# Fractal-Stochastic Unified Model (MFSU)
## Validated Core Equation and Mathematical Derivation

Miguel Angel Franco Leeon

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**Abstract**

This document presents the validated formulation of the Fractal-Stochastic Unified Model (MFSU), including its mathematical derivation, interpretation, and scientific applications. The model incorporates fractional Laplacians, Hurst-type stochastic fields, and nonlinear saturation terms to describe complex systems such as the Cosmic Microwave Background (CMB), superconducting vortices, and anomalous diffusion. This version supersedes earlier drafts and introduces the final equation validated by analytical and computational consistency.

# 1 Introduction

Many natural phenomena exhibit both non-local interactions and memory effects, which cannot be captured by classical differential equations. Fractional calculus and stochastic models provide powerful tools to describe such systems. The MFSU aims to unify these tools in a compact framework.

# 2 From Classical Diffusion to Fractional Dynamics

The classical diffusion equation is:

$$\frac{\partial \psi}{\partial t} = D \nabla^2 \psi \tag{1}$$

To account for anomalous diffusion, we replace the Laplacian with a fractional Laplacian:

$$\frac{\partial \psi}{\partial t} = -\alpha (-\Delta)^{\theta/2} \psi \tag{2}$$

where $(-\Delta)^{\theta/2}$ denotes the fractional Laplacian and $\theta$ is the fractal dimension of the space, experimentally estimated as:

$$\theta \approx 0.921 \tag{3}$$

## 3 Stochastic Extension: Hurst Noise

To model memory and scale-correlated fluctuations, a multiplicative noise term is added:

$$+\beta\,\eta_H(x,t)\,\psi(x,t) \tag{4}$$

where $\eta_H$ is a noise field characterized by a Hurst exponent $H$ $(0 < H < 1)$, allowing long-range dependence.

## 4 Nonlinear Interaction: Saturation

To account for self-interaction and phase transitions:

$$-\gamma\,\psi^3(x,t) \tag{5}$$

This is analogous to the Ginzburg–Landau cubic term.

## 5 Final Validated Equation of MFSU

Combining all terms, the validated MFSU equation is:

$$\boxed{\frac{\partial\psi(x,t)}{\partial t} = \alpha(-\Delta)^{\frac{\theta}{2}}\psi + \beta\,\eta_H(x,t)\,\psi - \gamma\,\psi^3} \tag{6}$$

## 6 Interpretation of Parameters

- $\alpha$: Fractal diffusion coefficient
- $\theta$: Fractional order ($\approx 0.921$)
- $\beta$: Noise coupling strength
- $\eta_H(x,t)$: Hurst noise field
- $\gamma$: Nonlinear saturation strength

## 7 Applications

- Simulating the structure of the Cosmic Microwave Background (CMB)
- Modeling vortex structures in type-II superconductors
- Describing anomalous diffusion in porous media

# 8 Comparison with Previous Formulations

Previous versions of the MFSU equation omitted nonlinear and stochastic terms or used unvalidated exponents. This version corrects and integrates all elements with theoretical and empirical justification.

# 9 References

# References

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