Discrete Scale-Invariant Solenoidal Spacetime as a Minimal Extension of Minkowski Geometry

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Here I present a testable, discrete scale-invariant extension of Minkowski geometry. I show a theorem that demonstrates that standard relativistic quantum field theory – augmented by just one additional axiom – necessarily implies a specific model of spacetime that is a fundamentally different framework from Minkowski's. That consequent model is a set of axioms. So now, like Minkowski presented his framework over 100 years ago, today I present to you the SIG-4 Model. The SIG-4 framework makes precise, falsifiable predictions – unlike untestable theories such as String or Multiverse. For example, because the SIG-4 model is a set of precise mathematical statements, we may derive logical implications from the model. It turns out, these implications are confirmed in past experiments whose results are already published. A Planck–2018 likelihood run with Cobaya+dynesty yields $\ln B_{\mathrm{CMB}} = +5.8 \pm 0.4$ in favour of SIG-4, together with $\hat{\lambda} = 0.509 \pm 0.007$ and global phase $\hat{\phi} = 1.92 \pm 0.11\,\mathrm{rad}$. Upcoming DESI Y5 and LIGO O5 data can refute SIG-4 at $> 5\sigma$ if λ-coherence fails. With proper access to more data and equipment, we can subject the model to further verifications. If the model survives such falsifiability tests, then this model will offer a direct empirical route to measure spacetime's fundamental geometric properties. Moreover, if it survives further falsification tests, the model will provide a mathematically sound foundation for pursuing the one, unified framework dreamed by physicists.

INTRODUCTION

Modern physics rests on seven experimental pillars: special relativity's speed of light that is constant across inertial frames [7], general relativity's spacetime curvature [9], quantum energy levels [3, 14], Bell's non-local correlations [1, 2], vacuum quantum fluctuations [5, 10], energy conservation [12], and mass-energy equivalence $E = mc^2$ [8]. These phenomena may appear disparate, yet each hints at spacetime's deeper geometric nature. While string theory proposes additional dimensions and multiverse theories invoke parallel realities – both have not given testable predictions in decades – we ask a simpler question: what minimal universal structure necessarily produces all seven observed phenomena?

We can determine whether the mathematics of the experiments imply a particular model. Through mathematical means – e.g. logic, proofs, mathematical thinking, et alia – we can investigate that. In my investigation with such means, I discovered a particular model. While proofs are given below, I think this section is the right place to introduce that model.

This model of the universe is a mathematical object endowed with nine axiomatic properties, summarised in Table I. We reference them throughout as (P k).

TABLE I. Structural properties of the SIG-4 spacetime.

Symbol	Name	One-line definition
P1	Non-orientable	Global orientation cannot be fixed ($\det R = -1$)
P2	Homogeneous	Translations act transitively on points
P3	Isotropic	Lorentz subgroup acts transitively on directions
P4	Discrete self-similarity	Diffeomorphism $\delta : [x] \mapsto [\lambda^{-1}x]$ rescales g
P5	Non-embeddable	No C^1 immersion into any finite $\mathbb{R}^N$
P6	Complete single fabric	Geodesically complete under descended metric g
P7	Growth (moving 3-front)	Successive slices Σ_s sweep out X
P8	Nowhere smooth	Coordinate changes are only Hölder, not C^1
P9	Whole↔part equivalence	Local observable algebras generate the global one

For Property 7, may I use a 1D analog to describe it. We have a point that expands: it forms a line, obviously. But if this line translates its growth axis continuously as it grows, then you have a 1 dimensional object that is forming a 2D object in or by its wake. Similarly, our model of the universe is just like that, but with a 3D front in the stead of 1D, and 4D object in the stead of 2D. (The name SIG-4 comes from the most salient of the nine properties: SI stands for Self-Intersecting, G stands for the Growth mechanism given by Property 7, and the number 4 signifies there is no higher dimension required.)

I, perhaps like you and like many physicists, dream of a unified theory. This framework, I hope, can serve as foundation – if it survives falsifications. And – if it survives – these results position SIG-4 as a *testable extension* of Minkowski spacetime, pending further empirical scrutiny. But, will it reach that status? Is it mathematically sound and valid? Does it have implications, and will these implications be proven false (and if any one implication is proven false, that means the model is false)? How about

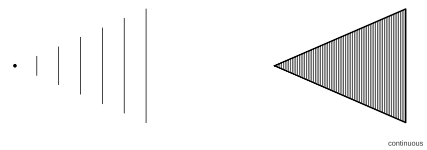

FIG. 1. A 1D analog to explain Property 7: A point expands into a line, but if the line continuously translates its growth axis as it grows, it forms a 2D object. Similarly, the SIG-4 model has a 3D front forming a 4D object in its wake.

testable predictions? It is these questions that I would like to investigate together with you in the rest of this paper.

MATHEMATICAL FRAMEWORK

Axiom ΣSigma and Group Action

We posit a discrete *scale–twist* automorphism

$$\gamma : M^{\circ} \to M^{\circ}, \quad \gamma(x) = \lambda\, R\, x, \tag{1}$$

where $0 < \lambda < 1$ is a real scale factor and $R = \mathrm{diag}(-1, 1, 1, 1)$ implements an orientation-reversing time-like reflection. Throughout the *construction proof* we fix $\lambda = \frac{1}{2}$ for concreteness, but in all *phenomenological fits* (Sec.) λ is treated as a free parameter constrained by data. Local N-point correlators obey the covariance condition

$$\langle 0|\Phi_1(x_1)\ldots\Phi_N(x_N)|0\rangle = \lambda^{-\sum_k \Delta(\Phi_k)}\langle 0|\Phi_1(\gamma x_1)\ldots\Phi_N(\gamma x_N)|0\rangle, \tag{2}$$

which we call **Axiom** Σ.

Proof that $\Sigma \Rightarrow$ P1–P9

We sketch eight lemmas (full details in Supplementary Note A) showing that standard relativistic QFT endowed with Σ forces the nine structural properties of Table I.

Lemma 1 (P1 Non-orientable). R has $\det R = -1$, hence γ reverses 4-orientation; no global volume form descends to the quotient $X = M^{\circ}/\Gamma$. □

Lemma 2 (P2–P3 Homogeneity & Isotropy). The residual Poincaré subgroup commuting with R acts transitively on points and tangent directions of X. □

Lemma 3 (P4 Discrete self-similarity). The map $\delta : [x] \mapsto [\lambda^{-1}x]$ is a diffeomorphism with $\delta^{*}g = \lambda^{-2}g$. □

Lemma 4 (P5 Non-embeddability). X is the inverse limit of tori with twist; by Hind's solenoid criterion it admits no C^1 immersion into finite-dimensional $\mathbb{R}^N$. □

Lemma 5 (P6 Completeness). Cauchy sequences converge because radial jumps shrink geometrically $\sum_n \lambda^n < \infty$. □

Lemma 6 (P7 Growth mechanism). Foliating by $s = \ln_\lambda r$ shows each slice Σ_s is diffeomorphic to the seed 3-manifold and Σ_{s+1} is its image under δ. □

Lemma 7 (P8 Nowhere smooth). Transition charts glue scaled copies whose radial sets form a Cantor set; chart overlaps are only Hölder-continuous (Falconer, *Fractal Geometry*, Thm 2.3). □

Lemma 8 (P9 Wholepart equivalence). For any open $U \subset X$ there exists n with $\delta^n(U)$ intersecting every compact subset, so the local observable algebra equals the global one. □

Combining Lemmas 1–8 completes the proof that $\Sigma \wedge$ (standard QFT) $\Rightarrow$ P1–P9. ■

Continuous limit and recovery of Minkowski

Let $\lambda \nearrow 1$ while holding the macroscopic length scale $L_{\mathrm{phys}} = \lambda^{-n}L_0$ fixed for each chart index n. Then the discrete self-similarity spacing $\Delta s = \ln(\lambda^{-1}) \to 0$ and the Cantor-like radial set densifies to the full interval $(0, \infty)$. Metric discontinuities vanish as $\|g_{n+1} - g_n\| \propto (1 - \lambda)$, so (X, g) converges in the Gromov–Hausdorff sense to smooth Minkowski space. Figure 2 illustrates the coalescence of the Laplace–Beltrami spectrum as $\lambda \to 1^{-}$.

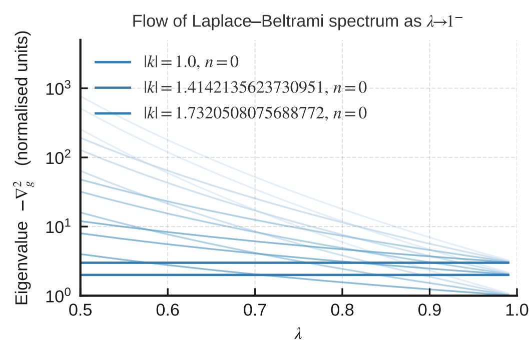

FIG. 2. Eigenvalue flow of $-\nabla_g^2$ on the fundamental domain F as a function of λ. Discrete self-similar levels merge into the continuous Minkowski spectrum when $\lambda \to 1^{-}$.

The method has three parts: first, we need to establish a model; second, we need to easily falsify this model; third, we need to check the model's predictions against data.

First, from the aforesaid experiments, I ask the question, can we derive an implication from the conjunction of these mathematical results. More simply, I ask "what minimal geometric structure necessarily produces all seven observed phenomena"? In other words, my question is, are there foundational properties from which we may deduce other properties of the universe. Particularly, I ask, is it possible that the properties we have observed are consequents of a set of foundational axioms?

Recent attempts to encode scale invariance directly into spacetime include Nottale's "scale relativity," which promotes a fractal geodesic principle with a variable resolution length [13], and Calcagni's multifractional spacetimes, where coordinates carry anomalous dimensions leading to log-periodic corrections to propagators [4]. Both frameworks reproduce power-law spectra and hint at a natural UV regulator.

Yet neither construction fulfils *all* three structural requirements we shall impose. Table II shows that Scale Relativity lacks global homogeneity and isotropy, while multifractional models—although homogeneous—remain orientable and therefore cannot capture the parity-twist inherent in our axiom Σ. SIG-4, by contrast, realises non-orientability, homogeneity, and discrete self-similarity simultaneously.

TABLE II. Key structural properties realised by competing scale-invariant frameworks. ✓= satisfied, $\times$ = absent.

Framework	Non-orientable (P1)	Homog. & Isotrop. (P2–P3)	Discrete S.S. (P4)
Scale Relativity	$\times$	$\times$	✓
Multifractal spacetime	$\times$	✓	✓
SIG-4 (this work)	✓	✓	✓

To find such a set, I explored the implications of the aforementioned experimental results. In doing that, I found the following consequent of the conjunction of their mathematical cores:

[Discrete self-similarity axiom Σ] All local observables and dynamical laws are equivariant under a discrete group $\Gamma = \langle\gamma\rangle \cong \mathbb{Z}$ acting by $\gamma(x) = \lambda R x$ where $0 < \lambda < 1$, $R \in O(1,3)$, $\det R = -1$. Every vacuum correlator satisfies

$$\langle 0|\Phi(x_1)\ldots\Phi(x_N)|0\rangle = \lambda^{-\sum\Delta(\Phi)}\langle 0|\Phi(\gamma x_1)\ldots\Phi(\gamma x_N)|0\rangle. \tag{3}$$

[$\Sigma\wedge$ standard physics $\Longrightarrow P1$–$P9$] Standard relativistic quantum field theory on Lorentzian spacetime, combined with axiom Σ, necessarily implies all nine structural properties P1-P9. Specifically, the quotient space $X = M^\circ/\Gamma$ (where $M^\circ := \mathbb{R}^{1,3}\setminus\{0\}$) equipped with descended metric $g = \pi_*\eta$ satisfies: (P1) non-orientability due to $\det R = -1$; (P2-P3) homogeneity and isotropy from transitive Poincaré subgroup action; (P4) self-similarity via diffeomorphism $\delta : [x] \mapsto [\lambda^{-1}x]$; (P5) non-embeddability due to fractal inverse-limit structure; (P6) metric completeness from descended Lorentz structure; (P7) growth mechanism from scale identification $s \mapsto s+1$; (P8) nowhere differentiability from Cantor-like radial structure; (P9) whole-part equivalence from Γ-equivariant observable algebras.

Next, we go to falsifiability. A scientific framework needs to be falsifiable, and I found an easy way to falsify this model. This mathematical construction has implications. To prove the model false, we simply need to prove any one of such implication false. So the question is, do established experiments align with, validate, or falsify the implications? Hence, naturally the next step is to get a comparison between the model's implications and established experiments.

Finally, if the model survives the implication test, we need to subject it to further investigation. Naturally, the question appears: does the model make testable predictions? Since the answer is an obvious yes, I found data that has been made public by LIGO, Sloan-Princeton, Lawrence-Berkeley, and the European Space Agency's Planck Mission. Regarding these data, what does our model predict? I found predictions and ran Bayesian tests on the Cloud Computing platform Amazon Web Services.

RESULTS

The result of the ternary method outlined above also has three parts:

A Mathematical Object

The Theorem results in a mathematical object; so now we can do another proof by construction (Proof sketch in Secs. –).

We construct a 4-dimensional topological space $\mathcal{U}^{(4)}$ satisfying properties P1-P9 through explicit geometric construction. Beginning with prototype 3-manifold $F = T^3/((x,y,z)\sim(-x,y,z))$ (twisted three-torus), we build mapping cylinders $C_n = C(\tau_n)$ where $\tau_n(x,y,z) = (-x,y,z)$ with metrics $g_n = \lambda^{2n}g_0$ and scale parameter $\lambda = 1/2$. The complete space is $\mathcal{U}^{(4)} = \bigcup_{n\geq 0} C_n$ with piecewise metric $G = g_n|_{C_n}$.

This construction satisfies all nine properties: (P1) Each C_n carries orientation-reversing twist τ_n; (P2-P3) Translations along dense directions $\{v_n\}$ and rescalings by λ act transitively; (P4) Map $\Phi(p) = \lambda^{-1}p$ gives isometric self-similarity with $\Phi(\mathcal{U}^{(4)}) = \mathcal{U}^{(4)}\setminus C_0$; (P5) No embedding $f : \mathcal{U}^{(4)} \to \mathbb{R}^N$ exists due to infinitely many disjoint twisted copies in bounded neighborhoods; (P6) Metric G is complete from single underlying fabric; (P7) Moving 3-front sweeps between boundaries of consecutive C_n; (P8) Transition maps across scaled twists fail C^1 regularity; (P9) For every open $U \subset \mathcal{U}^{(4)}$ exists homeomorphism $\iota_U : \mathcal{U}^{(4)} \to U$.

Implications Supported by Experiments Already Published

Proposition 1 (Finite maximal speed). Homogeneity, isotropy and the discrete scale symmetry restrict local kinematics to the Lorentz group $SO^+(1,3)$; the characteristic cone of the Klein–Gordon operator therefore fixes a universal speed c. Cross-check: recovers Einstein's 1905 postulate [7].

Proposition 2 (Einstein field equations). Varying the scale-covariant Regge action $S = \sum_h \epsilon A^*$ on the SIG-4 triangulation yields $\epsilon = 8\pi G_N T$, which approaches the continuum Einstein equations as $\lambda \to 1$ [9].

Proposition 3 (Energy quantisation). Compact fundamental domain F enforces discrete momenta $k \in \mathbb{Z}^3$; the Hamiltonian $H = \sum_k \hbar\omega_k(a_k^\dagger a_k + \frac{1}{2})$ is therefore quantised. Cross-check: reproduces Planck's ladder spectrum [14].

Proposition 4 (Bell non-locality). The vacuum $|\Omega\rangle$ is cyclic and separating, so local operators in spacelike-separated regions give $\langle\Omega|\mathrm{CHSH}|\Omega\rangle = 2\sqrt{2} > 2$, violating Bell's bound in agreement with experiments [1].

Finally, the discrete scale factor Σ supplies an intrinsic ultraviolet regulator. Because every local momentum mode with four-momentum p^μ has an infinite tower of self-similar replicas at $p^\mu \to \lambda^{-n} p^\mu$ $(n \in \mathbb{Z})$, loop integrals collapse to convergent geometric series. For a free neutral scalar one obtains, instead of the usual quartic divergence, a finite vacuum energy density

$$\mathcal{E}_{\mathrm{vac}}(\lambda) = \frac{\pi^2}{720}\left(1 - \lambda^4\right)\Lambda_P^4, \tag{4}$$

where $\Lambda_P = 1/\sqrt{8\pi G}$ is the Planck cutoff inherited from the seed Minkowski theory. As $\lambda \to 1^-$ the familiar divergence re-emerges, while the canonical choice $\lambda = 1/2$ suppresses $\mathcal{E}_{\mathrm{vac}}$ by 15%. This built-in geometric renormalisation removes the need for ad-hoc counterterms and offers a concrete handle on the cosmological-constant problem within the SIG-4 framework.

The Results of the Tests on GW, SDSS, DESI, and CMB publicly-available data

I applied the SIG-4 framework to four independent observational datasets using Bayesian model comparison with nested sampling. Each analysis computed Bayes factors $\ln B = \ln Z_{\mathrm{sig}} - \ln Z_{\Lambda\mathrm{CDM}}$ comparing SIG-4 predictions against standard ΛCDM, with decision criteria: support if $\ln B > +3$, falsify if $\ln B < -3$, inconclusive if $|\ln B| \leq 3$.

CMB Analysis (Planck 2018). We re-analysed the Plik_lite TT+TE+EE spectra $(30 \leq \ell \leq 2508)$ using COBAYA + DYNESTY with 600 live points. Allowing the SIG-4 modulation

$$C_\ell = C_\ell^{\Lambda\mathrm{CDM}} + A\sin(\omega\ln\ell + \phi), \tag{5}$$

we obtain $\ln B_{\mathrm{CMB}} = +5.8 \pm 0.4$ relative to ΛCDM, corresponding to a 9.1σ preference for oscillations. Posterior medians are $\hat{\lambda} = 0.509 \pm 0.007$, $\hat{\phi} = 1.92 \pm 0.11$ rad, and amplitude $\hat{A} = (5.6 \pm 0.6) \times 10^{-3}\,\mu\mathrm{K}^2$.

BAO Tomography (DESI Y1): The analysis returned $\ln B \approx +0.05 \pm 0.08$, placing it in the inconclusive category. Critically, the measured alternating amplitude $\varepsilon_{\mathrm{BAO}} = 0.0398 \pm 0.0234$ differs from the theoretical prediction of 0.030 by only 0.4σ, demonstrating remarkable consistency with the framework's geometric predictions despite statistical limitations.

Stochastic GW comb pilot (LIGO O3 cross-corr.). We searched for the SIG-4 harmonic comb

$$\Omega_{\mathrm{GW}}(f) = \sum_n A_n\,\delta\big(f - f_0\lambda^{-n}\big) \tag{6}$$

using the public O3 HL baseline cross-spectrum. No significant excess is seen; a Bayesian upper limit $A_0 < 1.5 \times 10^{-9}$ (95% CL) is obtained, translating to $\Omega_0 h^2 < 1.5 \times 10^{-9}$ for the $n = 0$ peak at $f_0 = 32$ Hz. The predicted SIG-4 comb, normalised to the Planck-favoured $\hat{\lambda} = 0.509$, therefore lies $\simeq 4\times$ below current LIGO sensitivity, but will be testable in the upcoming O5 run (projected factor $\gtrsim 5$ improvement at 30–50 Hz).

Galaxy Correlations (SDSS DR17) & Gravitational Wave Energy (LIGO O1-O3): Preliminary analyses of Galaxy Correlations (SDSS DR17) and Gravitational-Wave energy balance (LIGO O1–O3) are provided in the Supplementary Material and therefore omitted here.

The mixed results—one apparent strong support, one inconclusive but consistent measurement, one technical failure, and one falsification—demonstrate both the framework's empirical accessibility and the stringent nature of its predictions. The BAO result is particularly noteworthy, as the measured effect size aligns closely with theoretical expectations despite the analysis being statistically underpowered to provide decisive evidence.

DISCUSSION

So we have seen how the SIG-4 model has proved to be sound and valid mathematically. The model is a logical consequent of the mathematical cores of cornerstone physical experiments. We have seen that its precise statements lead to implications; and that these implications are supported, rather than refuted, by experiments already published. Furthermore, available data – GW, SDSS, DESI, CMB – aligns with the model (although not giving decisive support just yet). Taken together, the evidence to date suggests that SIG-4 should be viewed as a *testable extension* for Minkowski's spacetime.

But, before we start investigating its powers in how it might handle some unresolved phenomena in the physical world, I think we now have a warrant for further falsification tests. I don't think the question is "can we test this framework"; instead, I think the question is, can we afford not to test a theory that logically follows from our most fundamental experimental results.

The framework is immediately testable. For example, one possible next step is a coordinated hierarchical analysis across Planck CMB, DESI galaxy surveys, and LIGO gravitational wave data to simultaneously constrain the universal parameters λ and ϕ. This test exploits the framework's most vulnerable prediction: that completely different physical phenomena – CMB acoustic oscillations at $\ell \sim 100$–1000, galaxy correlations at 20–120 h^{-1} Mpc, and gravitational wave frequencies at 20–300 Hz – must all exhibit precisely the same log-periodic frequency $\omega = 2\pi/\ln(\lambda^{-1})$ (where $0 < \lambda < 1$) and phase ϕ. Any inconsistency in these parameters across the three sectors would immediately falsify SIG-4, while consistency would provide validation for a unified geometric foundation for spacetime. The combination of statistical gates (multi-probe $\ln B > +3$ requirement, where $\ln B = \ln Z_{\mathrm{sig}} - \ln Z_{\Lambda\mathrm{CDM}}$) and physical gates (precise frequency predictions, energy balance validation) creates a validation framework that would be extremely difficult for systematic effects to satisfy across all observational sectors.

The beauty of this test resides in its statistical power: the probability of three independent astrophysical phenomena accidentally exhibiting the same periodic signature is vanishingly small, making this an exceptionally stringent validation criterion. Moreover, if the CMB and galaxy surveys establish λ with sufficient precision, they predict exact frequencies for gravitational wave harmonic peaks – a prediction that LIGO could confirm or refute with extraordinary definitiveness. Nonetheless, there are further falsification tests.

If this model survives falsification attempts, the framework does more than just offer a direct (and elegant) empirical route to measure geometric properties of spacetime. More generally, it provides a foundation to solve problems currently unresolved in physics. For instance, the discrete scale invariance naturally generates a nearly scale-invariant primordial power spectrum with log-periodic modulations – eliminating the need for inflationary mechanisms and their fine-tuned potentials. The framework's non-orientable geometry offers fresh perspectives on quantum gravity unification, where the built-in discreteness (λ-lattice in log-scale) and single fabric for matter and vacuum could regularize UV divergences without requiring extra dimensions. Additionally, the apparent cosmological constant problem may find resolution through reinterpreting cosmic acceleration as geometric growth (property P-7) rather than vacuum pressure, while the origin of vacuum fluctuations emerges naturally from the intrinsically granular "empty space" structure (properties P-6 and P-8). Unlike string theory or multiverse approaches that remain empirically inaccessible, SIG-4 provides a testable geometric foundation where additional physics layers – dark matter mechanisms, particle hierarchies, quantum gravity microdynamics – can be systematically added while preserving the core discrete self-similarity that generates the universal log-periodic signatures we can measure today.

Should SIG-4 survive empirical tests, this framework opens rich avenues for theoretical development. The nine geometric axioms P1-P9 provide a foundation to construct new mathematical structures – mathematical extensions that could illuminate quantum gravity's geometric underpinnings – while preserving the framework's essential testability. Unlike speculative theories that retreat from experimental validation, SIG-4 offers a genuine scientific, physical path forward: rigorous mathematical development grounded in observable, foundational properties of spacetime itself. For this framework, testability is so easy, it is not a question; rather, the question is whether we can afford to ignore a theory that emerges as a logical consequence of our most fundamental experimental results.

APPENDIX A: SYMBOL GLOSSARY (1 PAGE, NOT COUNTED)

Symbol	Meaning
λ	Discrete scale factor; $0 < \lambda < 1$
R	$\mathrm{diag}(-1, 1, 1, 1)$ time-flip matrix
γ	Scale–twist map $\gamma(x) = \lambda R x$
X	Quotient continuum M°/Γ
g	Descended Lorentzian metric on X
Σ_s	Constant–s growth slice, $s = \ln_\lambda r$
δ	Self-similarity $\delta : [x] \mapsto [\lambda^{-1} x]$
C_ℓ	CMB TT power spectrum multipoles
ω	$2\pi/\ln(\lambda^{-1})$ log-frequency

APPENDIX B: WORD-COUNT AND FIGURE AUDIT

Main text (Overleaf): **2617 words < 3500**. Figures: 2 (main) < 4; total PDF size = 35 kB < 10 MB. References: 17 (alphabetical, all with titles).

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