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III. *The Dielectric Strength of Air.*

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TABLE OF CONTENTS.

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* Read November 24, 1905.

1. *Introduction.*

PHYSICISTS generally attempt to deduce the dielectric strength of air, at a given barometric pressure, from the results of experiments on the disruptive voltages between equal metal electrodes at given distances apart. They calculate the maximum value of the electric intensity between the metal electrodes on the assumption that the electric field round them is similar to that existing at low voltages. Figures obtained in this way were found, greatly to the disappointment of the early experimenters, to vary widely with the distance apart of the electrodes. Lord Kelvin, however, as far back as 1860 *, deduced from the results of his experiments with large electrodes that it was "most probable" that the numbers obtained in this way at higher voltages would be "sensibly constant." An examination of the results, which are given below, obtained recently by electricians will show that experiment has amply justified Lord Kelvin's conclusion. The author finds, by considering experimental results obtained both with direct and alternating pressures, that the limiting value to which the numbers approach is 38 kilovolts per centimetre.

When the electrodes are small, or when the disruptive voltages are only a few kilovolts, the numbers obtained in this way differ largely from 38 kilovolts per centimetre. It is therefore necessary to explain why this is the case. It will be shown in what follows that when the electrodes are small, the air surrounding them may have broken down and become a conductor at voltages which are only a fraction of the disruptive voltage. In this case luminous effects are generally observed at the electrodes. When a high alternating pressure, less than the disruptive voltage, is maintained between small electrodes a few inches apart, each electrode, when the P.D. is sufficiently high, is seen surrounded by a faintly luminous enveloping cloud of a bluish colour, which apparently does not touch the conductor it envelopes. We shall call this cloud the corona. As the pressure is increased, short violet streamers are seen issuing outwards from the

* Proc. Roy. Soc. April 12, 1860: 'Reprint,' p. 259.

corona, the space immediately outside it being the seat of great electrical activity. At higher pressures the streamers are longer, and a hissing noise is heard. When the potential-difference between the electrodes approaches the disruptive value, sparks take place between them, and finally, when all the air is broken down, an arc is suddenly established.

Now, when luminous effects make their appearance, it is obvious that the boundaries of the Faraday-tubes are altered, and, consequently, that the electric field is different from what it is at low voltages. We cannot apply formulæ, therefore, which have been obtained on the assumption that the distribution of the tubes is the same as that for low pressures. We have not attempted to deduce formulæ which will give the dielectric strength of air from the disruptive voltage between two electrodes surrounded with coronæ, as the space occupied by the brush discharges is not clearly defined. There are many cases, however, when there are no luminous effects and where a disruptive discharge ensues the moment that the dielectric stress attains the breaking-down value. We have deduced the dielectric strength of air from the experimental results obtained in these cases.

Many electricians consider that a disruptive discharge always occurs the moment the electric stress at any point of the dielectric between the two electrodes attains a certain maximum value. In what follows, however, we show that in many cases, when some of the air round an electrode breaks down, the new value of the "maximum electric intensity" at the boundary of the broken-down air is less than the old value at the boundary of the metal, and so there is equilibrium, a corona being formed.

The explanation of the varying numbers obtained when large electrodes are used and the disruptive voltages considered are small, is more difficult. When the minimum distance x between the electrodes is less than 3μ, the sparking potentials are practically independent of the nature of the gas between the electrodes *. Since the material of which the electrodes is made exerts an important influence on the

* G. M. Hobbs, "The Relation between P.D. and Spark-length for Small Values of the latter." Phil. Mag. [6] x. p. 617 (Dec. 1905).

sparking potential V, at these small distances, it is highly probable that the carriers of the discharge come from the metal and not from the gas. For a certain distance greater than 3μ, G. M. Hobbs finds in some cases that V remains constant and equal to the minimum spark-potential which in air is about 350 volts *. For slightly greater distances V increases uniformly with x.

It is obvious, therefore, that when the electrodes are very close together, we cannot assume that we have a homogeneous medium bounded by rigid equipotential surfaces. Hence, as the equipotential surfaces are unknown, we cannot apply the ordinary electrostatic equations. For these reasons we have in the following paper only considered experimental results obtained for values of x greater than one millimetre. If we had only considered distances greater than half a centimetre (one fifth of an inch), it would have been unnecessary to make any assumptions about the actions that take place at the end of the tube subjected to the maximum electric stress, as the maximum values of the electric intensity, at the instant of discharge, are found to be in satisfactory agreement. In order, however, to include in our formulæ the sparking potentials for values of x lying between 0·1 and 0·5 cm., we have found that it is necessary to make the following assumption. At the moment of the disruptive discharge, the pressure on the ends of the Faraday tube subjected to the maximum stress is $V-\epsilon$, where ϵ represents what we shall call the lost volts. When the electrodes are surrounded with coronæ an assumption of this nature must be made †, but in this case ϵ will be a function of V and x. In the cases we consider we assume that ϵ is constant and equal to 0·8 of a kilovolt. Making this assumption and, for reasons given above, only considering experiments with large electrodes, at appreciable distances apart, we find that the maximum values of the electric intensity at the moment

* The Hon. R. J. Strutt, "On the Least Potential-Difference required to produce Discharge through Various Gases." Phil. Trans. vol. 193. A. p. 377 (1899-1900).

† H. J. Ryan, "The Conductivity of the Atmosphere at High Voltages." Trans. Am. Inst. El. Eng. vol. xxiii. p. 101 (1904).

of the disruptive voltage is practically constant for distances varying from a millimetre up to 15 centimetres, and for voltages varying between 4 and 160 kilovolts.

2. *Historical.*

Nearly all experimenters have used equal spherical electrodes. It is therefore necessary to be able to write down at once the value of the electric intensity between two spheres whatever may be their potentials. Kirchhoff* in a very valuable paper has shown how to obtain from Poisson's† equations an expression for the maximum value of the electric intensity in the form of an infinite series. Unfortunately this paper can only be understood by those who are thoroughly familiar with Jacobi's theorems in Elliptic Functions, and so the important results contained in it are known to few physicists. In 1890, Professor A. Schuster ‡ published a table giving the value of the maximum electric intensity between two spheres when one was at potential V and the other at potential zero. He gives, however, no proof of the formula, merely referring to Kirchhoff's work. He reduces the infinite series formula for the case of two spheres close together, given by Kirchhoff, into a remarkably simple algebraical form, and shows that, when the spheres are at potentials V and 0, it applies with sufficient accuracy for practical purposes up to a distance between them equal to one-fifth of their radius. In what follows it will be shown that this Kirchhoff-Schuster formula applies with very considerable accuracy, when the potentials are $+V/2$ and $-V/2$, up to a distance apart equal to their radius. This is proved by actually calculating the values of the series, as it is difficult to see from Kirchhoff's method of proof what are the limitations of his formula. By considering the equipotential surfaces round two particles having equal and opposite

* Crelle's *Journal*, 1860, "Ueber die Vertheilung der Elektricität auf zwei leitenden Kugeln," p. 89; *Gesammelte Abhandlungen*, p. 78.

† *Mémoires de l'Institut Impérial de France*, "Sur la Distribution de l'Electricité à la Surface des Corps Conducteurs." Read 9th May and 3rd Aug. 1812.

‡ A. Schuster, "The Disruptive Discharge of Electricity through Gases," Phil. Mag. vol. xxix. p. 192 (Feb. 1890).

charges, the author shows how the first two terms of the Kirchhoff-Schuster formula can be found very simply.

Professor A. Heydweiller* carried out a valuable set of experiments on sparking distances in 1892. He also uses Kirchhoff's formula without, however, proving it. Tables of the numerical values of the electric intensity when the spheres are various distances apart and when they are at equal and opposite potentials are calculated. The formulæ are applied to his own experimental results, but as he does not discriminate between the cases when they are and when they are not applicable, and neglects the 'lost volts,' the results vary widely. The experimental results analysed in Table VI. below are taken from this paper. In Mascart and Joubert's 'Leçons sur l'Electricité et le Magnetisme,' vol. ii. p. 610 (1897), a neat proof of a series formula for the maximum electric intensity between two unequal spheres is indicated. Kirchhoff's results are also quoted, and the formulæ are applied with, however, indifferent success.

In this paper the author gives a simple proof by Kelvin's method of images of Kirchhoff's series formula. He shows by elementary algebra that it can be expressed quite approximately enough for all practical purposes by a simple formula. He has also calculated complete tables which enable any one to write down at once the maximum value of the electric intensity between two equal spheres whatever may be their potentials.

In some of the experiments analysed below cylindrical electrodes are used; it is therefore necessary to get the formula for this case also. It will, however, be more instructive to consider the very simplest mathematical cases first, and thus we shall be able to form a clearer picture of the phenomena that happen in the more difficult practical cases.

3. *The Electric Intensity between Two Concentric Spheres.*

In the case of a spherical condenser we have a metallic sphere concentric with a metallic spherical envelope. If the

* Wiedemann's *Annalen*, vol. xlviii. p. 785 (1893).

radius of the inner sphere be a and the inner radius of the outer sphere be b, we have

$$-\frac{dv}{dr}=\frac{q}{r^2},$$

where v is the potential at a distance r from their common centre, and q is the charge on the inner sphere. Hence we easily find that

$$-\frac{dv}{dr}=\frac{\mathrm{V}ab}{r^2(b-a)},$$

where V is the P.D. between the spheres. Now dv/dr has obviously its maximum value R_m when $r=a$, and thus we have

$$\mathrm{R}_m=\frac{\mathrm{V}b}{a(b-a)}.$$

If we suppose that b and V are fixed and a is a variable, we see that R_m increases from $a=0$ to $a=b/2$ and diminishes for greater values of a. Hence, if a be greater than $b/2$ and we gradually increase V, the moment the electric intensity attains a certain value the air immediately in contact with the inner sphere breaks down and becomes conducting. The electric intensity at the surface of this stratum of conducting air round the inner sphere will be greater than the old maximum electric intensity, and hence a new stratum will be broken down. It is unlikely that the boundaries of the strata successively broken down will be exactly spherical, but any lack of symmetry will accelerate the discharge and an arc between the two spheres will certainly be established. Thus when a is greater than $b/2$ the sparking voltages between the two spheres may be used to calculate $\mathrm{R}_{\max.}$ the dielectric strength of air.

On the other hand, if a be less than $b/2$, an increase in its value will diminish R_m, and thus equilibrium is possible with a conducting stratum of air round the inner sphere. The outside of this stratum is what we call the corona. As the voltage V is increased the corona grows until its radius is nearly equal to $b/2$, when a disruptive discharge will ensue. We see therefore that the size of the inner sphere has no practical effect on the disruptive voltage provided that its radius be less than $b/2$.

When the radius is greater than $b/2$ we should expect no luminous effects until the final discharge took place. This would occur at the instant when

$$R_{\text{max.}} = \frac{V-\epsilon}{b-a} \cdot \frac{b}{a},$$

where ϵ represents the lost volts.

4. *The Electric Intensity inside a Concentric Main.*

Let us now consider the important practical case of a concentric main. A hollow conducting cylinder of inner radius b contains a coaxial conducting cylinder of radius a. If the cylinders be separated by air, the electric intensity R at a point P in the air at a distance r from the axis of the cylinders is given by*

$$R = -\frac{dV}{dr} = \frac{V}{r \log (b/a)},$$

where V is the P.D. between the cylinders. R has obviously its maximum value R_m when $r=a$. We also have

$$\frac{dR_m}{da} = \frac{V}{\{a \log (b/a)\}^2} \{1 - \log (b/a)\},$$

if V and b remain constant. Hence, if a be less than b/ϵ, where ϵ is the base of Neperian logarithms, R_m will diminish as a increases. In this case a corona will be formed. When the radius of the corona is nearly equal to b/ϵ a disruptive discharge will ensue.

If the radius of the inner cylinder be greater than b/ϵ, a disruptive discharge ensues whenever the intensity at the surface of the inner cylinder equals $R_{\text{max.}}$. This was verified roughly by Gaugain †.

The same formulæ apply when the dielectric coefficient of the insulating material between the conductors is not unity. We see, therefore, that the "factor of safety" of concentric mains is not necessarily increased by diminishing the radius of the inner conductor. This result is of considerable practical importance.

* Russell, 'Alternating Currents,' vol. i. p. 95.

† *Annales de Chimie et de Physique*, viii. p. 75 (1866).

5. *The Corona round a Cylinder.*

The luminous effects produced when a cylinder is maintained at a very high alternating potential from earth have been investigated experimentally by E. Jona*. He found that a cylindrical wire supported by high-tension insulators becomes luminous when the voltage between it and surrounding objects attains a definite value, which depends mainly on the diameter of the cylinder. The corona in this case is practically a concentric cylinder, the diameter d of which varies with the voltage V. In the following table V is in kilovolts and d is in millimetres.

TABLE I.—E. Jona's experiments on the diameter of the corona round a thin wire at various voltages.

V...	12·3	18·2	28	42	57	81	106	127	152	185	196
d...	0·12	0·25	0·50	1·00	2·00	4·00	6·00	7·60	10·0	12·5	15·0

From 1 to 15 millimetres V is given roughly by the equation

$$V = 30 + 12d.$$

It will be seen that the diameter of the corona for a pressure of 18 kilovolts is 0·025 cm. Thus if the diameter of the wire is less than 0·025 cm., in a dark room it will be seen surrounded with a corona when the pressure between it and the earth is greater than 18 kilovolts. Approaching an earthed conductor to the wire will increase the luminous effects. E. Jona found that the diameter of the corona was 1·5 cm. whether the wire were 0·01 or 1·4 cm. in diameter, when the pressure was 196 kilovolts.

6. *The Stress in the Dielectric round Two Particles having equal and opposite charges of Electricity.*

Let there be a charge $+q$ of electricity at P (fig. 1) and a charge $-q$ at N. The potential v at a point distant r_1 and r_2 from P and N respectively is given by

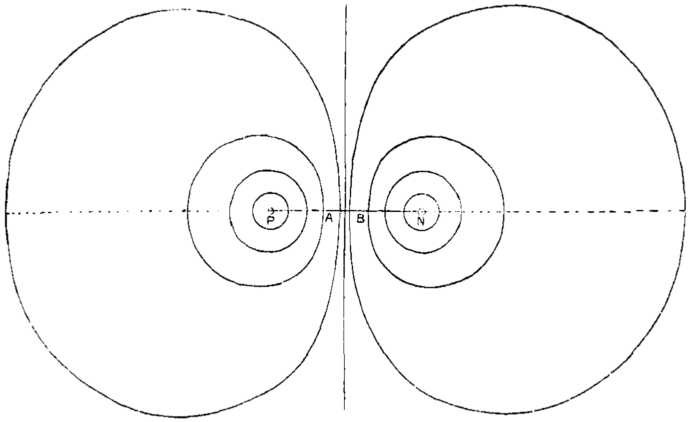
$$v = q/r_1 - q/r_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

The locus of the points, the bipolar coordinates of which

* E. Jona, *Elettricistà*, Rome, xiii. pp. 113-115, April 15, 1904; Science Abstracts, vol. vii. B, p. 605.

satisfy the equation (a) for a given value of v, will give the surface on which the potential is v. Hence these surfaces (fig. 1) can be easily constructed. We see that the equipotential surfaces near P and N are practically spheres.

Fig. 1.



Equipotential surfaces round two points having equal and opposite charges.

Hence the equipotential lines shown in the figure are approximately the same as those of two spheres at a considerable distance apart.

Let V/2 and $-$V/2 be the potentials on any two equal equipotential surfaces surrounding P and N respectively. We shall find a formula to determine the maximum value of the electric intensity between these two surfaces. From symmetry the maximum value of the electric intensity R_m in the space between the two will be at the points A and B, where the line joining P and N cuts the surfaces. If PN$=d$ and PA$=a$, we have

$$\frac{V}{2}=\frac{q}{a}-\frac{q}{d-a},$$

and therefore

$$\frac{q}{V}=\frac{a(d-a)}{2(d-2a)}.$$

We also have

$$R_m=q\left\{\frac{1}{a^2}+\frac{1}{(d-a)^2}\right\},$$

and hence

$$R_m=(V/x)f, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

where x, which equals $d-2a$, is the minimum distance between the two surfaces and f is given by

$$f=\tfrac{1}{2}(1+x/a)+\frac{1}{2(1+x/a)}. \quad . \quad . \quad . \quad . \qquad (c)$$

Now V/x is the average value of the electric intensity along the line joining the nearest points of the surfaces, and is the number which electricians ordinarily give as a measure of the dielectric stress on the insulating medium. We see that f is the factor required to convert this number into the maximum electric intensity.

When x/a is large the surfaces are very approximately spheres of radius a, and (c) can therefore be used to calculate the value of f for two spheres when their distance apart is large compared with the radius of either.

When x/a is small we can show that

$$f=1+x/3\rho, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (d)$$

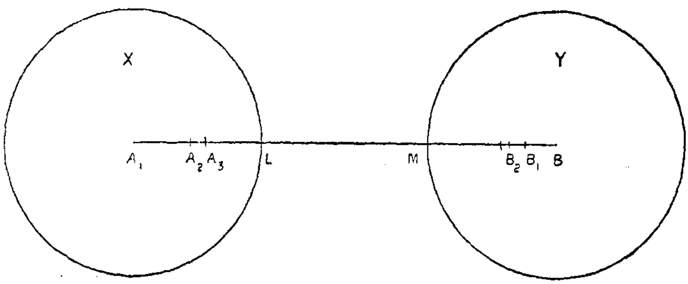
approximately, where ρ is the radius of curvature of the equipotential surfaces at the points where the intensity is a maximum. We should expect therefore that, if we had two spheres the radius of each of which was ρ, (d) would give the value of f approximately when x/ρ was small. We shall show later on that (d) gives the value of f in this case, to an accuracy of one in a thousand when x/ρ is 0·1 or less. Even when x/ρ is unity the error is only about 2 per cent.

The two terms given on the right-hand side of (d) are the first two terms in the important Kirchhoff-Schuster formula quoted below.

7. *Proof of the Series-Formula for the Maximum Electric Intensity between Two Equal Spheres.*

Let us suppose that the radii of the conducting spheres X and Y (fig. 2) are each equal to a, that the distance between their centres A_1 and B is d, and that the minimum distance LM between them is x, so that $d=x+2a$. Let us suppose also that these spheres are at potentials V_1 and V_2. We picture Faraday-tubes starting from their surfaces. If their potentials are of opposite sign some of these tubes connect the two spheres and others connect them with neighbouring conductors. We suppose that these other conductors are so far away that they do not appreciably affect the distribution of the tubes in the field between the two spheres. Now, if the spheres be removed we shall show that this field can be exactly reproduced by a series of point charges placed at definite points on the lines AL and BM (fig. 2). The

Fig. 2.



$A_1B=d$; $A_1L=BM=a$; $LM=x=d-2a$.

point charges will have the spherical surfaces X and Y for the equipotential surfaces V_1 and V_2 respectively. We can therefore write down at once the potentials and the electric intensities at all external points.

We shall first consider the series of points A_1, A_2 . . . B_1, B_2 . . . (fig. 2) which are connected by the following relations,

$$BA_1 . BB_1 = a^2 = A_1A_2 . A_1B_1$$

$$BA_2 . BB_2 = a^2 = A_1A_3 . A_1B_2.$$

. . . .

We see that the points A_2, B_1 ; . . . A_{n+1}, B_n, are conjugate with respect to the sphere X and the points B_1, A_1 ; . . . B_n, A_n, are conjugate with respect to the sphere Y. Let

$$A_1 A_{n+1} = u_{n+1} \text{ and } B B_n = u'_n,$$

then the above equations may be written

$$(d-0)u_1' = a^2 = u_2(d-u_1')$$

$$(d-u_2)u_2' = a^2 = u_3(d-u_2')$$

. . . .

In general, we have

$$(d-u_{n-1})u'_{n-1} = a^2 = u_n(d-u'_{n-1}),$$

and thus

$$u_n\{d-a^2/(d-u_{n-1})\} = a^2,$$

or

$$u_n u_{n-1} - \{(d^2-a^2)/d\}u_n - (a^2/d)u_{n-1} = -a^2.$$

This form of difference equation is well known* and is readily solved by assuming that $u_n = v_{n+1}/v_n + (d^2-a^2)d$. Making this assumption we find that

$$v_{n+1} + \{(d^2-2a^2)/d\}v_n + (a^4/d^2)v_{n-1} = 0,$$

a linear difference equation with constant coefficients. Hence solving in the ordinary way† we get

$$v_n = \mathrm{A}a^n(a/d-q)^n + \mathrm{B}a^n(a/d-1/q)^n,$$

where A and B are constants and

$$2q = d/a - \sqrt{d^2-4a^2}/a, \quad . \quad . \quad . \quad . \quad (1)‡$$

and

$$2/q = d/a + \sqrt{d^2-4a^2}/a, \quad . \quad . \quad . \quad . \quad (2)$$

so that

$$1/q + q = d/a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$1/q - q = \sqrt{d^2-4a^2}/a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Now when n is unity $u_1=0$, and thus

$$v_2/v_1 = -(d^2-a^2)/d = -a(1+q^2+q^4)/\{q(1+q^2)\}.$$

Substituting for v_2 and v_1 their values, in terms of A and B, in this equation we find that $\mathrm{B} = -\mathrm{A}q^2$. We thus find on substituting for v_{n+1} and v_n their values and simplifying, that

$$u_n = aq\frac{1-q^{4n-4}}{1-q^{4n-2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$u_n' = \frac{a_n}{d-u_n}$$

$$= aq\frac{1-q^{4n-2}}{1-q^{4n}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

* Boole's 'Finite Differences,' 3rd ed. p. 233.

† Boole's 'Finite Differences,' chap. xi.

‡ I have called this expression q so as to introduce elliptic function notation. It is a pure number and has nothing to do with an electric charge.

Let us now suppose that charges $Q_1, \ldots Q_n$, are placed at the points $A_1, \ldots A_n$ (fig. 2), and that charges $Q_1', \ldots Q_n'$, are placed at the points $B_1, \ldots B_n$. We shall find the values of these charges so that the potential of the spherical surface X is V_1 and that of the spherical surface Y is zero.

Consider the potential at a point P′ at a distance a from B. The potential at this point will obviously be

$$\Sigma\left(\frac{Q_n}{P'A_n} + \frac{Q_n'}{P'B_n}\right).$$

If therefore we choose the ratio of Q_n to Q_n' so that $Q_n/Q_n' = -P'A_n/P'B_n =$ a constant for every point on Y, the potential at P′ will be zero, and therefore also the potential of the spherical surface Y will be zero. Since A_n and B_n are conjugate points with respect to the sphere Y we have *

$$\frac{Q_n'}{Q_n} = -\frac{P'A_n}{P'B_n} = -\frac{BB_n}{a} = -\frac{u_n'}{a} \quad . \quad . \quad . \quad . \quad (7)$$

Again, the potential at a point P distant a from A is given by

$$\frac{Q_1}{a} + \Sigma\left(\frac{Q_{n+1}}{PA_{n+1}} + \frac{Q_n'}{PB_n}\right).$$

This will be V_1 if we make $Q_1 = V_1 a$ and

$$\frac{Q_{n+1}}{Q_n'} = -\frac{PA_{n+1}}{PB_n} = -\frac{A_1A_{n+1}}{a} = -\frac{u_{n+1}}{a} \quad . \quad . \quad . \quad (8)$$

Hence if we determine the charges Q_n and Q_n' by means of (7) and (8) the potential of the spherical surface X will be V_1 and that of the spherical surface Y will be zero.

From (7) and (8), we have

$$\frac{Q_{n+1}}{Q_n} = \frac{u_{n+1}u_n'}{a^2}$$

$$= q^2\frac{1-q^{4n-2}}{1-q^{4n+2}},$$

and thus, since $Q_1 = V_1 a_1$, we have

$$Q_n = aV_1\frac{(1/q-q)q^{2n-1}}{1-q^{4n-2}} \quad . \quad . \quad . \quad . \quad (9)$$

Hence also

$$Q_n' = -aV_1\frac{(1/q-q)q^{2n}}{1-q^{4n}} \quad . \quad . \quad . \quad . \quad (10)$$

* Russell, 'Alternating Currents,' vol. i. p. 101.

The electric intensity between the two spheres will obviously have its maximum values R_m at L and M, and thus,

$$R_m = \frac{Q_1}{a^2} + \frac{Q_2}{(a-u_2)^2} + \ldots + \frac{Q_{n+1}}{(a-u_{n+1})^2} + \ldots$$
$$- \frac{Q_1'}{(d-a-u_1')} - \ldots - \frac{Q_n'}{(d-a-u_n')^2} - \ldots$$

Now by (8)

$$\frac{Q_n'}{(d-a-u_n')^2} = -\frac{Q_{n+1}(u_{n+1}/a)}{(a-u_{n+1})^2},$$

and hence

$$R_m = \frac{Q_1}{a^2} + \Sigma \frac{Q_{n+1}}{a} \cdot \frac{a+u_{n+1}}{(a-u_{n+1})^2}.$$

Substituting for u_{n+1} and Q_{n+1} their values from (5) and (9) we get

$$R_m = \frac{V_1}{a} \frac{(1+q)^2}{1-q} \sum_1^{\infty} \frac{1-q^{4n-3}}{(1+q^{4n-3})^2} q^{2n-2}. \quad . \quad . \quad (11)$$

The value of the electric intensity R_m' at M is given by

$$R_m' = \frac{Q_1}{(d-a)^2} + \ldots + \frac{Q_n}{(d-a-u_n)^2} + \ldots$$
$$- \frac{Q_1'}{(a-u_1')^2} - \ldots - \frac{Q_n'}{(a-u_n')^2} - \ldots$$

Noticing that $d-u_n = a^2/u_n'$ and that $Q_n/Q_n' = -a/u_n'$, we find that

$$R_m' = -\frac{V_1}{a} \frac{(1+q)^2}{1-q} \sum_1^{\infty} \frac{1-q^{4n-1}}{(1+q^{4n-1})^2} q^{2n-1}. \quad . \quad (12)$$

We can write down the values of R_m and R_m' when the spheres X and Y are at potentials 0 and V_2 in a similar manner. Hence by the principle of superposition we find that the electric intensity at L when the spheres are at potentials V_1 and V_2 is given by

$$R_m = \frac{V_1}{a} \frac{(1+q)^2}{1-q} \sum_1^{\infty} \frac{1-q^{4n-3}}{(1+q^{4n-3})^2} q^{2n-2}$$
$$- \frac{V_2}{a} \frac{(1+q)^2}{1-q} \sum_1^{\infty} \frac{1-q^{4n-1}}{(1+q^{4n-1})^2} q^{2n-1} \quad . \quad . \quad (13)$$

The most important case is when

$V_1 = -V_2 = V/2$, and in this case

$$R_m = \frac{V}{2a} \cdot \frac{(1+q)^2}{1-q} \sum_1^\infty \frac{1-q^{2n-1}}{(1+q^{2n-1})^2} q^{n-1} \quad . \quad . \quad (14)$$

$$= \frac{V}{x} \cdot f,$$

where

$$f = \frac{x}{2a} \frac{(1+q)^2}{1-q} \sum_1^\infty \frac{1-q^{2n-1}}{(1+q^{2n-1})^2} q^{n-1}, \quad . \quad . \quad . \quad (15)$$

and x is the minimum distance between the two spheres. It is convenient to tabulate f for various values of x/a. We see that f is the factor which converts the average electric intensity in the line joining the centres of the two spheres into the maximum electric intensity. In measuring dielectric strengths electricians as a rule merely give the average electric intensity, assuming that f is unity whatever may be the shape of the electrodes.

Another practical case is when one of the spheres is maintained at zero potential. In this case let us suppose that $V_1 = V$, and $V_2 = 0$. Hence by (13)

$$R_m = \frac{V}{x} f_1,$$

where

$$f_1 = \frac{x}{a} \frac{(1+q)^2}{1-q} \sum_1^\infty \frac{1-q^{4n-3}}{(1+q^{4n-3})^2} q^{2n-2}, \quad . \quad . \quad . \quad (16)$$

In practice it is very difficult to make certain that one sphere is at zero potential, and so this method of testing dielectric strengths is not advisable.

We may write (13) in the form

$$R_m = \frac{V_1}{x} f_1 - \frac{V_2}{x}(2f - f_1)$$

$$= \frac{V_1 - V_2}{x} f_1 + 2\frac{V_2}{x}(f_1 - f). \quad . \quad . \quad (17)$$

Thus if we can calculate the values of f and f_1 for any given value of x/a we have completely solved the problem.

When V_2 is zero or negative we see, since $f_1 - f$ is always positive, that for a given value of $V_1 - V_2$, R_m has its greatest value when V_2 is zero and has its least value when $V_2 = -V_1$.

We shall now give methods and formulæ for calculating f and f_1, and we shall also give tables of their values.

8. *Approximate Formulæ for the Maximum Electric Intensity between Two Equal Spheres.*

Formula (15) may be written in the form

$$f = \frac{x}{2a}\left\{1 + \frac{(1+q)^2}{1-q}\cdot\frac{1-q^3}{(1+q^3)^2}q + \dots + \frac{(1+q)^2}{1-q}\cdot\frac{1-q^{2n+1}}{(1+q^{2n+1})^2}q^n + \dots\right\}.$$

It is easy to see that the $(n+1)$th term of the series in the brackets equals

$$\frac{1+q+q^2+\dots+q^{2n}}{(1-q+q^2-\dots+q^{2n})^2}q^n = \frac{q^n+1/q^n+q^{n-1}+1/q^{n-1}+\dots}{\{q^n+1/q^n-(q^{n-1}+1/q^{n-1})+\dots\}^2}.$$

By formula (3)

$$q + 1/q = d/a = 2 + x/a = y \text{ (say)},$$

and thus *

$$q^n + 1/q^n = y^n - ny^{n-2} + \frac{n(n-3)}{\underline{|2}}y^{n-4} - \dots + (-1)^r\frac{n(n-r-1)\dots(n-2r+1)}{\underline{|r}}y^{n-2r}\dots$$

We can thus easily express f in terms of y. Substituting and simplifying we find that

$$f = \frac{y-2}{2}\left\{1 + \frac{y+1}{(y-1)^2} + \frac{y^2+y-1}{(y^2-y-1)^2} + \frac{y^3+y^2-2y-1}{(y^3-y^2-2y+1)^2} + \frac{y^4+y^3-3y^2-2y+1}{(y^4-y^3-3y^2+2y+1)^2} + \frac{y^5+y^4-4y^3-3y^2+3y+1}{(y^5-y^4-4y^3+3y^2+3y-1)^2} + \dots\dots\right\}.$$

Now y cannot be less than 2. Hence expanding by the binomial theorem and neglecting $1/y^9$ and higher powers of

* Todhunter's 'Theory of Equations,' 3rd ed. p. 183.

$1/y$, we find that

$$f=\frac{y-2}{2}\left\{1+\frac{1}{y}+\frac{4}{y^2}+\frac{9}{y^3}+\frac{17}{y^4}+\frac{33}{y^5}\right.$$
$$\left.+\frac{64}{y^6}+\frac{126}{y^7}+\frac{252}{y^8}+\ldots\right\}. \quad . \quad . \quad . \quad . \quad (18)$$

It will be seen that the coefficients of $1/y$ are rapidly getting larger, but it has to be remembered that f must equal unity when $y-2$ is zero. We therefore alter the above formula so as to make $f=1$ when y is 2, and yet make the expanded form of the altered formula agree with (18) as far as the coefficient of $1/y^8$. By this means we secure that the formula (19) gives the correct value of f when y is 2, and again when we can neglect the ninth term in the series formula (15). Expanding $y/(y-2)$ in powers of $1/y$ as far as the term containing the eighth power, and substituting in (18) we get

$$f=\frac{y-2}{2}\left\{-\frac{1}{y}+\frac{y}{y-2}+\frac{1}{y^3}+\frac{1}{y^4}+\frac{1}{y^5}-\frac{2}{y^7}-\frac{4}{y^8}\right\}$$

approximately, or

$$f=\tfrac{1}{2}(y-1)+\frac{1}{y}+\frac{(y-2)}{2}\left\{\frac{1}{y^3}+\frac{1}{y^4}+\frac{1}{y^5}-\frac{2}{y^7}-\frac{4}{y^8}\right\}. \quad . \quad (19)$$

Substituting $2+x/a$ for y, we get

$$f=\tfrac{1}{2}(x/a+1)+\frac{1}{x/a+2}+\frac{x/a}{2(x/a+2)^3}+\frac{x/a}{2(x/a+2)^4}$$
$$+\frac{x/a}{2(x/a+2)^5}-\frac{x/a}{(x/a+2)^7}-\frac{2x/a}{(x/a+2)^8}. \quad . \quad . \quad (20)$$

The values of f are easily computed by this formula. For values of x/a less than 0·1 or greater than 0·7 the error is less than 1 in 1000, whilst for values of x/a between 0·1 and 0·7 the error is never as great as 2 in 1000. For practical purposes therefore the formula (20) gives the values of f with sufficient accuracy. We could have made it more accurate by taking more terms into account in the expansion (18), but we have not done so, as we have found by actual computation that the Kirchhoff–Schuster formula

$$f=1+\frac{1}{3}\cdot\frac{x}{a}+\frac{1}{45}\cdot\frac{x^2}{a^2}+\frac{73}{53760}\cdot\frac{x^3}{a^3} \quad . \quad . \quad . \quad (21)$$

sums the series with a most gratifying accuracy until x/a gets greater than 0·7. It therefore completely covers the part of the scale of our formula which is slightly inaccurate. The formulæ (20) and (21) therefore give the complete practical solution. It is not easy to give a simple proof of (21), but we have found above by elementary considerations the first two terms. If we expand the expression (20) in powers of x/a we get

$$f=1+\frac{11}{32}\cdot\frac{x}{a}-\frac{3}{256}\cdot\frac{x^2}{a^2}+\frac{11}{256}\cdot\frac{x^3}{a^3}\cdot \quad . \quad . \quad . \quad . \qquad (22)$$

The difference between the values of f given by (21) and (20) when x/a is small is roughly the hundredth part of x/a, and as f is greater than unity it will be seen that the percentage error made by using (22) instead of (21) is small.

In Table III. below the values of the column headed f have been found directly from the series-formula (15). In calculating this column I have to acknowledge the help I received from four of my pupils, Messrs. Hewitt, Hoggett, Ritter, and Taylor. In the second column the numbers are calculated by (21), and in the third column by (20).

I am indebted to Mr. Arthur Berry, of King's College, Cambridge, for showing me how the direct calculation can be greatly simplified. The formula (15) may be written

$$f=\frac{x}{2a}\cdot\frac{(1+q)^2}{(1-q)\sqrt{q}}\left\{\frac{\mathrm{K}k}{2\pi}-\frac{2}{q^{3/2}}\sum_1^\infty\frac{q^{3n}}{(1+q^{2n-1})^2}\right\},$$

for*

$$\frac{\mathrm{K}k}{2\pi}=\sum_1^\infty\frac{q^{(2n-1)/2}}{1+q^{2n-1}}.$$

We have used this theorem to check several of our results. For instance, when x/a is 0·5, q is also 0·5 by formula (1). Also

$$\begin{aligned}\dagger\ \sqrt{2k\mathrm{K}/\pi}&=2q^{\frac{1}{4}}(1+\sum_1^\infty q^{n^2+n})\\&=2q^{\frac{1}{4}}(1+q^2+q^6+q^{12}+q^{20}+\ \ldots)\\&=2^{\frac{3}{4}}(1+0{\cdot}25+0{\cdot}015625\\&\qquad\qquad+0{\cdot}000244+\ \ldots)\\&=2^{\frac{3}{4}}(1{\cdot}26587).\end{aligned}$$

* A. Enneper, *Elliptische Functionen*, p. 179.

† A. G. Greenhill, 'Elliptic Functions,' p. 303.

Therefore $$\frac{k\mathrm{K}}{2\pi}=2^{-\frac{1}{2}}(1{\cdot}26587)^2$$
$$=1{\cdot}1331.$$

Also $$\sum_1^\infty \frac{q^{3n}}{(1+q^{2n-1})^2}=0{\cdot}07001.$$

We thus find that $f=1{\cdot}1726$, when $x/a=0{\cdot}5$.

Knowing the values of f we can find the values of f_1 easily by means of an elliptic integral series which is quoted in Kirchhoff's paper. It can be shown that *

$$\sum_1^\infty (-1)^{n-1} q^{\frac{2n-1}{2}} \frac{1-q^{2n-1}}{(1+q^{2n-1})^2}=\frac{kk'\mathrm{K}^2}{\pi^2}.$$

Hence it follows from (15) and (16) that

$$f_1=f+\frac{x}{2a}\frac{(1+q)^2}{(1-q)\sqrt{q}}\left(\frac{kk'\mathrm{K}^2}{\pi^2}\right)$$
$$=f+\sqrt{\frac{x}{a}}\cdot\frac{x+4a}{2a}\cdot\frac{kk'\mathrm{K}^2}{\pi^2}. \quad . \quad . \quad . \quad (23)$$

The values of k, k', and K can easily be found by well-known formulæ. Let us suppose, for instance, that we wish to find the value of f_1 when x/a is 0·5. We have already found that f is 1·1726 and q is 0·5.

Now † $$\sqrt{2\mathrm{K}/\pi}=1+2\sum_1^\infty q^{n^2}$$
$$=1+1+0{\cdot}125+0{\cdot}003906$$
$$+0{\cdot}000031+\ldots$$
$$=2{\cdot}1289.$$

We also have

‡ $$\sqrt{2\mathrm{K}k'/\pi}=1+2\sum_1^\infty (-)^n q^{n^2}$$
$$=1-1+0{\cdot}12503-0{\cdot}00391+\ldots$$
$$=0{\cdot}12112.$$

Thus $$k'=(0{\cdot}12112/2{\cdot}1289)^2=0{\cdot}0032369$$

and $$k=\sqrt{1-k'^2}=0{\cdot}99999.$$

Hence $$kk'\mathrm{K}^2/\pi^2=0{\cdot}01662.$$

* A. Enneper, *Elliptische Functionen*, p. 180.
† A. G. Greenhill, 'Elliptic Functions,' p. 303.
‡ Greenhill, p. 303.

Thus finally by (16)

$$f_1 = 1{\cdot}1726 + \{9/(4\sqrt{2})\}(0{\cdot}01662)$$
$$= 1{\cdot}1990.$$

This agrees with the value of f_1 found by direct calculation from (16).

For values of x/a greater than unity the values of f_1 can be computed by the remarkably simple formula

$$f_1 = x/a + \frac{1}{x/a+1} + \frac{1}{(x/a+1)(x/a+2)^3}. \quad . \quad . \quad (24)$$

Hence it is unnecessary to tabulate the values of f_1 when x/a is greater than 4. The first row in the following table is taken from Schuster's paper, the second row is calculated by the formula

$$f_1 = x/a + \frac{1}{x/a+1}, \quad . \quad . \quad . \quad . \quad . \quad (25)$$

and the third row by (24).

TABLE II.—Values of f_1.

x/a.	4.	5.	6.	7.	8.
Schuster's values ...	4·200	5·172	6·144	7·126	8·111
f_1 by (25)	4·200	5·167	6·143	7·125	8·111
f_1 by (24)	4·201	5·167	6·143	7·125	8·111

The values of f_1 given in the last row are the correct values.

For values of x/a greater than 1·5 the values of f given in the last column are correct to four decimal figures. We have shown above by direct calculation that the value of f when x/a is 0·5 is 1·1726. The Kirchhoff-Schuster formula makes it 1·1724. This formula is therefore very accurate for values of x/a less than 0·5, and these are the values which it is so laborious to find by direct computation from (15).

TABLE III.—Values of f.

x/a.	q by (1).	f by (15).	f by (21).	f by (20).
0·0	1·0000	1·000	1·0000	1·0000
0·1	0·7298	1·034	1·0336	1·0343
0·2	0·6417	1·068	1·0676	1·0686
0·3	0·5821	1·102	1·1020	1·1032
0·4	0·5367	1·137	1·1370	1·1384
0·5	0·5000	1·173	1·1724	1·1735
0·6	0·4693	1·208	1·2083	1·2095
0·7	0·4431	1·245	1·2447	1·2460
0·8	0·4202	1·283	1·2814	1·2832
0·9	0·4006	1·321	1·3190	1·3210
1·0	0·3820	1·359		1·3594
1·5	0·3139	1·559		1·5594
2·0	0·2680	1·770		1·7704
3·0	0·2087	2·214		2·2149
4·0	0·1716	2·677		2·6777
5·0	0·1459	3·151		3·1513
6·0	0·1270	3·632		3·6317
7·0	0·1125	4·117		4·1165
8·0	0·1010	4·604		4·6044
9·0	0·0917	5·095		5·0946
10·0	0·0838	5·586		5·5865
100·0	0·0098	50·51		50·5098
1000·0	0·0012	500·5		500·5010

In the following table for the values of f_1 the first column is taken from Table III. The next column is calculated by the equation

$$\Delta=\sqrt{\frac{x}{a}}\cdot\frac{x+4a}{2a}\cdot\frac{kk'\mathrm{K}^2}{\pi^2},$$

and the last column for f_1 is got by the equation

$$f_1=f+\Delta.$$

TABLE IV.—Values of f_1.

x/a.	f from Table III.	Δ.	f_1.
0	1·000	0·00000	1·000
0·1	1·034	0·00001	1·034
0·2	1·0676	0·0008	1·068
0·3	1·102	0·004	1·106
0·4	1·137	0·013	1·150
0·5	1·173	0·026	1·199
0·6	1·208	0·045	1·253
0·7	1·245	0·068	1·313
0·8	1·283	0·095	1·378
0·9	1·321	0·125	1·446
1·0	1·359	0·158	1·517

The values of f_1 given in this table are in exact agreement with the numbers given by Professor Schuster *.

* Phil. Mag. vol. xxix. p. 192.

9. *The Disruptive Discharge between Two Spherical Electrodes.*

The formulæ and tables given above enable us to find the maximum value R_m of the intensity of the electric field round two spherical electrodes provided that the electrodes are not enveloped by coronæ; that is, provided that none of the air surrounding them is broken down. If no coronæ are formed before the disruptive discharge ensues, then we can calculate R_m at this instant, and so find $R_{max.}$ the dielectric strength of the air. As in the case of a concentric main or two concentric spheres, it is of importance to know in what cases coronæ can be formed. The problem is now much more difficult as the coronæ are only approximately spherical, the maximum thickness of the stratum of conducting air round each electrode being on the line joining the centre of the two spheres.

If we make the assumption that the surrounding air is broken down to the same depth at every point on the surface of either electrode, we can find whether the value of R_m increases or diminishes with this depth. In the former case a disruptive discharge will certainly ensue, and *a fortiori* it will ensue in the actual case of two spherical electrodes, as the actual breakdown begins at the centre of the spherical face, raising, as it were, a small blister at that point, and so R_m must be greater owing to the greater curvature.

When the distance between the spheres is greater than the radius a, we have, to an accuracy of 1 in a 1000,

$$R_m = \frac{V}{x}\left\{\tfrac{1}{2}(1+x/a) + \alpha/(2+x/a)\right\},$$

where α is 1·077, provided that x/a is less than 7.

Hence
$$R_m = \frac{V}{2}\left\{\frac{1}{d-2a} + \frac{1}{a} + \frac{2a\alpha}{d(d-2a)}\right\}$$
$$= \frac{V}{2}\left\{\frac{1+\alpha}{d-2a} + \frac{1}{a} - \frac{\alpha}{d}\right\},$$

and therefore
$$\frac{dR_m}{da} = \frac{V}{2}\left\{\frac{2(1+\alpha)}{(d-2a)^2} - \frac{1}{a^2}\right\},$$

when V and d are constants. Hence, for values of d less than $a(2+\sqrt{2(1+\alpha)})$, that is, for values of d less than $4·04a$, R_m increases as a increases, and thus, on our assumption, a disruptive discharge will ensue.

If the spheres be not further apart than twice their diameter we should therefore expect a disruptive discharge to ensue the moment R_m became R_{max}. For large spheres, experiment shows that this is the case up to a distance apart equal to about three times their diameter. For greater distances apart, the moment R_m attains the value R_{max} the air in the neighbourhood of that point is broken down and a partial corona is formed, the value of R_m at the surface of the corona being less than R_{max}. In these cases, as the equipotential surfaces are no longer spheres, we cannot apply our formulæ.

10. *The Maximum Electric Intensity between a Sphere and a Plane.*

When the plane is at zero potential, we see, by taking the image of the sphere in the plane, that

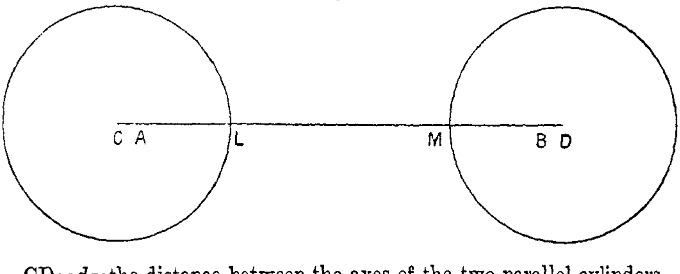
$$R_m = (V/x) f_p, \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where f_p is the value of the factor f given above corresponding to $2x/a$; x being the least distance of a point on the sphere from the plane and a being its radius.

11. *The Maximum Electric Intensity between two infinitely long parallel Cylinders.*

Let us consider the value of the electric potential at points

Fig. 3.



$CD = d =$ the distance between the axes of the two parallel cylinders.
$LM = x =$ the minimum distance between the cylinders.
$a =$ the radius of either cylinder.
A and B are inverse points. $CA \cdot CB = CL^2 = DA \cdot DB$.

between the two cylinders, the sections of which by the plane of the paper are shown in fig. 3. If q and $-q$ be the charges per unit length on the cylinders the axes of which pass through A and D respectively, the potential v at any point P external to them is given by

$$v=-2q\log(\mathrm{AP}/\mathrm{BP}),$$

where A and B are the inverse points of the circular sections. The maximum values R_m of the electric intensity will be at L and M. The potential at any point p on CD will be

$$v=-2q\log r+2q\log(c-r),$$

where r is Ap and c is the distance AB. Hence

$$R=\frac{2q}{r}+\frac{2q}{c-r}.$$

Now R has its maximum value R_m when r is AL.

Hence $$R_m=\frac{2qc}{\mathrm{AL}(c-\mathrm{AL})}=\frac{2qc}{a(d-2a)}=\frac{2qc}{ax},$$

where x is the minimum distance between the cylinders. Now, we have *

$$q=\frac{V}{4\log\{(d+c)/2a\}},$$

where V is the potential-difference between the cylinders and $c^2=d^2-4a^2$.

Hence we have

$$R_m=\frac{V}{x}\frac{c/2a}{\log\{(d+c)/2a\}}$$

$$=\frac{V}{x}f,$$

where $$f=\frac{y}{\log(1+x/2a+y)}, \quad . \quad . \quad . \quad . \quad (27)$$

and $$y=\{x/a+(x/2a)^2\}^{\frac{1}{2}}.$$

* Russell, 'Alternating Currents,' vol. i. p. 102.

Values of f are given in the following table:—

TABLE V.

x/a ...	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
f	1·00	1·00	1·00	1·01	1·01	1·01	1·01	1·01	1·015
x/a ...	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
f	1·02	1·03	1·05	1·065	1·08	1·10	1·11	1·13	1·14
x/a ...	1	2	3	4	5	6	7	8	9
f	1·16	1·315	1·46	1·61	1·74	1·88	2·01	2·14	2·24
x/a ...	10	20	30	40	50	100	1000		10,000
f	2·39	3·56	4·62	5·62	6·58	11·0	72·5		587

12. *The Disruptive Discharge between Two Parallel Cylinders.*

It is well known in practice that when we have two parallel wires with a high P.D. between them, then in certain cases coronæ envelope the wires. When they are close together, however, this effect is not produced, a disruptive discharge occurring directly the P.D. attains a certain value. It is important therefore to know what distance apart the wires must be in order that coronæ can be formed.

If we assume that $\sec\theta = d/2a$, we find that

$$R_m = \frac{V}{d(1-\cos\theta)} \cdot \frac{\tan\theta}{\log(\tan\theta+\sec\theta)}.$$

Let d be constant and let a vary, then, solving the equation $dR_m/d\theta=0$, we find that

$$\log\tan(\pi/4+\theta/2) = \frac{\sin\theta}{\sin^2\theta-\cos\theta}.$$

When θ is nearly 70° this equation is satisfied, and in this case $d=5{\cdot}85\,a$ nearly.

Hence making the assumption that the coronæ are cylindrical in shape, we see that R_m diminishes as a increases when d is greater than $5 \cdot 85\, a$. In practice, therefore, we should not expect coronæ to be formed when the wires were at a less distance apart than about three times their diameter.

13. *The Application of the Formulæ to Experimental Results.*

I. WITH DIRECT PRESSURES.

(i.) *Lord Kelvin's tests with large electrodes.*

Lord Kelvin * was the first to make accurate tests on the disruptive voltages between electrodes in air. He found that the apparent dielectric strength of a thin stratum of air was much greater than that of a thick one. The apparent dielectric strength in our notation being Vf/x, we have

$$Vf/x = R_{\text{max.}} + 0 \cdot 8\, f/x,$$

where V is in kilovolts, and $R_{\text{max.}}$, the dielectric strength of air, is a constant. In Kelvin's experiments f was practically equal to unity at all distances, and thus V/x increases rapidly as x diminishes.

From his experimental results† Lord Kelvin concludes that a battery of 5510 Daniell cells could produce a spark between two slightly convex electrodes when the minimum distance between them was 1/8th of a centimetre. Taking the E.M.F. of a Daniell cell as 1·07 volts, this makes the dielectric strength $R_{\text{max.}}$ of air to be 40·8 kilovolts per centimetre; a result which is only about 6 per cent. higher than the number we give as the average value of R_{max}.

(ii.) A. Heydweiller. *5 cm. spheres (2·5 cm. radius).*

In the valuable paper by A. Heydweiller, published in the *Annalen der Physik und Chemie*, vol. xlviii. p. 785 (1893), there are many tables of sparking-distances given both between equal and unequal electrodes. We consider merely the last table he gives, and we choose the 5 centimetre spheres as being likely to give the most accurate results.

* Proc. Roy. Soc. 1860, or 'Reprint,' p. 24.
† Proc. Roy. Soc. April 12, 1860, p. 259.

The height of the barometer was 74·5 cms., and the temperature 18° C. during the test. The columns headed x and V are taken from Heydweiller's paper, f_1 is calculated by the formulæ given above, and $R_{max.}$ is found by

$$R_{max.} = \{(V - 0{\cdot}8)/x\} f_1.$$

We have assumed that the potentials of the spheres are V and 0 at the instant of the discharge. The results seem to indicate that this was not the case when the electrodes were at their greatest distances apart.

TABLE VI.

Heydweiller's test with 5 cm. spheres ($a = 2{\cdot}5$).

x = distance apart in cms. V = disruptive pressure in kilovolts.

x.	x/a.	f_1 (calc.).	V (observed).	$R_{max.}$ (calc.).
0·5	0·2	1·068	18·36	37·5
0·6	0·24	1·081	21·60	37·5
0·7	0·28	1·102	24·54	37·3
0·8	0·32	1·116	27·33	37·0
0·9	0·36	1·132	30·09	36·9
1·0	0·40	1·150	32·85	36·9
1·1	0·44	1·169	35·58	37·0
1·2	0·48	1·188	38·31	37·0
1·3	0·52	1·209	41·01	37·4
1·4	0·56	1·231	43·68	37·7
1·5	0·60	1·253	46·23	37·9
1·6	0·64	1·277	48·66	38·2

The mean value of the numbers in the last column is 37·5, and none of them differ from the mean value by as much as 2 per cent. Hence this experiment gives 37·5 kilovolts per centimetre as the dielectric strength of air.

(iii.) J. Algermissen. 5 *cm. spheres* ($a = 2{\cdot}5$ *cm.*).

In the following table the values of x and V are taken from Dr. Zenneck's work 'Elektromagnetische Schwingungen und Drahtlose Telegraphie,' 1905, p. 1011. They are due to J. Algermissen, and are deduced from the average of the values obtained on different days under varying conditions. We have assumed that the potentials of the electrodes were $+V/2$ and $-V/2$ respectively at the instant of the discharge.

As the results in the last column are very approximately constant our assumption is justified.

TABLE VII.

J. Algermissen. 5 cm. spheres ($a = 2\cdot5$).

x is measured in cms. and V in kilovolts.

x.	x/a.	f(calc.).	V (obs.).	$R_{max.}$ (calc.).
1·5	0·6	1·208	46·2	36·6
1·6	0·64	1·223	48·6	36·5
1·7	0·68	1·238	51·0	36·6
1·8	0·72	1·253	53·4	36·6
1·9	0·76	1·268	55·8	36·7
2·0	0·80	1·283	58·2	36·8
2·1	0·84	1·298	60·6	37·0
2·2	0·88	1·312	62·8	36·9
2·3	0·92	1·326	65·0	37·0
2·4	0·96	1·342	67·0	37·0
2·5	1·00	1·360	69·0	37·1
2·6	1·04	1·374	70·8	37·0
2·7	1·08	1·390	72·6	37·0
2·8	1·12	1·406	74·4	37·0
2·9	1·16	1·421	76·2	37·0
3·0	1·20	1·437	78·0	37·0
3·1	1·24	1·452	79·7	37·0
3·2	1·28	1·469	81·3	37·0
3·3	1·32	1·484	83·0	37·0
3·4	1·36	1·500	84·7	37·0
3·5	1·40	1·515	86·4	37·1
3·6	1·44	1·533	88·0	37·1
3·7	1·48	1·549	89·6	37·2
3·8	1·52	1·566	91·2	37·3
3·9	1·56	1·583	92·7	37·3
4·0	1·60	1·599	94·2	37·4
4·1	1·64	1·616	95·7	37·4
4·2	1·68	1·632	97·2	37·4

The mean of the values of $R_{max.}$ in the last column gives the dielectric strength of air as 37·0 kilovolts per centimetre, and the greatest difference between any of the calculated numbers and this value is only about one per cent. It will be seen therefore, that the agreement between theory and experiment is quite satisfactory. The three final values for $R_{max.}$ obtained in this table closely agree with the mean of the values we deduced from Heydweiller's test. Considerable weight, therefore, must be attached to the results of these experiments in determining the value of $R_{max.}$.

(iv.) J. Joubert and G. Carey Foster. 1 *cm. and 2 cm. spheres* ($a=0\cdot5$ & 1).

In Foster and Porter's (Joubert's) 'Electricity and Magnetism,' p. 135, tables of the sparking-distances between 1 centimetre and 2 centimetre spheres are given. The results for the 1 cm. spheres are taken from Joubert's *Traité élémentaire d'électricité* (2nd edit.), and those for the 2 cm. spheres were obtained by G. Carey Foster. An analysis of the table for the 1 centimetre spheres shows that if we calculate $R_{max.}$ for sparking-distances of 5, 10, and 15 cms., on the assumption that the air round the electrodes is not broken down to any appreciable depth before the discharge occurs, the values are much too large. This is in accord with the conclusion of § 9. The mean of the values up to a distance of 2 cms. apart makes $R_{max.}$ 42·8. The mean of the values for the 2 cm. spheres makes $R_{max.}$ 42·9.

(v.) É. Hospitalier. 1 *cm. spheres.*

An analysis of the experimental results given by É. Hospitalier in the *Formulaire de l'Électricien*, 21st year, 1904, p. 289, for the sparking-distances between two electrodes, each one cm. in diameter, shows that the potentials of the spheres are not $+V/2$ and $-V/2$ at the instant of the discharge. The values of $R_{max.}$ calculated on this assumption diminish steadily from the maximum value 44·1 when the spheres are 0·6 of a cm. apart to 40·0 when they are 2 cms. apart. The values of f, however, are little affected by the absolute values of the potentials of the electrodes, provided that x/a is not greater than 0·3. Taking, therefore, the mean of the first three results given, we find that $R_{max.}$ is 42·2.

(vi.) Compagnie de l'Industrie Électrique. *Plate and sphere.*

The Compagnie de l'Industrie Électrique et Mécanique have published tests * on the disruptive voltages between a plate and a ball.

* Turner and Hobart, 'Insulation of Electric Machines,' p. 33 (1905).

TABLE VIII.

Compagnie de l'Industrie Électrique. Plate and 2 cm. ball.

x.	x/a.	f_p (calc. by § 10).	V (obs.).	$R_{max.}$
0·5	0·5	1·36	18	46·8
1·0	1·0	1·77	26	44·6
1·5	1·5	2·21	31	44·5
2·0	2·0	2·68	35·5	46·5
2·5	2·5	3·15	39	48·1
3·0	3·0	3·63	42·5	50·4
4·0	4·0	4 60	48·0	54·2
5·0	5·0	5·59	54·0	59·5
6·0	6·0	6·57	58·0	62·5

It will be seen that $R_{max.}$ is beginning to increase rapidly (see § 9). The mean of the first four values gives 45·6 kilo-volts per centimetre as the dielectric strength of air. In practice the plates used are not large, and so we are only justified in using our formula for f_p when the plate and the ball are close together. We do not attach much importance to this test.

II. WITH ALTERNATING PRESSURES.

(i.) C. P. Steinmetz. *2 inch spheres.*

In a paper on the "Dielectric Strength of Air," published in the Transactions of the American Institute of Electrical Engineers, vol. xv. p. 281, Professor C. P. Steinmetz gives the results of an elaborate and careful research on the disruptive voltages between pointed, spherical, and cylindrical electrodes. Alternating voltage was used of frequency 125, and the shape of the wave was practically identical with a sine curve when a particular smooth-core alternator was used. The ratio of the maximum to the effective voltage in all his experiments with this machine was practically 1·42. The spherical and cylindrical electrodes were put in nitrate of mercury and then rubbed with a clean cloth. When this was done it was found that the disruptive discharge, for a given distance apart of the electrodes, always took place at the same voltage. If the electrodes were merely polished, then at small distances apart the results were very erratic. The accuracy of the results obtained probably lies well within

4 per cent. in most cases. In the experiments the barometer varied from 75·2 to 76·2 cms. This variation introduces an uncertainty of about one per cent. The voltmeter readings may be one per cent. out, and there may be a one per cent. error in determining the ratio of the maximum to the effective potential-difference. An error is also due to the moisture in the air. This, however, was found to be small. When the electrodes were immersed in "live" steam at atmospheric pressure, the effect of the steam was to *increase* the apparent dielectric strength of the air, a greater voltage being required to produce the disruptive discharge. As pressures up to 160 effective kilovolts were employed, the sparking-distances were large and could be measured with great accuracy. We should expect that, with these high voltages, our formulæ would apply with considerable accuracy, as the disturbing effect of the cathode glow would be small and the field would be approximately symmetrical.

In the following table the results of tests when the electrodes were spheres 2 inches in diameter are analysed. The column headed $R_{max.}$ gives the values of the dielectric strength in kilovolts per centimetre, calculated by the formula

$$R_{max.} = \{(1{\cdot}42V - 0{\cdot}8)/x\}f.$$

TABLE IX.

C. P. Steinmetz. 2 inch spheres ($a = 2{\cdot}54$ cms.).
$\sim = 125$. $E/V = 1{\cdot}42$, where E is the maximum and V the effective value of the alternating voltage.

No. of Experiment.	x.	x/a.	f(calc.).	V (obs.).	$R_{max.}$(calc.).
1	0·318	0·125	1·04	8·95	39·0
2	0·635	0·25	1·08	15·9	37·1
3	1·25	0·49	1·17	26·7	34·7
4	2·74	1·08	1·39	51·0	36·2
5	3·69	1·45	1·54	65·2	38·3
6	4·29	1·69	1·63	70·8	37·9
7	5·72	2·25	1·88	83·8	38·9
8	7·62	3·00	2·21	94·0	36·7
9	8·74	3·44	2·42	102·0	39·9
10	10·0	3·95	2·66	101·5	38·0
11	12·9	5·08	3·19	108·0	37·7
12	14·2	5·60	3·44	114·5	39·1

The mean value of $R_{max.}$ obtained from the figures in the last column is 37·8. Considerable importance is attached to this test as the numbers actually observed are given.

The curve in fig. 4 (p. 82) gives the relation between V and x on the supposition that $R_{max.}$ is 38. Steinmetz's experimental results are plotted in this figure for purposes of comparison.

(ii.) Compagnie de l'Industrie Électrique. *2 cm. spheres.*

The Compagnie de l'Industrie Électrique et Mécanique of Geneva have published* a curve giving the sparking-distances between two spherical electrodes, each one centimetre in radius. The frequency of the alternating pressure employed was 50, and the ratio of the maximum to the effective voltage was 1·26. Calculating $R_{max.}$ by the formula

$$R_{max.} = \{(1{\cdot}26V - 0{\cdot}8)/x\}f$$

for values of x from 0·5 cm. to 5 cms., we find that the mean value of $R_{max.}$ is 37·9, which practically agrees with Steinmetz's result for 2 inch spheres.

(iii.) C. P. Steinmetz. *1, 0·5, and 0·25 inch spheres.*

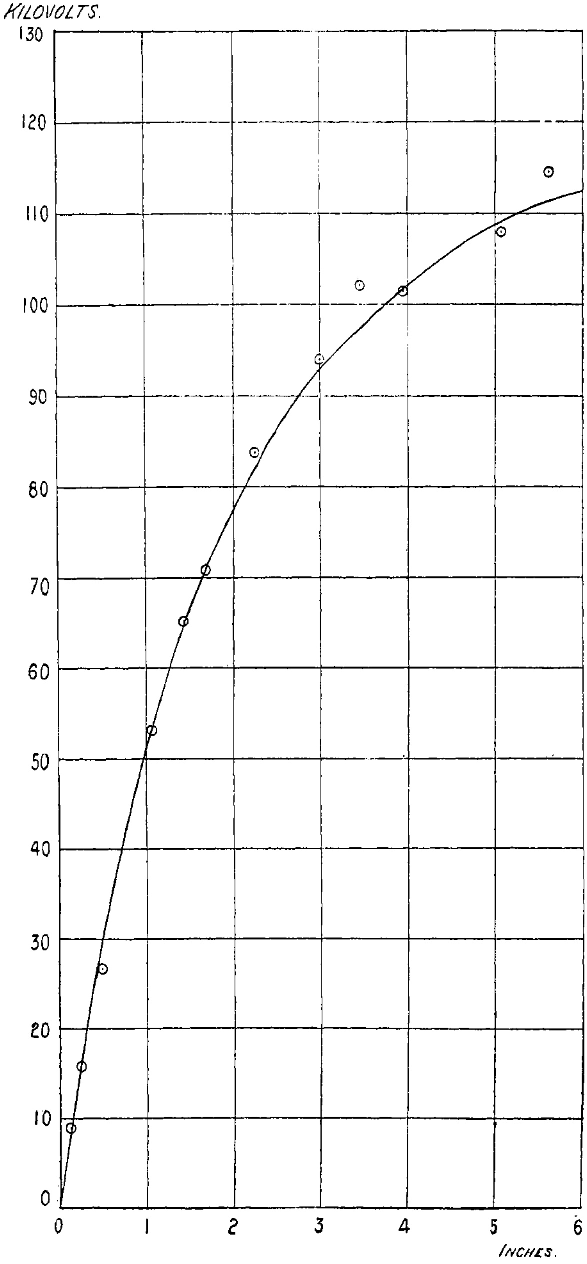
The analysis of Steinmetz's experiments with 1, 0·5, and 0·25 inch spheres are instructive, but for reasons explained in § 9 they do not give much assistance in obtaining $R_{max.}$ With the 1 inch spheres the mean of the values of $R_{max.}$ obtained up to pressures of 63·7 effective kilovolts is 41·3 kilovolts per centimetre. With the half-inch spheres the mean of the values for pressures up to 31·3 effective kilovolts is 43·1.

When the quarter-inch spheres were 28 cms. apart, the disruptive pressure was 112 effective kilovolts. If we calculate $R_{max.}$ for this pressure as if the spheres were in a vacuum, we find that it is more than six times the dielectric strength of air. In the experiment there must have been coronæ round each of the electrodes after the pressure was about 17 effective kilovolts.

In these experiments the frequency was 125 and E/V was equal to 1·42.

* Turner and Hobart, 'The Insulation of Electric Machines,' p. 35.

Fig. 4.—Sparking Voltages between 2 inch spherical electrodes. Points marked ⊙ are Steinmetz's experimental results.



(iv.) E. Jona. *Point and Plate.* *Two spheres.*

E. Jona has published * a table giving the sparking-distances between a point and a plate and also between two equal spherical electrodes each of 2 cms. diameter, for pressures varying from 15 to 240 kilovolts. When the electrodes are far apart it is obvious that the distribution of the Faraday-tubes is considerably affected by the supporting rods connecting the electrodes with the transformer terminals. An important result proved by these experiments is that for all distances greater than 23 cms. the sparking-voltages, with the electrodes used by Jona, were the same in the two cases. For instance, when the maximum value of the applied P.D. was 240 kilovolts ($\sim = 42$), the sparking-distance was 47 cms., whether the point and the plate or the two spherical electrodes were used. Faraday anticipated this result in his 'Experimental Researches,' § 1499:—" But as has long been recognized, the small body is only a blunt end, and, electrically speaking, a point only a small ball; so that when a point or blunt end is throwing out its brushes into the air, it is acting exactly as the small balls have acted in the experiments already described, and by virtue of the same properties and relations."

(v.) C. P. Steinmetz. 0·795 *cm. cylindrical electrodes* ($a=0{\cdot}3975$).

Professor Steinmetz in the paper referred to above also describes tests on the sparking-distances between cylindrical electrodes. In one case the electrodes were two copper rods 0·795 cm. in diameter and 71 cms. long. The rods were slightly curved, so that the sparks ensued across the minimum distance between them. The radius of curvature at this part was 198 cms., and so, provided the rods are not further apart than about 2 cms., we can neglect the curvature and assume that the field is very similar to that between two infinitely long parallel straight rods. We can therefore use the formula (27) for f given in § 11.

* E. Jona, *Atti dell' Associazione Elettrotechnica Italiana,* vol. vi. p. 3, " Distanze Esplosive nell' Aria, negli olii ed altri Liquidi Isolanti."

TABLE X.

C. P. Steinmetz. Two cylindrical rods slightly curved. Diameter of rods 0·795 cm. $\sim = 125$, $E/V = 1·42$.

No. of Experiment.	x.	x/a.	f (calc.).	V (obs.).	$R_{max.}$
1	0·165	0·42	1·07	4·78	38·9
2	0·203	0·51	1·08	5·70	38·8
3	0·433	1·15	1·18	10·85	38·9
4	0·881	2·21	1·35	18·55	39·1
5	1·115	2·81	1·43	23·7	42·2
6	1·194	3·01	1·46	22·9	38·8
7	1·435	3·61	1·55	23·0	34·3
8	1·753	4·40	1·66	30·5	40·3
9	2·134	5·38	1·80	34·3	40·4
10	2·362	5·96	1·88	33·1	36·8

The mean of the values of $R_{max.}$ given in the last column makes the dielectric strength of air 38·8 kilovolts per centimetre.

(vi.) C. P. Steinmetz. 1·11 *inch cylindrical electrodes.*

Experiments were also made with large cylinders 1·11 inch in diameter and 20 inches long. Up to a distance apart of about one-third of an inch we may assume that our formula applies approximately.

We have neglected therefore the experimental results for greater distances. The mean of the values of $R_{max.}$ deduced from the first five experiments is 34 kilovolts per centimetre. We do not attach so much importance to this result as to the preceding as our formula does not apply so accurately.

14. *Table of the Numbers obtained for the Dielectric Strength of Air from the Direct Pressure Experiments.*

TABLE XI.—Direct Pressures.

	Nature of Electrodes.	Authority.	$R_{max.}$
Table VII...	5 cm. spheres.	J. Algermissen.	37·0
Table VI. ...	5 cm. spheres.	A. Heydweiller.	37·5
§ 13, I., i. ...	Slightly convex surfaces.	Lord Kelvin.	40·8
§ 13, I., v....	1 cm. spheres.	É. Hospitalier.	42·2
§ 13, I., iv...	1 ,, ,,	J. Joubert.	42·8
,, ,,	2 ,, ,,	G. Carey Foster.	42·9

Of the above tests the first three seem to be the most accurate. The mean of the results obtained from these three tests makes the dielectric strength of air 38·4 kilovolts per centimetre.

15. *Table of the Numbers obtained for the Dielectric Strength of Air from the Alternating Pressure Experiments.*

TABLE XII.—Alternating Pressures.

	Nature of Electrodes.	Authority.	$R_{max.}$
§ 13, II., vi.	1·11 inch cylinders.	C. P. Steinmetz.	34
§ 13, II., ii.	2 cm. spheres.	Comp. de l'Ind. Élect.	37·9
Table IX.	2 inch spheres.	C. P. Steinmetz.	37·8
Table X.	0·313 inch cylinder.	„	38·8
§ 13, II., iii.	1 inch spheres.	„	41·3

An examination of the experimental results obtained and the methods of calculating $R_{max.}$ from them shows that the second, third, and fourth of the above tests are the only really satisfactory ones. The mean of the results obtained in these three tests is 38·2 kilovolts per centimetre, and this agrees closely with the number we obtained from the direct pressure experiments.

16. *Conclusion.*

We conclude therefore that the dielectric strength of the air at ordinary atmospheric pressures lies between 38 and 39 kilovolts per centimetre, which is about 30 per cent. greater than the value ordinarily given. J. J. Thomson * gives the value as approximately 30 kilovolts per centimetre and M. O'Gorman † as 27 kilovolts per centimetre.

The confidence of electricians who are responsible for the working of high-pressure net-works for the distribution of electric power ‡ on the working of their spark-gap safety-

* J. J. Thomson, 'Electricity and Magnetism,' p. 59 (1904).

† M. O'Gorman, "Insulation on Cables," Journal of the Inst. of Elect. Eng. vol. xxx. p. 666 (1901).

‡ Dusaugey, *Soc. Int. Elect.*, Bull. 5, pp. 109–132. "Méthode de Protection contre les Surtensions actuellement employée dans les Réseaux de Transport d'Energie," Feb. 1905.

valves at the moment the pressure attains a definite value, and the extensive use they make of micrometer spark-gaps * for measuring high voltages, prove that under ordinary working conditions they find that the dielectric strength of air is approximately constant. In ordinary work we may take its value as 38 kilovolts per centimetre.

Appendix I.

The Disruptive Voltages for Large Spherical Electrodes.

Table XIII. gives the disruptive pressures in kilovolts between equal spherical electrodes when their radii are 1, 10, 100, and 1000 cms. respectively. The dielectric strength of air has been taken as 38 kilovolts per centimetre, and V and V′ are calculated by the formulæ

$$V = 0{\cdot}8 + R_{\text{max.}}(x/f) \quad \text{and} \quad V' = V/\sqrt{2},$$

respectively. V′ therefore gives the effective value of the disruptive voltage when the pressure is alternating and sine-shaped.

Table XIII.

Calculated Values of the Disruptive Voltages between large Spherical Electrodes.

V = kilovolts (direct pressure). V′ = effective kilovolts (alternating pressure) when $V' = V/\sqrt{2}$.

x in cms.	2 cm. spheres.		20 cm. spheres.		200 cm. spheres.		2000 cm. spheres.	
	V.	V′.	V.	V′.	V.	V′.	V.	V′.
0·1	4·5	3·2	4·6	3·25	4·6	3·25	4·6	3·25
0·5	17·0	12·0	19·4	13·9	19·8	14·0	19·8	14·0
1·0	28·8	20·3	37·7	26·7	38·8	27·4	38·8	27·4
5·0	61·1	43·3	163	115	187	132	191	135
10·0			280	198	370	262	381	269
50·0			604	427	1625	1150	1860	1320
100·0					2795	1980	3690	2610
500·0					6030	4270	16200	11500
1000·0							28000	20000
5000·0							60000	42500

* P. H. Thomas, Amer. Inst. Electr. Engin. Proc. xxiv. pp. 705–742. "An Experimental Study of the Rise of Potential on Commercial Transmission Lines due to Static Disturbances caused by Switching, Grounding, etc.", July, 1905.

If the electrodes were infinite planes, the direct pressure required to produce the disruptive discharge when they were 50 metres apart would be 190 million volts.

With spherical electrodes of 10 metre or less radius, about 60 million volts would be sufficient to spark over the same distance. We suppose of course that the P.D.'s are established sufficiently slowly to allow the Faraday-tubes to attain their positions of statical equilibrium approximately before the discharge occurs.

Appendix II.

The Capacity Currents to the Electrodes.

The formulæ (9) and (10) enable us to find at once the analytical expressions for the electrostatic coefficients of two equal spheres. If Q, Q′ denote the charges and V_1 and V_2 the potentials of the electrodes, we have *

$$Q = K_{1\cdot1} V_1 + K_{1\cdot2} V_2$$

and

$$Q' = K_{2\cdot2} V_2 + K_{2\cdot1} V_1.$$

In our case $K_{1\cdot1} = K_{2\cdot2}$. By making $V_2 = 0$, we get by (9)

$$K_{1\cdot1} = a\,\frac{1-q^2}{q}\sum_1^\infty \frac{q^{2n-1}}{1-q^{4n-2}},$$

and by (10)

$$K_{1\cdot2} = -a\,\frac{1-q^2}{q}\sum_1^\infty \frac{q^{2n}}{1-q^{4n}}.$$

Denoting Lambert's series $\sum_1^\infty \frac{q^n}{1-q^n}$ by $F(q)$ we get

$$K_{1\cdot1} = a\,\frac{1-q^2}{q}\{F(q) - 2F(q^2) + F(q^4)\}$$

and

$$-K_{1\cdot2} = a\,\frac{1-q^2}{q}\{F(q^2) - F(q^4)\}.$$

By a known transformation due to Clausen † we have

$$F(q) = \sum_1^\infty q^{n^2}\frac{1+q^n}{1-q^n};$$

and this series can be very readily computed.

* Russell, 'Alternating Currents,' vol. i. p. 89 *et seq.*

† Crelle's *Journal*, vol. iii. p. 95, quoted in Jacobi's *Fundamenta Nova*, pp. 187–188.

For instance, when x is 0·5 we find by (1) that q is also 0·5, and

$$F(0{\cdot}5)=1{\cdot}6067,\ F(0{\cdot}25)=0{\cdot}4210,\ \text{and}\ F(0{\cdot}0625)=0{\cdot}0709.$$

Hence we find that

$$K_{1{\cdot}1} = 1{\cdot}2534a \text{ and } K_{1{\cdot}2} = -0\ 5252a.$$

The capacity between the two spheres * is

$$(K_{1{\cdot}1} - K_{1{\cdot}2})/2 = 0{\cdot}8893a$$

and the capacity † for equal potentials is

$$2(K_{1{\cdot}1} + K_{1{\cdot}2}) = 1{\cdot}4565a.$$

To reduce these values to microfarads we divide by 900,000, a being measured in centimetres.

When the potentials follow the harmonic law we have

$$A_1 = (K_{1{\cdot}1}V_1 + K_{1{\cdot}2}V_2)\omega$$

and

$$A_2 = (K_{1{\cdot}1}V_2 + K_{1{\cdot}2}V_1)\omega,$$

where A_1 and A_2 are the effective values of the capacity currents flowing to the two spheres respectively; V_1, V_2 the effective values of their potentials, and $\omega/2\pi$ the frequency of the alternating pressures.

For instance, suppose that we have two 20 cm. spherical electrodes 5 cms. apart, and suppose that the effective value of the P.D. between them is 90 kilovolts. Then, if ω be 1000, so that the frequency is nearly 160, and the potentials of the electrodes be equal and opposite at every instant, we have

$$A = \omega KV = 1000 \times 0{\cdot}889 \times 10 \times 90 \times 10^3/(9 \times 10^5 \times 10^6)$$
$$= 0{\cdot}000889 \text{ ampere.}$$

If the potentials of the electrodes be not equal and opposite at every instant, the difference between the effective values of the capacity currents to the electrodes equals the effective value of the current in the earth connexion. Our formulæ are not applicable when the electrodes are surrounded with coronæ. In this case the capacity between them is considerably increased.

* Russell, 'Alternating Currents,' vol. i. p. 92.
† Russell, 'Alternating Currents,' vol. i. p. 393.

The author is indebted to Mr. Arthur Berry for suggesting the use of Clausen's theorem as an aid in calculating the capacity coefficients of two equal spheres.

Postscript.

I have received from Principal G. Carey Foster the results of experiments made in his laboratory in 1876 on the sparking distances between 2·6 cm. brass knobs. As these results are of considerable interest I have obtained his permission to publish them.

G. Carey Foster.—2·6 cm. spheres ($a=1{\cdot}3$).

x.	x/a.	f.	V (obs.).	$R_{max.}$
0·05	0·0385	1·013	3·09	46·4
0·1	0·0769	1·026	5·04	43 5
0·2	0·1538	1·051	8·43	40·0
0 3	0·2307	1·077	11·46	38·3
0·4	0·3076	1·105	14·61	38·1
0·5	0·3846	1·131	17·49	37·7
0·6	0·4614	1·159	20·43	37·9
0·7	0·5383	1·185	23·37	38·2
0·8	0·6152	1·213	26·25	38·5
0·9	0·6921	1·242	29·13	39·0

A 'home-made' absolute electrometer was used to measure the voltage. The attracted disk was hung from an ordinary balance and the attraction weighed directly. Up to 30 kilo-volts it gave trustworthy readings.

Neglecting the first value of $R_{max.}$ as the formula given in the paper is only roughly applicable when the distances are less than one millimetre, we find that the mean value of $R_{max.}$ is 39. If we neglect the first two readings the mean value of $R_{max.}$ is 38·5. Both of these results agree very closely with the final conclusions at which we arrived. It will be seen that the experimental results obtained during the last thirty years on the sparking distances in air at ordinary pressures, when no coronæ are formed, could have been predicted with considerable accuracy from the above results.

Principal Carey Foster also suggested the formula

$$V = 2{\cdot}13 + 30{\cdot}6\,x,$$

for the sparking potentials between 2·6 cm. knobs. It is interesting to notice that Baille and many other experimenters subsequently suggested linear formulæ for the relation between V and x.

In reply to a question by Professor Poynting, I have worked out the values of $R_{max.}$ for the case of Heydweiller and Algermissen's tests on the assumption that the ordinary electrostatic equations hold, without modification, at the instant of breaking down. In this case we have

$$R'_{max.} = (V/x)f,$$

where f can be found by formula (20) given above. The values of $R_{max.}$ are those found in Tables VI. and VII.

Heydweiller's Test.

	Maximum value.	Minimum value.	Mean value.
$R_{max.}$	37·5+0·7	37·5−0·6	37·5
$R'_{max.}$	38·35+0·85	38·35−0·55	38·35

Algermissen's Test.

	Maximum value.	Minimum value.	Mean value.
$R_{max.}$	37+0·4	37−0·5	37·0
$R'_{max.}$	37·4+0·3	37·4−0·3	37·4

The last result is so remarkable that I give the complete table.

Algermissen's Test.

x.	$R'_{max.}$	x.	$R'_{max.}$	x.	$R'_{max.}$	x.	$R'_{max.}$
1·5	37·2	2·2	37·5	2·9	37·3	3·6	37·5
1·6	37·1	2·3	37·5	3·0	37·4	3·7	37·5
1·7	37·1	2·4	37·5	3·1	37·3	3·8	37·6
1·8	37·2	2·5	37·5	3·2	37·3	3·9	37·6
1·9	37·2	2·6	37·4	3·3	37·3	4·0	37·7
2·0	37·3	2·7	37·4	3·4	37·4	4·1	37·7
2·1	37·5	2·8	37·4	3·5	37·4	4·2	37·5

Hence, Algermissen's experimental results give us the ratios of all the values of f, from x equal to 1·5 a to x equal to 4·2 a, with a maximum inaccuracy of less than 1·6 per cent. To fully appreciate this result it is necessary to try and sum the series (15) for any two values of x lying between the given limits.

For sparking distances greater than half a centimetre (one fifth of an inch), therefore, when no coronæ, and consequently no brush discharges, are formed, the error made in assuming that the boundaries of the electrodes form the equipotential surfaces is negligibly small. The disruptive discharge ensues as soon as the maximum value of the electric intensity attains a definite value which is the measure of the dielectric strength of the air between the electrodes under the given atmospheric conditions.

Discussion.

Dr. H. A. Wilson expressed his interest in the Author's explanation of the brush discharge and the formation of coronas. When the distance between the electrodes was not too small, it was known that the sparking P.D could be expressed as $V=\alpha+\beta d$, where d was the distance between the electrodes and α and β were constants. This constant β the Author had called the dielectric strength of air, but he did not think he was justified in doing so.

Prof. J. H. Poynting, referring to Table V. in the paper, asked if the rise in value of the dielectric strength as the distance apart of the electrodes increased was due to the formation of coronas.

Mr. Russell, in reply, remarked that Mr. Strutt had shown that the P.D. between the cathode and the negative glow was 341 volts whatever the atmospheric pressure. We were therefore quite justified in assuming that at ordinary pressures the electric pressure on the Faraday tube subject to the maximum stress is $V-\epsilon$, where ϵ is greater than 341. The experimental results analysed in the paper indicate that ϵ is 0·8 of a kilovolt. In answer to Prof. Poynting, he stated that the slight rise in the values of the dielectric strength in Table V. was probably due to the potentials of the electrodes not being V and zero at the instant of discharge.