Scrutinizing magnetic fields of dipole clusters

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Introduction

The particles in magnetic fluids tend to form magnetic clusters, e.g., in the aging process. Macroscopic models in the form of spherical magnets, and computer animations as presented here, mimic the dipole-dipole interaction of the particles and are thus useful tools for understanding the characteristics of such dipole clusters.

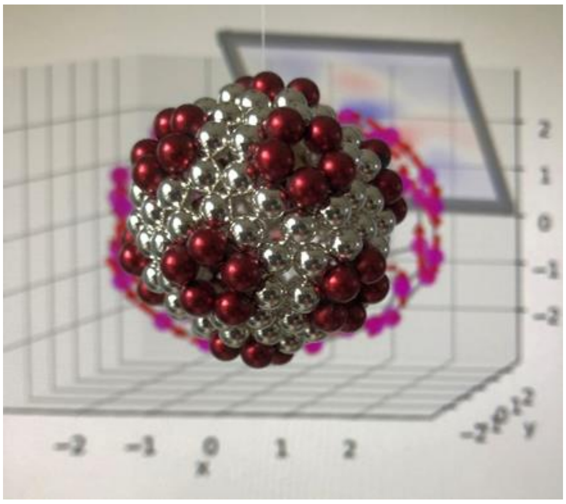

Figure 1. Dipole cluster build from magnetic spheres. The background is a screenshot from Ref. [1].

The app advertised here allows the interactive exploration of more than 500 arrangements of dipoles [1], including:

- Platonic & Archimedean solids,
- simple cubic, FCC & HCP packings,
- 3d-cuts of a tetraplex,
- several stacks of rings.

Figure 2. A dipole cluster from 5 mm neodymium spheres with dodecahedral symmetry. The magnetic order of the dipole orientation will in general reduce that symmetry to a subgroup, which can be interactively explored [1].

The clusters can be scrutinized concerning their

- dipole arrangement & field,
- binding energy & mutual force.

Moreover, the app helps to design tailored magnetic fields from permanent magnets – dipoles and rods – useful for the external manipulation of magnetic fluids [2].

Methods

The app comes as an open source Python script, and as an executable file [1]. For the demonstration experiments, spherical neodymium magnets proofed to be almost perfect dipoles [3].

Results for dipole rings

The ring configuration [4] might be the most prominent for the coagulation of magnetic particles. It is the energetically favored cluster of N dipoles, if $3 < N < 14$ [5]. For larger particle numbers, a tube – formed by a stack of S rings from L dipoles each – is more stable. The field $B(r)$ within these tubes increases with the distance r according to

$$B(r) \sim r^{L-1+mod(S+1,2)}. \tag{1}$$

The field decreases far from the tube as

$$B(r) \sim r^{-L-2-mod(S+1,2)}. \tag{2}$$

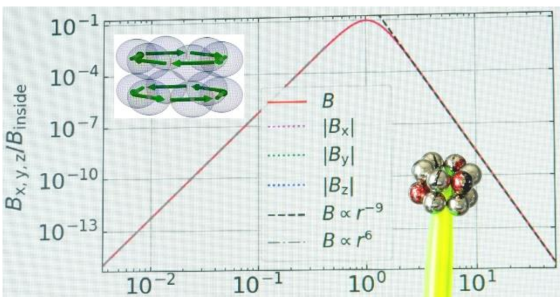

Figure 3. B vs. r, where r is the distance from the centre. The field inside a stack of 2 rings from 6 dipoles each increases with a power of 6, and decreases with a power of -9 outside the arrangement

The force between such tubes decays with

$$F(r) \sim r^{2L-2(1+mod(S+1,\ 2))}. \qquad (3)$$

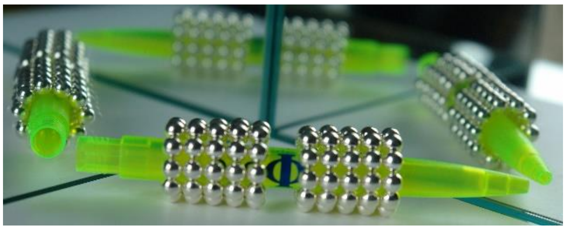

Figure 4. The force between these two S=5 stacks of L=10 rings decreases with a power of -22.

General Result for all dipole clusters

The sum of the powers for the field increase in the center of the cluster and the decrease in the far field is a universal number, namely -3.

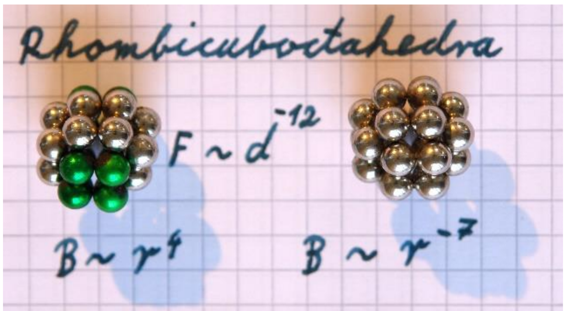

Figure 5. The field of a rhombicuboctahedron increases with a power of 4 inside and decreases with a power of -7 on its outside, fulfilling the -3-rule. The force between two such clusters decays with -12th power of the distance [1].

Result for Halbach rings

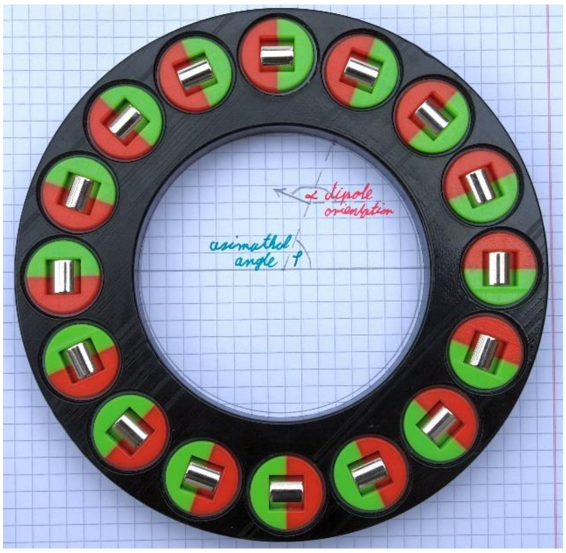

Figure 6: 16 neodymium 1 cm × 1 cm cylinders in red-green coloured holders, which glide on ball bearings and are thus free to rotate. The photo shows the ground state with $\alpha = \varphi + 90°$.

Special arrangements of the permanent magnets along a ring yield an almost homogenous field inside the ring, which is desirable for applications [2]. A dipole orientation $\alpha = 2\varphi$, as originally introduced by Halbach for magnetic rods [6], yields a fairly homogenous field [7], even for the cylindrical magnets shown here. A slight modification of this arrangement, namely

$$\tan \alpha = \frac{3\sin(2\varphi)}{6\cos^2 \varphi - 2}, \qquad (4)$$

increases the field strength by about 5%. The area of sufficiently constant field (see V1.2.1 of [1]) enlarges by about 40%.

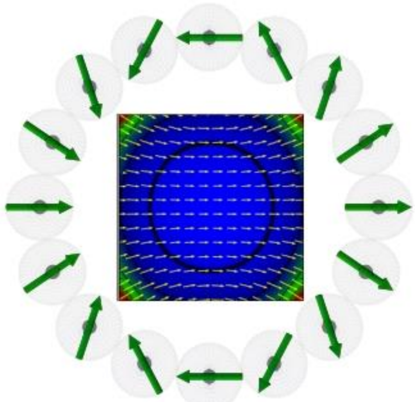

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Figure 7. A calculation of the magnetic field within the dipole arrangement of eq. (4) using the animation app [1].

Acknowledgments

It is a pleasure to thank Günter Auernhammer, Boyd Edwards, Kyongwan Kim, Jan Lellmann, Andreas Menzel, René Messina, Ivan Novikau, Johannes Schönke, Matthias Schröter, and Igor Stancović for encouraging comments.

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