

ART. XXXV.—*Results of Experiments on the Set of bars of Wood, Iron, and Steel, after a Transverse Stress*; by W. M. A. NORTON, Professor of Civil Engineering in Yale College.

AT intervals, during the last two years, I have carried on a systematic series of experiments, with the view of determining the laws of the set of materials resulting from a transverse stress under varied circumstances. The experiments were made with the testing machine which I devised several years since, for the purpose of experimenting on the deflection of bars under a transverse stress. A detailed description of this

machine is given in the Proceedings of the American Association for the Advancement of Science, Eighteenth Meeting, Aug., 1869, (p. 48). The depressions of the middle of the bar experimented on,—while under a transverse stress, or remaining after the stress has been withdrawn—are measured by it to within $\frac{1}{10000}$ of an inch. The experiments on set have been fully discussed in two papers read before the National Academy of Sciences, Washington, (April, 1874 and April, 1875). The first paper set forth the results of the experiments on bars of wood, and contained a detailed account of the course of experiments instituted for the purpose of detecting instrumental errors, and of the precautions taken to reduce the incidental errors, from variations of temperature and other causes, to a minimum. The second paper discussed the experiments on the set of bars of wrought iron and steel; which gave results generally similar, under corresponding circumstances, to those obtained with wood. I propose, in the present communication, to give a succinct statement of the general conclusions that follow from the whole discussion.

The experimental investigation was prosecuted under three general heads:

I. Sets from momentary strains.

II. Sets from prolonged strains.

III. Duration of set; and variation of set with interval of time elapsed after the withdrawal of the stress.

Each of these embraced several special topics of inquiry. The bars used in most of the experiments consisted of one of white pine, 3 in. by 3 in. and 4 ft. long; another of wrought iron, $\frac{1}{4}$ in. wide, 1 in. deep, and 4 ft. long; and a third of steel of the same dimensions. The discussion of the entire series of experiments has brought out the following results, as alike applicable to bars of wrought iron, steel, and white pine.

1. The immediate set,—that is, the residual deflection which obtains immediately after the transverse stress is withdrawn,—increases in nearly the same proportion as the stress applied; until this exceeds a certain amount, beyond which the set increases according to a more rapid law than that of proportionality to the strains. It is to be understood here that the varying strains are applied at considerable intervals of time.

2. The immediate set augments with the duration of the stress, up to a certain interval of time. In the experiments with white pine, the duration of strain which gave the maximum immediate set, varied, with the strain, from ten minutes to one hour. The immediate set resulting from a prolonged strain, was found to be from five to nine times as great as that which succeeded a momentary strain.

3. The residual depression below the original line of the bar,

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is greater if the stress is reached by a series of increasing weights than if the full stress is directly applied.

4. When the same strain is repeated on the same bar, after a short interval of time, the set first obtained is not augmented, unless the load applied exceeds a certain amount, varying with the material and dimensions of the bar. With loads greater than this limit each repetition of the load augments the total set. The amount of the increase varies with the interval of time since the previous application of the load and the number of previous applications.

5. The set, or residual depression of the middle of the bar, experiences marked variations as the interval of time subsequent to the removal of the stress increases. When the immediate set is less than about 0.0005 in. it passes off in a few minutes (10 m. or less). When it is greater than this it habitually varies as follows: it invariably decreases for a short interval of time, and then ordinarily increases for a longer interval, with moderate fluctuations. The period of decrease varies from about 5 m. to 20 m.; and is the longer in those instances in which the stress is prolonged. The subsequent increased set, or augmented depression of the line of the bar, may attain in less than an hour to an amount even greater than the set observed immediately after the stress is withdrawn. In some of the experiments the depression increased until it came to be about double that first observed. The proportionate increase of set is usually, however, much less than this. This increase of set is eventually succeeded by another decrease. These remarkable fluctuations observed in the line of the bar were more conspicuous in the experiments with white pine, than in those with iron and steel. The difference was, however, only in degree. Under similar conditions the general character of the fluctuations was the same whichever material was used. The fluctuations observed with the bars of iron and steel, as well as with the wooden bar, far exceeded any errors to which the observations were liable. They were also much too slow, and too prolonged, to be regarded as simple vibrations of the bar, consequent on the removal of the downward pressure.

6. Abnormal variations from the general law of variation of the set just noticed, may occur under especial circumstances. Such deviations were observed after the bar had been subjected to repeated strains from day to day. Under these circumstances the bar may be in such an abnormal condition that the set observed immediately after the stress is withdrawn may pass off rapidly, and the line of the bar may even rise considerably above the position held when the stress was applied—though not above its original line some days previously, before any strain was applied.

7. When the load, or stress at the middle of the bar, exceeds a certain amount, the set resulting from one or more applications of the load on any one day is not only still discernible on the following day, but the actual result may be that the middle of the bar may be lower than at the close of the observations on the previous day. Such effects were observed, in the experiments with white pine, when the load was sufficient to produce a longitudinal strain on the upper or lower fibers of 500 lbs. per square inch; and in the experiments with the steel bar, resting edgewise on its supports, when the strain on the outer fibers amounted to 1500 lbs. per square inch.

8. Repeated applications of the same load, from day to day, are attended with an indefinite augmentation of the residual depression of the middle of the bar, if the load exceeds a certain amount. When a smaller load is similarly applied, the set attains after a few days to a maximum, and subsequently subsides more or less. The load answering to the critical point here referred to, is obviously the maximum safe value for a variable load that can be applied, with an indefinite number of repetitions, to the bar. In the case of a white pine stick (3 in. by 3 in., and 4 ft. long) the experiments show it to be less than $\frac{1}{4}$ the theoretical breaking load. Under repeated applications of 500 lbs. (or about $\frac{1}{4}$ the theoretical breaking weight) the set steadily increased from day to day—that is, the middle of the stick became more and more depressed—during the entire period (seven days) that the prolonged effects were noted. Under daily repetitions of a load equivalent to $\frac{1}{6}$ the breaking weight, the depression increased for three days, and after another interval of three days the stick had recovered its original line. The depressions here referred to are those which obtained on the morning of each day just before the first application of the stress on that day.

9. In connection with the phenomena of set which have been signalized, it is important to note that during any interval in which a bar was kept under a transverse stress, the resulting deflection commonly experienced a continual variation. In general the deflection increased as the strain was prolonged. But the deflection of the steel bar in some instances diminished, under the prolonged strain. This unusual result was apparently dependent on some molecular condition of the bar, induced by previous strains. The comportment of the wrought iron bar, as regards varying deflection under a continual strain, was not particularly examined.

It is also noteworthy, in this connection, that the deflection resulting from any single stress was found to be more or less dependent on the previous strains to which the bar had been subjected. The wooden bar, when it had been exposed to a

cross strain not long before, was generally in a condition to suffer a greater deflection than it had before experienced under the same load. The same was true of the steel bar during several successive days of experiments with loads of 4 lbs. and 6 lbs.; but as the result of these repeated strains the bar came eventually to be in a condition in which each renewal of the stress gave, for the most part, a less and less deflection.

10. It is apparent from the foregoing experimental results, that every application of a transverse stress to a bar must induce some change in its molecular condition, which continues, with variations that may be either progressive or fluctuating, for a greater or less interval of time. The duration of sensible influence varies with the amount and duration of the stress. For the smaller strains it is but a few minutes; for the larger several days. The prolonged influence of strains applied from day to day to a bar, was apparent from the fact that the same stress did not on different days produce either the same deflection or the same set. It was strikingly shown in the experiments with the steel bar by causing the bar, to which loads had been repeatedly applied for several previous days, to rest on its opposite side, and comparing the deflection and set with those obtained immediately before the reversal. It was found that the deflection produced by $18\frac{1}{2}$ pounds was $\frac{1}{15}$ greater than the deflection produced by the same weight just before the reversal; and the set obtained was now many times greater than before. The deflection also now increased with a prolongation of the strain, whereas it before decreased. Also the set now increased for a considerable interval of time after the withdrawal of the strain, whereas it before decreased.

11. There was no discernible limit of elasticity, revealed by the experiments, with either wood, iron, or steel. A perceptible set obtained, with each material, immediately after the stress was removed, however small its amount, until the set fell below the lowest possible determination of which the apparatus was capable (viz: $\frac{1}{10000}$ of an inch, as the experiments were ordinarily conducted.) To test the question still farther, the delicacy of the measuring apparatus was largely increased, by the adaptation of a device for magnifying the movements to be observed; and it was found that the least perceptible immediate set was still limited only by the capability of detecting, with the apparatus, minute displacements.

If we take for the limit of elasticity the condition of things at which a *permanent* set is obtained, the case is different. Thus it was found that the set which subsisted after the pine stick (3 in. by 3 in. and 4 ft. long), had been loaded at its middle with 200 pounds ($\frac{1}{15}$ the theoretical breaking weight), eventually passed off entirely. This was the case whether the stress was momentary or prolonged, and whether it was applied but once

or repeatedly. But with a load of 500 lbs. a permanent set was obtained, as the result of a single application of the stress; and repetitions of the stress were attended with a continual increase in the depression of the middle of the bar. It may accordingly be affirmed that a practical limit of elasticity exists, but not a theoretical one.

12. If a bar, on the withdrawal of a transverse stress, fails to recover its original line of position, or, technically speaking, has a set, it is plain that its integrant molecules have not returned precisely to their original positions, and that the distances between contiguous molecules have either increased or diminished—increased in the line of the longitudinal fibers that have experienced a tensile strain, and decreased in the line of those which have experienced a compressive strain. Now we have seen that, as the result of a series of increasing transverse stresses, the set increases continuously with the stress, from the lowest amount capable of detection with the measuring apparatus employed. We must therefore conclude that, after the application of a series of increasing strains, in which the molecules are relatively displaced by minute fractions of their intervening distances, they take up, when the strain is removed, a series of new positions of equilibrium, differing by excessively minute degrees from those previously occupied. We may draw the same conclusion from the experiments on the set produced by a series of direct tensile and compressive stresses, made by Hodgkinson, Chevandier and Wertheim, and other experimenters. This general conclusion, to which experiments on set, under every variety of strain, conduct, leads to the inevitable inference that *the effective forces exerted by the molecules on one another have suffered some change of intensity, in consequence of the stress applied to the bar under experiment.* Viewing the residual displacement of the molecules, in their relative positions, as a mechanical problem, we are constrained to regard the effective molecular forces, that take effect at a given distance, as having acquired a different intensity. We have confirmatory evidence of this induced molecular condition of the bar in the fact that all the diverse effects, which may ensue on subsequent applications of a transverse stress, are found to be either less or greater than those previously observed under similar conditions.

13. The fluctuations that have been noticed as occurring in the set with the lapse of time, reveal the fact that the change in the intensities of the effective molecular forces, which results from the temporary application of the stress, is not permanent but fluctuating; and may, according to the amount of the stress applied, rapidly pass off, or, after a partial collapse, be slowly recovered again. It should be observed, however, that the curious fact of the increase of set which ordinarily succeeds the first sudden fall, may be in part attributable to the gradual

propagation inward of the greater disturbed condition of the molecules of the upper and lower fibers.

14. The general correspondence in the phenomena of set and altered deflection, that obtain with different materials altogether precludes the idea that they may result, either wholly or in a considerable degree, from irregular strains subsisting in certain parts of the bar before the stress is applied, and which are more or less modified by the stress; as some persons have conjectured. The change that supervenes must be a general one, or one in which all the molecules participate, though in diverse degrees according to the amount of molecular displacement. The especial character of the change, for each individual molecule, must depend upon the kind of strain to which the molecule is exposed, whether tensile, compressive, or shearing; and not on the nature of the material subjected to strain.

15. If, as experiment has established, when the distance between two contiguous molecules has been forcibly altered, the molecules, when again left to their mutual actions, no longer exert, at the same distance, effective actions of the same intensity as before, it is apparent that *the molecules in the act of displacement have experienced some change, either in their dimensions, or in their internal mechanical condition.* This change must result from the change that took place in the mutual action of the molecules when they were urged nearer to each other, or separated to greater distances. It must be experienced by the ultimate molecule, whether this be identical with the integrant molecule or not—that is whether we regard the integrant molecule as a single ultimate molecule, or as a group of ultimate molecules. For it is plain that a group of ultimate molecules could not undergo an internal change, that abides after all external actions have ceased, unless its constituent molecules have suffered a change, by reason of which they no longer act upon one another with the same intensities of force as before.

It is well known that with Physicists the “chemical atom” has come to be replaced by the “ultimate molecule.” Of the probable physical constitution of the ultimate molecule different conceptions have been formed. To those Physicists who regard it as made up of a limited number of precisely similar atoms, endued with unvarying forces—of attraction at certain distances, and repulsion at other distances—I leave it to reconcile this conception with the legitimate inference to be drawn from experimental results, that the ultimate molecule is liable to a change of mechanical or physical condition, with every slight displacement it may experience—a change which subsists after the constraining cause of the displacement has ceased to act; and may, under different conditions, either be permanent, or gradually subside with fluctuations.