Long-Distance Telephony

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XXVII. *Long-Distance Telephony.* *By* Prof. PERRY, *F.R.S., assisted by* H. A. BEESTON *.

WHEN resistance, capacity, self-induction, and leakage are taken into account, this subject is one of considerable difficulty. It is given to very few men to be able to discuss complicated mathematical formulæ without making mistakes—the proceedings of Scientific Societies possess many such mistakes detected and undetected—and consequently I instruct my students to experiment with their formulæ, using numerical values for their variables. The consideration of the most general problem of long-distance telephony, involving certain terminal conditions, has been taken up by Mr. Heaviside; but the ordinary mathematical physicist must find great difficulty in understanding the investigation (Phil. Mag. January 1887). Some of my students have recently obtained numerical results, neglecting the terminal conditions, which seem to me to be very instructive, and I think that the Tables will have a permanent value.

* Read June 23, 1893.

As a matter of fact, the line is supposed to be of infinite length, and we consider the state of a signal as it gets farther and farther away from the origin.

By comparing the current c at a section x centimetres from the origin with the current at $x+dx$, and properly disposing of the difference, we arrive at the equation :—

$$\frac{d^2c}{dx^2}=kl\frac{d^2c}{dt^2}+(kr+sl)\frac{dc}{dt}+src, \quad . \quad . \quad . \quad . \quad (1)$$

where (per unit length of conductor) k is the capacity, r the resistance, l the self-induction, and s the leakage conductivity. The solution which suits telephonic conditions is

$$c=a\epsilon^{-hx}\sin(pt-gx),$$

where

$$\sqrt{\frac{kpr}{2}}\sqrt{\sqrt{\left(1+\frac{p^2l^2}{r^2}\right)\left(1+\frac{s^2}{k^2p^2}\right)}\mp\left(\frac{pl}{r}-\frac{s}{pk}\right)}$$

gives the value of h if the minus sign is taken, and gives the value of g if the plus sign is taken, and $c=a\sin pt$ is the current at the origin. Of course $p=2\pi f$, where f is the frequency. Any number of such functions of any frequencies may exist simultaneously.

Two conditions must be satisfied in telephony. Taking the shrillest and gravest notes of the human voice as being of frequencies f and f', and taking therefore currents of these frequencies :—let X be the distance at which the ratio of the amplitudes of the shrill and grave currents is increased by $1/m$th of itself; let Y be the distance at which one of the currents has altered in lag behind the other by $1/n$th of the periodic time of the more rapid one; then it is easy to see that

$$X=1\Big/\{m(h-h')\}\ *,$$

$$Y=2\pi\Big/\left\{np\left(\frac{g}{p}-\frac{g'}{p'}\right)\right\}.$$

* This is approximate. If m is not large, the true expression ought to be used,

$$X=\log_\epsilon\left(1+\frac{1}{m}\right)\Big/(h'-h).$$

The letters with dashes indicate that p' or $2\pi f'$ must be taken instead of p. It is easy to see that X and Y become infinite if $\frac{l}{r}=\frac{k}{s}$.

I do not know what values of m and n would produce confusion of sound in the telephone. But as an exercise we have taken $m=4$ and $n=6$. We have also taken $p=6000$, $p'=600$.

For the first French Atlantic Cable the capacity and resistance were 0·43 microfarad and 2·93 ohms per nautical mile, so that

$$k=2{\cdot}3215\times10^{-12}\text{ farads per centim.}$$

$$r=1{\cdot}582\times10^{-5}\text{ ohms per centim.}$$

Mr. Beeston has calculated the distances X and Y, the lesser of which may be taken as the limiting distance for good telephony for various values of l and s.

Table I.—Limiting distances X in millions of centimetres for various amounts of leakage and self-induction. (One million centimetres are equivalent to about six miles.)

Values of $s\times10^{10}$.	Values of $l\times10^{10}$.						
	0	2·6373	26·373	79·118	131·863	184·61	263·73
0	0·983	1·054	1·963	5·169	10·178	17·500	33·839
·01	...	...	1·961				
·10	...	...	1·969				
1	...	...	1·999				
5	1·049	1·130	2·187	6·339	13·944	41 532	75·59
10	1·130	1·224	2·356	8·462	21·98	59·03	328·2
20	1·319	1·444	3·357	18·009	128·17	very large	131·65
40	1·754	1·965	6·250	390·11	67·95	18·94	8·75
70	2·527	2·935	17·671	39·73	7·727	4·161	2·573
100	3·447	4·134	72·03	10·00	0·650	2·111	1·617
150	5·314						
200	7·536						
250	10·090						

TABLE II.—Limiting distances Y in millions of centimetres for various amounts of leakage and self-induction.

Values of $s \times 10^{10}$.	Values of $l \times 10^{10}$.						
	0	2·6373	26·373	79·118	131·863	184·61	263·73
0	1·459	1·484	1·778	2·665	3·719	4·996	7·239
·01	...	...	1·783				
·10	...	...	1·800				
1	...	...	1·893				
5	1·889	1·927	2·424	4·142	6·674	10·472	20·29
10	2·426	2·482	3·297	6·639	14·130	28·88	116·36
20	3·752	3·874	5·796	18·700	82·59	very large	75·12
40	7·292	7·649	15·152	566·36	76·55	18·184	7·214
70	15·216	17·342	57·965	76·304	11·493	5·266	2·815
100	27·078	30·147	283·49	22·70	0·354	3·267	1·810
150	58·35						
200	106·43						
250	170·36						

It will be noticed that although X and Y are derived in different ways, by taking certain values for m and n they could be made much the same in value, and altering s or l seems to produce the same sort of effects on X and Y.

Mr. Beeston has drawn curves from the calculated numbers, but these need not be published.

If there is no self-induction, increasing the leakage increases the distance to which we can telephone. If there is no leakage, increasing the self-induction increases the distance.

When the amounts of s and l are not too great, increasing either increases the distance. These and other important facts are visible in the tables.

Without such tables as these and this method of study it would be almost impossible for the average mathematician to make anything of his mathematical results. Thus, for example, when equation (1) applies to such a function as $\sin pt$ it is just the same as

$$\frac{d^2c}{dx^2} = \left(kl - \frac{sr}{p^2}\right)\frac{d^2c}{dt^2} + (kr + sl)\frac{dc}{dt} \quad . \quad . \quad . \quad . \quad (2)$$

Hence we see that the effect of leakage is to *diminish* the self-induction by the amount sr/kp^2, and to increase the resistance by the amount sl/k. But it is easy to see that if we diminish self-induction or increase resistance we do harm in telephony, and yet this kind of diminution through leakage does good. On going into the matter carefully, it is seen that it is the p being in the denominator of sr/kp^2 which produces the good effect. In fact, if l and s are small, taking $p=6000$, $p'=600$, we find

$$X \propto 1 \Big/ \sqrt{kr}\left\{1-4246\frac{l}{r}-\frac{s}{3800k}\right\}.$$

So that increasing s or l produces a good effect. Having found the mathematical reason we have not far to go to find the physical reason.

It is evident from the tables that if we had no leakage we could completely get rid of the evil effects of capacity by introducing self-induction. It is also evident that if we had no self-induction, we could completely get rid of the evil effects of capacity by introducing leakage. But when there is some leakage and some self-induction, we can in practice only mitigate the evil effect of capacity; for it is obvious that, although certain values of l and s give infinite distances, doubling or halving these values produces enormous diminution in distance, and such a constant of a cable as s may alter very greatly.

About fifteen years ago, with Prof. Ayrton I made many experiments on signalling through bare copper wires lying at the bottom of the water in the moat of Yedo in Japan. Here k and s were both very great. We had much less success than we expected, and we abandoned, perhaps too readily, our idea of a very cheap submarine cable.

The following tables are of general application. Let the numbers given in Table III. be divided by the value of $\sqrt{kr}$ for any cable or conductor of a telephonic line, and let them also be divided by the value of m which is considered suitable*, and they will become the limiting distances X in centimetres for that conductor, for the various values of $\frac{l}{r}$ and $\frac{s}{k}$ given.

* It is more correct to say that the numbers are to be multiplied by $\log_\epsilon\left(1+\frac{1}{m}\right)$.

Table III.

Values of s/k.	Values of $10^5 \times l/r$.						
	0	1·667	16·67	50	83·33	116·7	166·7
0	0·0267	0·0286	0·0533	0·1404	0·2765	0·4753	0·9191
215·4	0·0285	0·0307	0·0594	0·1722	0·3787	1·1281	2·053
600	......	......	......	......	......	......	∞
430·8	0·0307	0·0332	0·0640	0·2298	0·5970	1·6034	8·913
857	......	......	......	......	......	∞	
861·5	0·0358	0·0392	0·0912	0·4891	3·4732	2755·0	3·576
1200	......	......	......	......	∞		
1723	0·0476	0·0534	0·1698	10·596	1·8455	0·5145	0·2377
2000	......	......	......	∞			
3015	0·0686	0·0797	0·4799	1·0792	0·2099	0·1130	0·0699
4308	0·0936	0·1123	1·9564	0·2717	0·0176	0·0573	0·0439
6000	......	......	∞				
60000	......	∞					

Table IV.

Values of s/k.	Values of $10^5 \times l/r$.						
	0	1·667	16·67	50	83·33	116·7	166·7
0	0·0531	0·0540	0·0646	0·0969	0·1352	0·1813	0·2633
215·4	0·0687	0·0700	0·0881	0·1506	0·2427	0·3808	0·7379
600	......	......	......	......	......	......	∞
430·8	0·0882	0·0903	0·1199	0·2414	0·5139	1·0502	4·231
857	......	......	......	......	.....	∞	
861·5	0·1364	0·1409	0·2108	0·6800	3·0033	111056	2·732
1200	......	......	......	......	∞		
1723	0·2652	0·2781	0·4377	20·596	2·7837	0·6613	0·2624
2000	......	......	......	∞			
3015	0·5533	0·6306	2·1079	2·775	0·4179	0·1915	0·1024
4308	0·9847	1·0963	0·1031	0·826	0·0129	0·1188	0·0658
6000	......	......	∞				
60000	......	∞					

Let the numbers in Table IV. be divided by the value of $\sqrt{kr}$ for any conductor of a telephonic line, and let them also be divided by the value of n which is considered suitable, and they will become the limiting distances Y in centimetres for that conductor, for the various values of $\frac{l}{r}$ and $\frac{s}{k}$ given.

Another way of putting the results in these tables is this :—Let K be the whole capacity of the line in farads, R its resistance in ohms, L its whole self-induction in secohms, S the whole leakage conductivity in mhos : L/R and S/K are the same as the l/r and s/k of the tables. For given values of these find the number in either table ; square it and divide by any chosen m^2 or n^2 ; this gives the product of the whole capacity K and the whole resistance R of the conductor.

Of course in all lines which have the same values of L/R and S/K, the product KR is constant. I must again draw attention to the fact that we have neglected the terminal conditions.

I have not hitherto said anything about the amplitude of the current ; in fact the receiving apparatus has been supposed to be infinitely delicate. It is obvious that $1/h$ is the

TABLE V.

Values of s/k.	Values of $\frac{e}{r} \times 10^5$.						
	0	1·667	16·67	50	83·3	116·7	166·7
0	·0127	·0167	·0197	·0314	·0402	·0476	·0567
215·4	·0124	·0130	·0188	·0283	·0340	·0389	·0331
430·8	·0122	·0128	·0177	·0257	0295	·0315	·0329
861·5	·0118	·0123	·0167	·0218	·0233	·0236	·0232
1723	·0110	·0114	·0145	·0166	·0164	·0158	·0147
3015	·0099	·0102	·0121	·0124	·0115	·0106	·0096
4308	·0091	·0093	·0105	·0100	·0090	·0081	·0072
6462	·0079						
8616	·0071						
10770	·0065						

distance in which the amplitude becomes $1/\epsilon^{\text{th}}$ of its initial amount. If the numbers in Table V. be divided by $\sqrt{kr}$ for any line, they give the distances in which a current of frequency 955 per second has its amplitude *halved.*

The effect of leakage is in every case to diminish the amplitude of the current, making it necessary to have more sensitive receiving-instruments or more powerful sending-instruments.

If some of our clever mathematicians would for a while put aside the ambition to write original papers and would give us in one paper, however long, an exposition of Mr. Heaviside's views on the subject, he would confer great benefits upon the average electrician. Mr. Heaviside can discover new truths, and we all believe in his results when we understand them, but he seems unable to lower his reasoning to our mathematical levels. Since writing this paper I have tried to understand Mr. Heaviside's numerous papers on this subject, but I am sorry to say that I am not yet able to express a certain opinion as to the practical value, or want of value, of the preceding tables.

Discussion.

Mr. Blakesley said that some eight years ago he discussed the subject when capacity and resistance were alone considered, and now pointed out that when self-induction and leakage were introduced, the equations were still of the same form. He also suggested how terminal conditions on lines of finite length might be easily taken into consideration.

Prof. Perry, in reply, said the introduction of self-induction and leakage rendered the calculations much more laborious, and that the terminal conditions were much more complicated than Mr. Blakesley supposed.