MIXER SKEWNESS CALIBRATION

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LDC

So, you want to calibrate mixer skewness:

Statement of the problem:

Ideally, the mixer multiplies the I input by $\cos(\omega t)$ & the Q input by $\sin(\omega t)$, adding the results to the RF output:

$$V_{rf} \propto I_{in}(t)\cos(\omega t) + Q_{in}(t)\sin(\omega t). \quad \text{[ideal]}$$

However, in practice there are amplitude & phase imperfections, so that the mixer outputs:

$$V_{rf} \propto I_{in}(t)\cos(\omega t) + Q_{in}(t)\,\alpha\,\sin(\omega t + \varphi).$$

The mixer imperfections are characterized by two real valued parameters

$$\alpha \quad \text{and} \quad \varphi$$

"calibrating" a mixer means finding α & φ.

Achieving a single sideband

- In the lab, we iterate w/ I and Q of the form

$$I_{in} = \cos(\omega_{if} t)$$

$$Q_{in} = A\sin(\omega_{if} t + \theta)$$

We vary A & θ (keeping I fixed) to minimize the unwanted sideband. How do these two parameters (A, θ) relate to the mixer skewness parameters (α, φ)?

$$V_{rf} \propto C_{\omega_{if}t}\, C_{\omega t} + A\, S_{\omega_{if}t+\theta}\, \alpha\, S_{\omega t+\varphi}$$

$$= \frac{1}{2}\left(C_{\omega_+} + C_{\omega_-}\right) - \frac{1}{2}A\alpha\left(C_{\omega_+ + \theta + \varphi} - C_{\omega_- + \varphi - \theta}\right)$$

$$= \frac{1}{2}\left(C_{\omega_+} - A\alpha\, C_{\omega_+ + \theta + \varphi}\right) + \frac{1}{2}\left(C_{\omega_-} + A\alpha\, C_{\omega_- + \varphi - \theta}\right)$$

where $\omega_\pm = \omega \pm \omega_{if}$

The conditions for cancelling ω_+ are $A_+ = 1/\alpha$, $\theta_+ = -\varphi \pmod{2\pi}$

The conditions for cancelling ω_- are $A_- = -1/\alpha$, $\theta_- = \varphi \pmod{2\pi}$

- Note that $\boxed{|A_+| = |A_-|}$

We can absorb the sign difference in the amplitudes into the phases:

$$\theta_+ = -\varphi \mod (2\pi)$$
$$\theta_- = \varphi + \pi \mod (2\pi)$$

$$\boxed{\theta_- - \theta_+ = \pi + 2\varphi \quad \mod (2\pi)}$$

So now we have an experimental way to find α & φ:

- Compensating for mixer skewness:

Let's use phasors:

$$V_{rf} = \begin{pmatrix} 1 & a \\ 0 & b \end{pmatrix} \begin{pmatrix} I_{in} \\ Q_{in} \end{pmatrix}, \text{ where } \begin{matrix} a = -\alpha \sin(\varphi) \\ b = \alpha \cos(\varphi) \end{matrix}$$

We'd like to pretransform I & Q to give use phasors that are purely real & purely imaginary: ie, find M s.t.

$$V_{rf} \begin{pmatrix} 1 & a \\ 0 & b \end{pmatrix} M \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & a \\ 0 & b \end{pmatrix} M \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

evidently,
$$M = \begin{pmatrix} 1 & a \\ 0 & b \end{pmatrix}^{-1} = \frac{1}{b} \begin{pmatrix} b & -a \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -a/b \\ 0 & 1/b \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \tan(\varphi) \\ 0 & \frac{1}{\alpha} \sec(\varphi) \end{pmatrix}$$

Thus: $\boxed{M = \begin{pmatrix} 1 & \tan(\varphi) \\ 0 & \frac{1}{\alpha} \sec(\varphi) \end{pmatrix}}$

- Pulse transformations to compensate mixer skewness:

suppose I_q & Q_q are the envelopes you want at the qubit frequency

<u>example</u>: $R_x(t)$: I_q = 

Q_q = 

- consider the following identities:

$$C_{\omega_q} = C_\omega C_{\omega_{if}} - S_\omega S_{\omega_{if}}$$
$$S_{\omega_q} = S_{\omega t} C_{\omega_{if} t} + C_{\omega t} S_{\omega_{if} t}$$

$$\begin{aligned} V_{rf} &= I_q C_{\omega_q t} + Q_q S_{\omega_q t} \\ &= I_q (C_{\omega_t} C_{\omega_{if} t} - S_{\omega t} S_{\omega_{if} t}) + Q_q (C_{\omega t} S_{\omega_{if} t} + S_{\omega t} C_{\omega_{if} t}) \\ &= (I_q C_{\omega_{if} t} + Q_q S_{\omega_{if} t}) C_{\omega t} \\ &\quad + (-I_q S_{\omega_{if} t} + Q_q C_{\omega_{if} t}) S_{\omega t} \end{aligned}$$

- Thus, for an ideal mixer:

$$\begin{pmatrix} I_{in}(t) \\ Q_{in}(t) \end{pmatrix} = \begin{pmatrix} C_{\omega_{if} t} & S_{\omega_{if} t} \\ -S_{\omega_{if} t} & C_{\omega_{if} t} \end{pmatrix} \begin{pmatrix} I_q(t) \\ Q_q(t) \end{pmatrix}$$

- For a real mixer, we must apply the pre-transforming matrix:

$$\boxed{\begin{pmatrix} I_{in}(t) \\ Q_{in}(t) \end{pmatrix} = M \begin{pmatrix} C_{\omega_{if} t} & S_{\omega_{if} t} \\ -S_{\omega_{if} t} & C_{\omega_{if} t} \end{pmatrix} \begin{pmatrix} I_q(t) \\ Q_q(t) \end{pmatrix}}$$

Note that M is applied at the very end!