.

The air-meters used in connection with the above scheme are arranged to register the energy in kilowatt-hours necessary to compress the air supplied from the mean atmospheric pressure and temperature at Johannesburg to the pressure of delivery by an isothermal compression process of overall efficiency E.

Assuming that the mean atmospheric pressure and temperature at Johannesburg are 12·086 lbs. per square inch and 519·5° F. abs. respectively, that the gas constant $\left(\frac{P}{WT}\right)$ for air is 53·337, and that 1 kilowatt-hour is equivalent to 2,654,155 foot-pounds, we have—

Kilowatt-hours required to compress 1 lb. of air from 12·086 lbs. per square inch and 519·5° F. (abs.) to pressure p_1 by means of an isothermal compression process of overall efficiency E.

$$= \frac{2 \cdot 302585 \times 53 \cdot 337 \times 519 \cdot 5}{2,654,155 \text{ E}} \times \log \frac{p^1}{12 \cdot 086}$$

$$= \frac{0 \cdot 024038}{\text{E}} \log \frac{p_1}{12 \cdot 086} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

APPENDIX V.

The Meter Formulas.

The air discharge through a Venturi tube in pounds per second is given by equation (24).

If U be the number of kilowatt-hours (as calculated on the basis of the Victoria Falls and Transvaal Power Company's contract) which are required to compress the air delivered per second, then, by (24) and (30)—

$$\text{U} = 1 \cdot 099\, \Omega \text{N} \phi \left[(p_1 - p_2)\frac{p_1}{\text{T}_1}\right]^{\frac{1}{2}} \times \frac{0 \cdot 024038}{\text{E}} \cdot \log \frac{p_1}{12 \cdot 086}$$

$$= 0 \cdot 02641 \frac{\Omega \text{ N } \phi}{\text{E}} \left[p_1 - p_2\right]^{\frac{1}{2}} \cdot \left[\frac{1}{\text{T}_1}\right]^{\frac{1}{2}} \cdot \left[p_1^{\frac{1}{2}} \log \frac{p_1}{12 \cdot 086}\right]$$

$$= 0 \cdot 02641 \frac{\Omega \text{ N}}{\text{E}} \left[\left(1 - \xi_m \frac{p_1 - p_2}{p_m}\right)\left(p_1 - p_2\right)^{\frac{1}{2}}\right] \cdot \left[\frac{1}{\text{T}_1}\right]^{\frac{1}{2}} \left[p_1^{\frac{1}{2}} \log \frac{p_1}{12 \cdot 086}\right] \cdot (31)$$

where $\xi_m = \left(\frac{1-\phi}{1-R}\right)_m$ is calculated for the particular value of n used at the mean pressure p_m and at the mean value of $(p_1 - p_2)$, at which the meter has to work.

The slight error caused by the variation of ϕ with p_1 is neglected.

For the gate meters—

$$U = f(\theta) \cdot \left[\frac{1}{T}\right]^{\frac{1}{2}} \left[p_1^{\frac{1}{2}} \log \frac{p_1}{12 \cdot 086}\right] \quad . \quad . \quad . \quad . \quad . \quad (32)$$

where $f(\theta)$ is the weight of air that would be passed at gate opening θ at unit pressure and temperature, on the assumption that for any gate opening θ, the weight of air passing per second varies as the square root of the product of the density of the fluid and the moment tending to close the gate.

APPENDIX VI.

The Shaped Float and Bell.

Let w_l = weight of 1 cubic inch of oil.

w_m = ,, ,, ,, mercury.

$a_1\ a_5\ a_6$ etc. = areas in square inches.

$dh\ dh_5\ dh_6$ = heads in inches of oil.

dc = downward displacement of float relative to bell chamber due to increase dh in the Venturi head.

db_l = rise of average oil level due to the downward displacement dc of the bell.

db_m = rise of mercury due to the downward displacement dc of the float.

$dl = db_m + dc$

= increase in the depth of float immersed due to increase dh in the Venturi head.

It will be seen on reference to equations (22) and (31), that if p_1 and T_1 are assumed constant the air-units per second passing a Venturi tube are given, with negligible error, by an equation of the form—

$$U = Ah^{\frac{1}{2}}(1 - Bh) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

where $h = \frac{p_1 - p_2}{w_l}$ = Venturi head in inches of oil of density w_l,

$$A = 0 \cdot 02641\ \frac{\Omega N}{E} \left[\frac{1}{T_1}\right]^{\frac{1}{2}} \left[p_1^{\frac{1}{2}} \log \frac{p_1}{12 \cdot 086}\right] w_l^{\frac{1}{2}},$$

$$B = \frac{w_l}{p_m}\ \xi_m = \frac{w_l}{p_m} \left(\frac{1-\phi}{1-R}\right)_m$$

Let a = cross-sectional area of the shaped float in square inches at distance l inches from the level at which it floats when the differential pressure is h_0.

It is desired to proportion the float so that when the differential pressure on the bell is increased from h_0 to h it shall descend a distance—

$$c = G\,(\log U - \log U_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Referring to *Fig. 26*, it will be seen that, for a small increment dh in the differential pressure, on equating forces and buoyancies—

$$w_l \, dh \, a_1 = w_l \, a_7 \, (dc + db_l + dh_6) + w_m \, . \, a \, . \, dl \quad . \quad . \quad . \quad (35)$$

where

$$db_l = \frac{a_7}{a_5 + a_6} dc,$$

$$dh_6 = \frac{a_5}{a_5 + a_6} dh,$$

$$dl = (db_m + dc) \quad . \quad . \quad . \quad . \quad (36)$$

$$= db_m \frac{a_8}{a} \quad . \quad . \quad . \quad . \quad . \quad (37)$$

These values in (34) give—

$$dl = \mathrm{D}dh + (1 - \mathrm{L}) \, dc \quad . \quad . \quad (38)$$

where

$$\mathrm{D} = \left(a_1 - \frac{a_5 a_7}{a_5 + a_6}\right) \frac{w_l}{w_m a_8},$$

$$\mathrm{L} = \left(\frac{a_7 a_9}{a_5 + a_6}\right) \frac{w_l}{w_m a_8},$$

Putting (33) in (34) and differentiating,

$$dc = 0 \cdot 4343 \, \frac{\mathrm{G}}{2} \left\{\frac{1 - 3 \, \mathrm{B}h}{h - \mathrm{B}h^2}\right\} dh \quad (39)$$

Substituting (39) in (38) and integrating,

$$l = \Big[\mathrm{D}h + (1 - \mathrm{L}) \, \mathrm{G} \log \{h^{\frac{1}{2}} (1 - \mathrm{B}h)\} \Big]_{h=h_o}^{h=h} \quad . \quad (40)$$

By (36), (37), (38) and (39)

$$a = \left\{\frac{a_8}{\frac{\mathrm{F} - h}{\mathrm{H}h - \mathrm{K}h^2 - \mathrm{Y}} + 1}\right\} \quad . \quad (41)$$

where

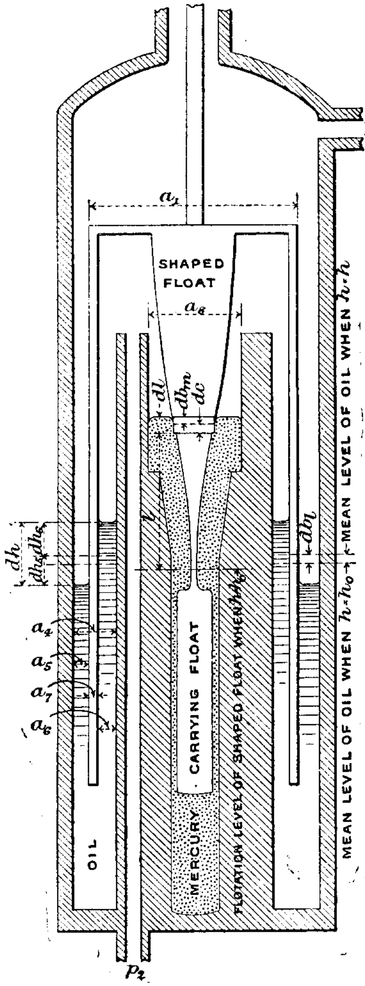
$$\mathrm{F} = \frac{1}{3 \, \mathrm{B}},$$

$$\mathrm{H} = \frac{2 \, \mathrm{D}}{0 \cdot 4343 \, \mathrm{G} \, . \, 3 \, \mathrm{B}} + \mathrm{M},$$

$$\mathrm{K} = \frac{2 \, \mathrm{D}}{0 \cdot 4343 \, \mathrm{G} \, . \, 3},$$

$$\mathrm{Y} = \frac{\mathrm{M}}{3 \, \mathrm{B}}.$$

Fig. 26.



Equations (40) and (41) enable the contour of the shaped float to be calculated.

APPENDIX VII.

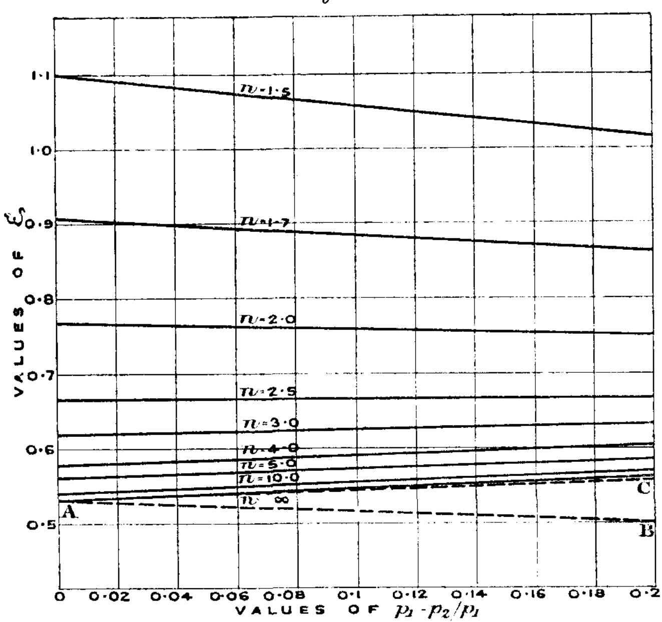
On p. 243 of vol. cxcvii of the Proceedings, Mr. Holbrook Gaskell, Jun., published some notes on his repetition of the Author's work on the square-edged orifices which the Author designed in 1910.*

In re-deriving the discharge equation in the form which it had been communicated to him by the Author, Mr. Gaskell made a slip in the value which he assigned to ξ.

This should be $$\frac{3}{4\gamma}\left(1+\frac{3}{8\gamma}\cdot\frac{p_1-p_2}{p_1}+\quad . \quad . \quad .\right)$$

instead of $$\frac{3}{4\gamma}\left(1-\frac{3}{8\gamma}\cdot\frac{p_1-p_2}{p_1}+\quad . \quad . \quad .\right).$$

Fig. 27.



If the above form of the discharge equation is used, it should be borne in mind that, while ξ varies very little with $\frac{p_1-p_2}{p_1}$, its variation with n is considerable.

This is shown in *Fig. 27*, in which the full curves give the relation between ξ and $\frac{p_1-p_2}{p_1}$ for various values of n. The dotted line AB shows the value given by Mr. Gaskell, while the dotted line AC shows Mr. Gaskell's value when corrected as above. It will be obvious from this Figure that when evaluating ξ, its variation with n must in most cases be taken into account.

* British patent No. 22,183 of 1910.

APPENDIX VIII.

Investigation of the Value of the Coefficient by which the Discharge, as Calculated from the Reading of a Damped Manometer, must be Multiplied in Order to Obtain the True Flow.

Referring to *Fig. 28*—

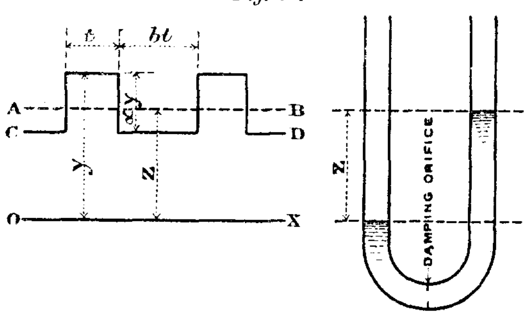
Let y be the maximum difference of pressure due to any flow-measuring device in which the flow at each instant is proportional to the square root of the differential pressure.*

,, z be the corresponding mean reading observed on a manometer which is damped by means of a small orifice placed between the two limbs. This orifice is assumed to have the same coefficient of discharge for flows in either direction.

,, ay be the difference between the maximum and the minimum values of the differential pressure. In order to simplify the investigation, it is assumed that the differential pressure changes suddenly from its maximum to its minimum value and vice versa, instead of gradually, as would usually occur in practice.

,, t and bt be the time in seconds during which the differential pressure remains steady at the maximum and minimum values respectively.

Fig. 28.



Assuming that the rate of flow through the damping orifice placed between the limbs of the manometer varies as the *square root of the differential pressure*, and that the wave-form repeats itself identically so that the differential pressure observed on the manometer assumes a steady value, then, since the exchange of liquid between the two limbs during any complete pulsation is zero—

$$t\sqrt{(y-z)} = bt\sqrt{z-(y-ay)},$$

whence

$$z = \frac{y(1+b^2) - ayb^2}{1+b^2}.$$

* It should be noted that for pulsations of very high frequency the flow at any instant may not be proportional to the square root of the differential pressure at that instant.

The apparent discharge per second, as calculated from the observed mean reading z . . $\left.\right\} = K\sqrt{z}$

$$= K\sqrt{\frac{y(1+b^2) - ayb^2}{(1+b^2)}}.$$

The true discharge per second, as obtained by considering each part of the wave-form separately $\left.\right\} = \frac{Kt\sqrt{y}}{t+bt} + \frac{Kbt\sqrt{y-ay}}{t+bt};$

therefore

$$\eta = \frac{\text{true discharge per second}}{\text{apparent discharge per second}} = \frac{\{1 + b\sqrt{1-a}\}\ \{\sqrt{(1+b^2)}\}}{\{\sqrt{(1+b^2) - ab^2}\}\ \{1+b\}} \quad . \quad . \quad . \quad (42)$$

If a *viscous* liquid is used in the manometer, so that the rate of exchange of liquid between the limbs is *directly proportional to the differential pressure*,

$$t(y-z) = bt[z - (y - ay)],$$

and equation (42) becomes

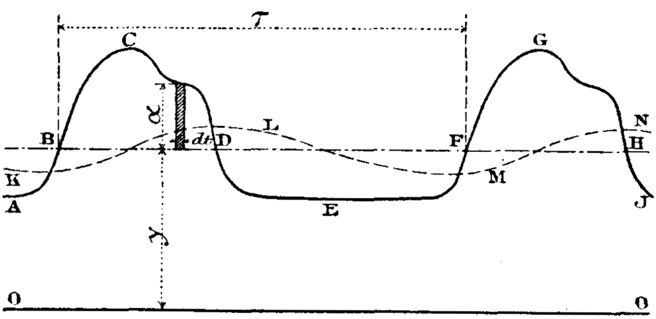
$$\eta = \frac{\text{true discharge per second}}{\text{apparent discharge per second}} = \frac{1 + b\sqrt{(1-a)}}{\sqrt{(1+b)^2 - ab(1+b)}} \quad . \quad . \quad . \quad . \quad (43)$$

APPENDIX IX.

Theory of the Pulsating Flow Meter.

Referring to *Fig. 29*, let it be assumed that ACEGJ represents the wave-form of the flow, that this repeats itself identically, and that the damped pointer takes up a steady position which is represented by the line BH * distant y from OO.

Fig. 29.



Let it further be assumed that the gate has a small moment of inertia, and the reaction due to the flexible coupling is negligible, so that its movements follow the wave-form exactly.

* Actually the pointer will always oscillate slightly about this mean position. The curve KLMN represents its motion to an enlarged scale.

Let α be the angular displacement between the gate and the pointer at any instant, and T the time of one complete period.

Then, since the turning moment which moves the pointer is proportional to the angle α, being produced by the hair-spring which connects the gate to the pointer spindle ; and the couple resisting motion is proportional to the velocity of the pointer, being produced by vanes moving in thick oil, the rate of motion of the pointer at any instant will be proportional to α, and $K\alpha dt$ will represent its actual motion in time dt.

Further, since the net movement of the pointer during a complete cycle is zero,

$$\sum_{t=0}^{t=\tau} K\alpha dt = 0$$

i.e.,
$$\sum_{t=0}^{t=\tau} \alpha dt = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

That is, the area BCD is equal to the area DEF, and the ordinate y gives the mean height of the periodic curve and represents the true mean flow for the wave-form considered. The pointer will therefore indicate the true mean flow corresponding to *any* periodic wave-form.

It will be obvious that if the law of damping of the pointer is such that the couple resisting motion is not directly proportional to the velocity, the pointer can still be made to indicate the true mean of the periodic curve, provided that the relation between the turning moment which moves it and the angle α is altered in such a way that its rate of motion is at each instant proportional to α.

APPENDIX X.

Theory of the Fan Proportional Meter.

Referring to *Fig. 20*, p. 141, if n/ψ for both the impulse-wheel jet and for the obstruction in the main is > 4, and if the discharge is such that the value of the ratio p_2/p_1 lies between 0·98 and 1, equations (9) and (16) show the volume per second passing in the main, and hence the velocity of discharge through the impulse-wheel jet, is proportional to—

$$\left(\frac{p_1 - p_2}{W_2}\right)^{\frac{1}{2}},$$

The linear velocity of the impulse-wheel blades is proportional to M, the number of revolutions per minute. If this is not negligible in comparison with the velocity of discharge through the jet,

$$\text{Torque driving impulse wheel} = K_1 W_2 \left(\frac{p_1 - p_2}{W_2}\right)^{\frac{1}{2}} \left[K_2 \left(\frac{p_1 - p_2}{W_2}\right)^{\frac{1}{2}} - K_3 M\right] \quad . \quad (45)$$

where K_1, K_2, K_3, etc., are numerical constants.

If W_2 be the density of the medium in which the impulse wheel and the damping fan rotate, and the resisting torque due to pivot and counter friction is q,

$$\text{Torque resisting rotation of impulse wheel} = K_4 W_2 M^2 + q \quad . \quad . \quad (46)$$

M 2

Equating (45) and (46) and solving the resulting quadratic,

$$M = -K_5\left(\frac{p_1 - p_2}{W_2}\right)^{\frac{1}{2}} + \sqrt{K_6\left(\frac{p_1 - p_2}{W_2}\right) - K_7\frac{q}{W_2}} \quad . \quad . \quad . \quad (47)$$

This equation shows that for flows higher than those at which the constant term $K_7 \frac{q}{W_2}$ becomes negligible in comparison with $K_6\left(\frac{p_1 - p_2}{W_2}\right)$, M is proportional to $\left(\frac{p_1 - p_2}{W_2}\right)^{\frac{1}{2}}$, and hence to the volume per second passing in the main.

For these higher flows equation (47) reduces to

$$M = \left(\frac{p_1 - p_2}{W_2}\right)^{\frac{1}{2}} (K_6^{\frac{1}{2}} - K_5) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

The meters are designed so that the friction term $K_7 \frac{q}{W_2}$ is negligible below about one-fifteenth of full load.

If M is required to vary in any given way with W_2, this can be effected by making the movement of the deflecting plates dependent upon W_2, and then suitably combining this movement and the cross-sectional shape of the jets which drive the impulse wheel.

APPENDIX XI.

Theory of the Venturi Gas-Meter.

The gaseous discharge in cubic feet per minute through a Venturi tube is given by the relation

$$D_1 = K_1\left[\frac{(p_1 - p_2)\,T_1}{\Delta . p_2}\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

where T_1 is the absolute temperature and p_1 the absolute pressure at the Venturi up-stream, p_2 the absolute pressure at the Venturi throat, K a numerical constant, and Δ the specific gravity of the gas relative to air.

Referring to *Fig. 30*, F is a small-displacement gas-meter which is rotated by the gas that escapes to atmosphere through the pipe A. Attached to the spindle of this meter is a heart-shaped cam G, which lifts the feeler H.

The distance between the zero position of the point of this feeler and the surface of the cam E is made proportional to $[p_1 - p_2]^{\frac{1}{2}}$ for each position which the cam is caused to take up by the bell B.

Each time the feeler is raised from the surface of the cam, an amount proportional to this distance is added on to the counter-train by means of a pawl and ratchet. The counter thus registers an amount proportional to

$$[p_1 - p_2]^{\frac{1}{2}} \times M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

where M is the number of revolutions per minute of the meter F.

The rate of rotation of this meter depends upon the discharge through the orifice O, which is given by the relation

$$D_4 = K_4 \left[\frac{(p_4 - p_5)\, T_4}{\Delta . p_5}\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

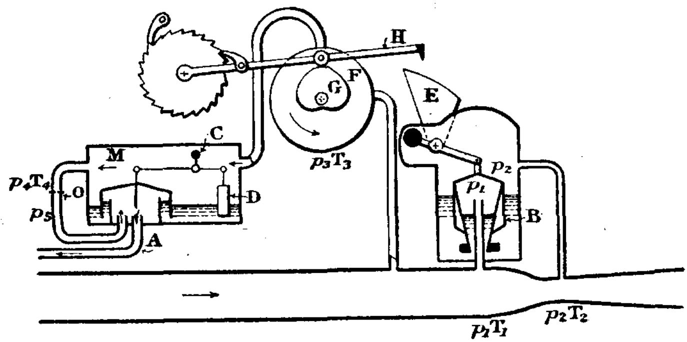
where p_4 and p_5 are the pressures on the up-stream and down-stream sides of the orifice respectively and T_4 the up-stream temperature.

If D_3 is the number of cubic feet per minute passing through the meter F at temperature T_3 and pressure p_3,

$$D_3 = D_4 \frac{T_3}{T_4} . \frac{p_4}{p_3}$$

$$= K_4 \left[\frac{(p_4 - p_5) . T_4}{\Delta . p_5}\right]^{\frac{1}{2}} . \frac{T_3}{T_4} . \frac{p_4}{p_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

Fig. 30.



Putting $M = K_3 D_3$ in (50),

Quantity registered $= K\,[p_1 - p_2]^{\frac{1}{2}} \times K_3 D_3$

$$= K . K_3 . K_4 [p_1 - p_2]^{\frac{1}{2}} \left[\frac{(p_4 - p_5) . T_4}{\Delta . p_5}\right] . \frac{T_3}{T_4} . \frac{p_4}{p_3} \quad . \quad . \quad . \quad (53)$$

Now if $(p_4 - p_5)$ is kept constant by means of the regulating valve M, and if, by suitably arranging the apparatus, the temperatures T_3 and T_4 are made sensibly equal to T_1, and the pressures p_3, p_4 and p_5 to p_2, equation (53) becomes

$$\text{Quantity registered} = K_1 \left[\frac{(p_1 - p_2)\; T_1}{\Delta . p_2}\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

which is identical with equation (49).